Deteriorating Items Inventory Model with Different Deterioration Rates and Shortages

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Abstract: **An inventory model with linear trend in demand and time varying holding cost is developed. Different deterioration rates are considered in a cycle. Shortages are allowed and completely backlogged. Numerical example is provided to illustrate the model and sensitivity analysis is also carried out for parameters.**

Key Words: Inventory model, Varying Deterioration, Linear demand, Time varying holding cost, Shortages

I. INTRODUCTION

Deterioration of items is a general phenomenon in real life and effect of deterioration cannot be ignored in many inventory systems. EOQ models for deteriorating items are studied by many researchers. Ghare and Schrader [3] considered no-shortage inventory model under constant deterioration. Shah and Jaiswal [11] considered an order level inventory model for items deteriorating at a constant rate. Aggarwal [1] discussed an order level inventory model with constant rate of deterioration. Dave and Patel [2] developed the deteriorating items inventory model with linear trend in demand. They considered demand as linear function of time. Hill [5] considered inventory model with ramp type demand rate. Mandal and Pal [7] developed inventory model with ramp type demand with shortages. Other research work related to deteriorating items can be found in, for instance (Raafat [9], Goyal and Giri [4], Ruxian et al. [10]). Hung [6] considered inventory model with arbitrary demand and arbitrary deterioration rate. Mathew [8] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time.

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with different deterioration rates for the cycle time. Demand is considered as linear functions of time. Shortages are allowed and completely backlogged. To illustrate the model, numerical example is taken and sensitivity analysis for major parameters on the optimal solutions is also carried out.

II. ASSUMPTIONS AND NOTATIONS NOTATIONS

The following notations are used for the development of the model:

- D(t) : Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)
- A : Replenishment cost per order
- c : Purchasing cost per unit
- p : Selling price per unit
- T : Length of inventory cycle
- I(t) : Inventory level at any instant of time t, $0 \le t \le T$
- Q_1 : Order quantity initially
- Q_2 : Quantity of shortages
- Q : Order quantity
- HC : Holding cost per unit time is a linear function of time t ($x+yt$, $x>0$, $0<y<1$)
- c_2 : Shortage cost per unit
- θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta_1<1$
- θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta_2<1$
- π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- Deteriorated units neither be repaired nor replaced during the cycle time.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

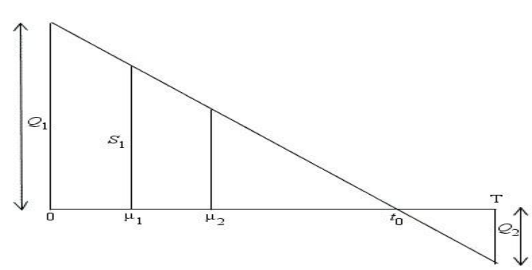


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) are given by

$$\frac{dI(t)}{dt} = -(a+bt), \qquad 0 \le t \le \mu_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt), \qquad \mu_1 \le t \le \mu_2 \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a+bt), \qquad \mu_2 \le t \le t_0 \qquad (3)$$

$$\frac{dI(t)}{dt} = -(a+bt), \qquad t_0 \le t \le T \qquad (4)$$

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$ and $I(T)=-Q_2$.

Solutions of these equations are given by:

$$I(t) = Q_1 - (at + \frac{1}{2}bt^2), \qquad (5)$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - t)\right] \qquad (6)$$

$$I(t) = \begin{bmatrix} a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta(t_0^3 - t^3) \\ +\frac{1}{8}b\theta(t_0^4 - t^4) - \frac{1}{2}a\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2(t_0^2 - t^2) \end{bmatrix}. \qquad (7)$$

$$I(t) = \left[-aT - \frac{1}{2}bT_2 + at_0 + \frac{1}{2}bt_0^2\right]. \qquad (8)$$

(by neglecting higher powers of θ)

From equation (5), putting $t = \mu_1$, we have

$$Q_1 = S_1 + \left(a\mu_1 + \frac{1}{2}b\mu_1^2\right). \qquad (9)$$

From equations (6) and (7), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \qquad (10)$$

$$I(\mu_2) = \begin{bmatrix} a(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) \\ +\frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \end{bmatrix}. \qquad (11)$$

