A Computational Study of A Multi Solid Wall Heat Conduction Made Up of Four Different Building Construction Materials Subjected to Various Thermal Boundary Conditions.[1]

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Abstract: **Heat transfer by conduction (also known as diffusion heat transfer) is the flow of thermal energy within solids and non-flowing fluids, driven by thermal non-equilibrium (i.e. the effect of a non-uniform temperature field), commonly measured as a heat flux (vector), i.e. the heat flow per unit time (and usually unit normal area) at a control surface. Consider a square building wall which is made up of four different materials (refer Fig. 1). The wall is subjected to a hot temperature of 100^{0}C (initial-condition) and is then subjected to various thermal boundary conditions and volumetric heat generation; as shown in a table below. By Computational Fluid dynamic (CFD) analysis i.e.Finite volume analysis for this state of Conduction ,we study the steady state temperature contours for the four different cases. We also analyse the heat transfer (positive for outward/heat-loss and negative for inward/heat-gain) from the different segments of the wall , total heat-gain, total heat-loss and energy balance; for the different cases. Finally we discuss the effect of volumetric heat generation on the results for the types [case C & D] of BCs and try to optimize our result**

Key words; Thermal Boundary, CFD, Conduction, Heat Transfer, Energy Balance. Finite volume method, : Optimization

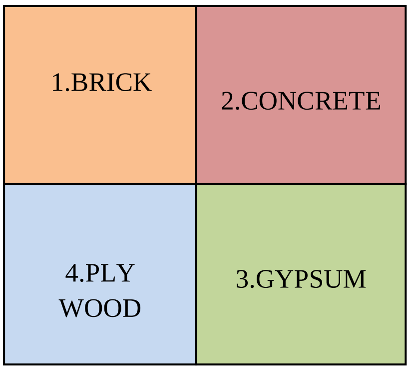


Fig 1: Multi Solid Wall

I. INTRODUCTION

Heat conduction is of increasing importance in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. A common example of heat conduction is heating an object in an oven or furnace. The material remains stationary throughout, neglecting thermal expansion, as the heat diffuses inward to increase its temperature. The importance of such conditions leads to analyze the temperature field by employing sophisticated mathematical and advanced numerical tools. The study considers the various solution methodologies used to obtain the temperature field.

The objective of conduction analysis is to determine the temperature field in a multi body analysis is how the temperature varies within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body. **Why** one need to know temperature field in a multi layered solid body is because, we can analyse and compute the heat flux at any location, compute thermal stress, expansion, deflection, design insulation thickness, and simplify heat treatment method and major design calculations.

The solution of conduction problems involves the functional dependence of temperature on space and time coordinate. The term steady implies no change with time at any point within the medium, while transient implies variation with time or time dependence. Obtaining a solution means determining a temperature distribution which is consistent with the conditions on the boundaries and also consistent with any specified constraints internal to the region. Further Transient heat conduction is encountered in metallurgical industries where metals and alloys are subjected to different heat treatment processes to enhance their physical and chemical properties (e.g. annealing). The actual heat treatment process involves complex heat transfer processes described and such problems are preferentially solved using numerical methods.

Numerical heat transfer is a broad term denoting the procedures for the solution, on a computer, of a set of algebraic equations that approximate the differential (and, occasionally, integral) equations describing conduction, convection and/or radiation heat transfer. Compared to the experimental method, numerical analysis provides a more direct way to enhance/reduce heat transfer effectively so as to improve the performance or to optimize the structure of a thermal device.

Cases	Boundary Conditions				Volumetric Heat Generation (W/m^3)
	Left	Bottom	Right	Top	
A	50^0C	Insulated	q_W=150 W/m^2	h=100W/m^2.K, T_∞=30^0C	0
B	Insulated	50^0C	q_W=150 W/m^2	h=100W/m^2.K, T_∞=30^0C	0
C	q_W=150 W/m^2	50^0C	Insulated	h=100W/m^2.K, T_∞=30^0C	250
D	50^0C	q_W=150 W/m^2	h=100W/m^2.K, T_∞=30^0C	Insulated	250

H.Baig et. al.[1] have analysed conduction/Natural convection by numerical analysis across multi layer buildings They carried out the heat leak for different numbers of air filled cavities. Thomas Bloomberg[2] studied heat conduction in two and three dimensions by numerical methods and analysed several cases of two dimensional heat conduction .Filipo de Monte, et al. [3] solved two dimensional Cartesian unsteady heat conduction problems for small values of time. Shidfara et al. [4] identified the surface heat flux history of a heated conducting body. The nonlinear problem of a non-homogeneous heat equation with linear boundary conditions is considered. F. de Monte [5] developed a new type of orthogonality relationship and used to obtain the final series solution of one-dimensional multilayered composite conducting slabs subjected to sudden variations of the temperature of the surrounding fluid. Arild Gustavsen et al.[6] analysed heat transfer and conduction in window frames with cavities by CFD method