So from equations (10) and (11), we get

$$S_1 = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) \\ +\frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \\ - a(\mu_1 - \mu_2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix}. \qquad (12)$$

Putting value of S_1 from equation (12) into equation (9), we have

$$Q_1 = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) \\ +\frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \\ - a(\mu_1 - \mu_2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + \left(a\mu_1 + \frac{1}{2}b\mu_1^2\right). \qquad (13)$$

Putting t = T in equation (8), we have

$$Q_2 = \left[aT - \frac{1}{2}bT^2 - at_0 - \frac{1}{2}bt_0^2\right]. \qquad (14)$$

Using (13) in (5), we have

$$I(t) = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) \\ +\frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \\ - a(\mu_1 - \mu_2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + a(\mu_1 - t) \ \frac{1}{2}b(\mu_1^2 - t^2). \qquad (15)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (16)

(ii) Holding cost (HC) is given by

$$HC = \int_0^{t_0} (x+yt)I(t)dt$$

$$= \int_{0}^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{t_0}(x+yt)I(t)dt$$

$$= \frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)t_0^5 + x\left(at_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}a\theta t_0^3 + \frac{1}{8}b\theta t_0^4\right)t_0^4$$

$$+\frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta t_0 - \frac{1}{4}b\theta t_0^2\right) - ya\right)t_0^3$$

$$+\frac{1}{2}\left(-xa + y\left(at_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}a\theta t_0^3 + \frac{1}{8}b\theta t_0^4\right)\right)t_0^2$$

$$+\frac{1}{30}yb\theta\mu_2^5 + \frac{1}{48}yb\theta T^6 - \frac{1}{30}yb\theta\mu_1^5$$

$$-\frac{1}{4}\left(\frac{1}{3}xa\theta + y\left(-\frac{1}{2}b - \frac{1}{2}a\theta t_0 - \frac{1}{4}b\theta t_0^2\right)\right)\mu_2^4$$

$$-\frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta t_0 - \frac{1}{4}b\theta t_0^2\right) - ya\right)\mu_2^3$$

$$-\frac{1}{2}\left(-xa + y\left(at_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}a\theta t_0^3 + \frac{1}{8}b\theta t_0^4\right)\right)\mu_2^2$$

$$-\frac{1}{48}yb\theta\mu_2^6 - x\left(at_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}a\theta t_0^3 + \frac{1}{8}b\theta t_0^4\right)\mu_2$$

$$-\frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)\mu_2^5$$

$$+x\left(\begin{array}{l} a\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}a\theta\mu_1^2 + \frac{1}{3}b\theta\mu_1^3 \\ + \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]} \\ \left(\left(\begin{array}{l} a(t_0-\mu_2) + \frac{1}{2}b(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) - \frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3-\mu_2^3) + a\theta\mu_2(\mu_1-\mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right)(1+\theta\mu_1)\right) \end{array}\right)\mu_2$$

$$+x\left(\begin{array}{l} \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]} \\ \left(\left(\begin{array}{l} a(t_0-\mu_2) + \frac{1}{2}b(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) - \frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3-\mu_2^3) + a\theta\mu_2(\mu_1-\mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right) \\ +a\mu_1 + \frac{1}{2}b\mu_1^2 \right) \end{array}\right)\mu_1$$

$$-\frac{1}{4}\left(\frac{1}{6}xb\theta + y\left(-\frac{1}{2}b + \frac{1}{2}a\theta\right)\right)\mu_1^4$$

$$-\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}b + \frac{1}{2}a\theta\right) \\ +y\left(\begin{array}{l} -a - \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]} \\ \left(\left(\begin{array}{l} a(t_0-\mu_2) + \frac{1}{2}b(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) - \frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3-\mu_2^3) + a\theta\mu_2(\mu_1-\mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right)\theta\right) \\ -\frac{1}{2}b\theta\mu_1^2 - a\theta\mu_1 \end{array}\right) \end{array}\right)\mu_1^3$$

$$-\frac{1}{2}\left(\begin{array}{l} x\left(\begin{array}{l} -a - \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]} \\ \left(\left(\begin{array}{l} a(t_0-\mu_2) + \frac{1}{2}b(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) - \frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3-\mu_2^3) + a\theta\mu_2(\mu_1-\mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right)\theta\right) \\ -\frac{1}{2}b\theta\mu_1^2 - a\theta\mu_1 \end{array}\right) \\ +y\left(\begin{array}{l} a\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}a\theta\mu_1^2 + \frac{1}{3}b\theta\mu_1^3 \\ + \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]} \\ \left(\left(\begin{array}{l} a(t_0-\mu_2) + \frac{1}{2}b(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) - \frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3-\mu_2^3) + a\theta\mu_2(\mu_1-\mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right) \\ (1+\theta\mu_1)\right) \end{array}\right) \end{array}\right)\mu_1^2$$

$$+\frac{1}{2}\left(\begin{array}{l}-xa+y\left(\begin{array}{l}\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\ \left(\left(\begin{array}{l}a\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ +a\mu_1+\frac{1}{2}b\mu_1^2\right)\end{array}\right)\end{array}\right)\mu_1^2$$

$$-\frac{1}{8}yb\mu_1^4-x\left(\begin{array}{l}a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\mu_1^3\\ +\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\ \left(\left(\begin{array}{l}a\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ \left(1+\theta\mu_1\right)\right)\end{array}\right)\mu_1 \quad (17)$$

(by neglecting higher powers of θ)

(iii) Deterioration cost (DC) is given by

$$DC=c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$$

$$=c\theta\left(\begin{array}{l}a\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}b\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{2}a\theta\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)\\ +\frac{1}{3}b\theta\left(\mu_1^3\mu_2-\frac{1}{4}\mu_2^4\right)-a\theta\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}b\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)\\ +\dfrac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(\left(\begin{array}{l}a\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)\\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)\\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ \left(\mu_2+\theta\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)\right)\right)\end{array}\right)$$

$$-c\theta\left(\begin{array}{l}\frac{1}{2}\mu_1^2+\frac{1}{3}b\mu_1^3+\frac{1}{6}a\theta\mu_1^3+\frac{1}{8}b\theta\mu_1^4\\ +\dfrac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(\left(\begin{array}{l}a\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)\\ -\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)\\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)\\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ \left(\mu_1+\frac{1}{2}\theta\mu_1^2\right)\right)\end{array}\right)$$

$$+c\theta\left(\begin{array}{l}\frac{1}{48}b\theta t_0^6+\frac{1}{15}a\theta t_0^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta t_0-\frac{1}{4}b\theta t_0^2\right)t_0^4\\ -\frac{1}{3}at_0^3+\frac{1}{2}\left(at_0+\frac{1}{2}bt_0^2+\frac{1}{6}a\theta t_0^3+\frac{1}{8}b\theta t_0^4\right)t_0^2\end{array}\right)$$

$$-c\theta\left(\begin{array}{l}\frac{1}{48}b\theta\mu_2^6+\frac{1}{15}a\theta\mu_2^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta t_0-\frac{1}{4}b\theta t_0^2\right)\mu_2^4\\ -\frac{1}{3}a\mu_2^3+\frac{1}{2}\left(at_0+\frac{1}{2}bt_0^2+\frac{1}{6}a\theta t_0^3+\frac{1}{8}b\theta t_0^4\right)\mu_2^2\end{array}\right) \quad (18)$$

(iv) Shortage cost (SC) is given by

$$SC=-c_2\left(\int_{t_0}^{T}I(t)dt\right)=-c_2\left(\int_{t_0}^{T}(-at-\frac{1}{2}bt^2+at_0+\frac{1}{2}bt_0^2)dt\right)$$

$$=-c_2\left(\begin{array}{l}-\frac{1}{2}a\left(T^2-t_0^2\right)-\frac{1}{6}b\left(T^3-t_0^3\right)\\ +at_0\left(T-t_0\right)+\frac{1}{2}bt_0^2\left(T-t_0\right)\end{array}\right) \quad (19)$$

$$\text{(v) } SR=p\left(\int_0^T(a+bt)dt\right)=p\left(aT+\frac{1}{2}bT^2\right) \quad (20)$$

The total profit during a cycle, $\pi(t_0,T)$ consisted of the following:

$$\pi(t_0,T)=\frac{1}{T}\left[SR-OC-HC-DC-SC\right] \quad (21)$$

Substituting values from equations (16) to (20) in equation (21), we get total profit per unit.

The optimal value of $t_0 = t_0^*$ and $T = T^*$ (say), which maximizes profit $\pi(t_0,T)$ can be obtained by differentiating it with respect to t_0 and T and equate it to zero i.e. $\dfrac{\partial \pi(t_0, T)}{\partial t_0} = 0, \ \dfrac{\partial \pi(t_0, T)}{\partial T} = 0$ (22)

provided it satisfies the condition

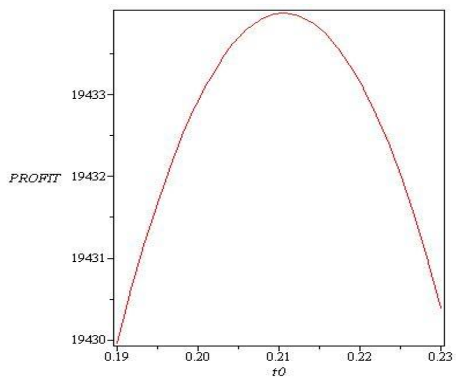
$$\begin{vmatrix} \dfrac{\partial^2 \pi(t_0, T)}{\partial t_0^2} & \dfrac{\partial^2 \pi(t_0, T)}{\partial t_0 \partial T} \\ \dfrac{\partial^2 \pi(t_0, T)}{\partial T \partial t_0} & \dfrac{d^2 \pi(t_0, T)}{\partial T^2} \end{vmatrix} > 0. \quad (23)$$

IV. NUMERICAL EXAMPLES

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, c_2= Rs. 8, in appropriate units. The optimal value of t_0^* = 0.2106, T* =0.3217, Profit*= Rs. 19433.9999 and optimum order quantity Q* = 176.2559.

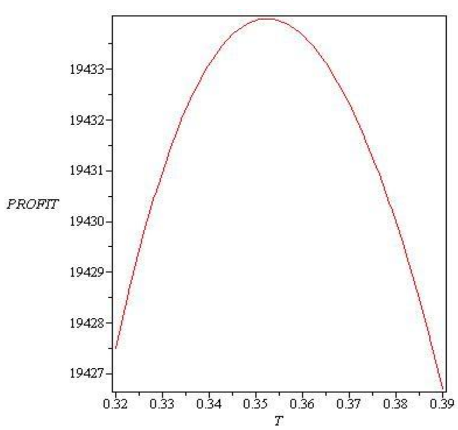
The second order conditions given in equation (23) are also satisfied. The graphical representation of the concavity of the profit function is also given.

t_0 and Profit



Graph 1

T and Profit



Graph 2

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Para-meter	%	t_0	T	Profit	Q
a	+20%	0.1924	0.3217	23380.0815	193.1739
	+10%	0.2009	0.3359	21406.4264	184.7492
	-10%	0.2218	0.3712	17462.9918	167.1970
	- 20%	0.2351	0.3936	15493.6465	157.5985
x	+20%	0.1868	0.3356	19405.3726	167.9213
	+10%	0.1979	0.3433	19419.0010	171.7868
	-10%	0.2251	0.3627	19450.6022	181.5294
	- 20%	0.2421	0.3750	19469.1014	187.7094
θ	+20%	0.2085	0.3507	19431.8087	175.5327
	+10%	0.2095	0.3515	19432.8998	175.9193
	-10%	0.2116	0.3530	19435.1092	176.6420
	- 20%	0.2127	0.3538	19436.2278	176.9043
A	+20%	0.2304	0.3856	19379.7948	192.9888
	+10%	0.2207	0.3693	19406.2845	184.8223
	-10%	0.1999	0.3343	19463.1304	167.2896
	- 20%	0.1886	0.3152	19493.9193	157.7234
c_2	+20%	0.2179	0.3401	19413.9927	170.2172
	+10%	0.2143	0.3457	19423.3461	173.0116
	-10%	0.2061	0.3601	19446.2516	180.1992
	- 20%	0.2009	0.3697	19460.5017	184.8546

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x and c_2, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity also decreases/ increases.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with linear demand under different deterioration rates and shortages. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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