In the present study we have used the grid dependent study and have used the finite volume approach method of computational heat conduction to study the CHC transient heat simulation of multi solid wall of 2D and say long (in *z*-direction) having Cartesian computational *x-y* domain of size *L=10 m* and *H=10m*, for CHC transient simulation. The multi solid wall (see **fig 1**) is made up of four different building construction materials. 1.**Brick** of density 1920 kg/m3, specific-heat: 790 J/Kg K, thermal-conductivity: 0.89 W/m-K 2. **Concrete** of density 2240 kg/m3, specific-heat: 840 J/Kg K, thermal-conductivity: 1.4 W/m-K 3.**Gypsum** of density 800 kg/m3, specific-heat: 1090 J/Kg K, thermal-conductivity: 0.16 W/m-K 4. **Plywood** of density 540 kg/m3, specific-heat: 1210 J/Kg K, thermal-conductivity: 0.12 W/m-K respectively. The multi solid wall is subjected to initial temperature of 100^0C (initial-condition) and is subjected to various thermal boundary conditions and volumetric heat generation(Case C & D); shown in a table A below.

Necessary Programs by using matlab/scilab have been generated to get the results and we plot the steady state temperature contours for the different cases. Other parameters like Heat transfer from left, Heat transfer from right, Heat transfer from top, Heat transfer from bottom, Total heat gain ,Total heat loss & Energy balance have been also studied from the different walls of the plate. Finally we discuss the effect of volumetric heat generation on the results for the types C&D of BCs and try to optimize our result.

TableA:

Fig 2A:

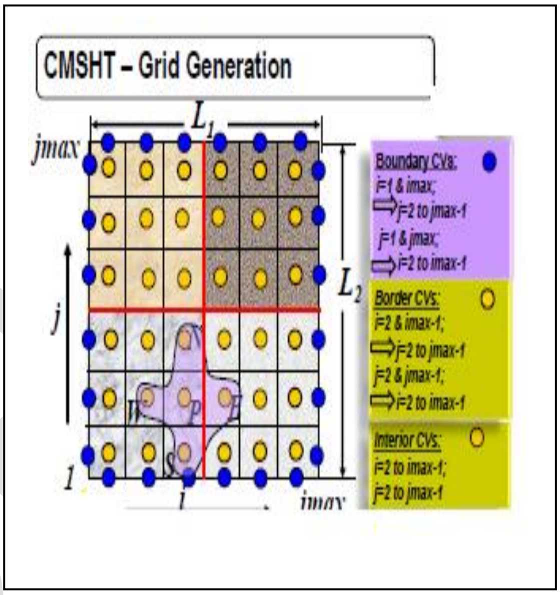
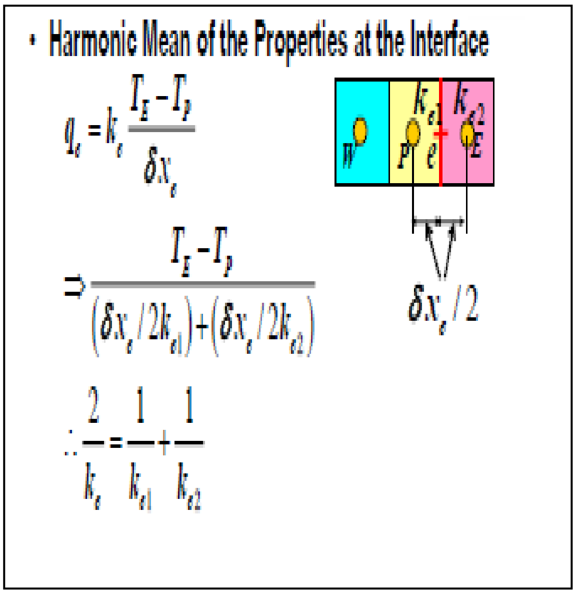


Fig 2B:-



II. MATHEMATICAL FORMULATION

The General 1 D Heat Conduction Equation is as follows which can be extended to our 2 D model is as follows.

Assume the density of the wall is ρ, the specific heat is C_p, and the area of the wall normal to the direction of heat transfer is A.
An energy balance on this thin element during a small time interval t can be expressed as:

(Rate of Heat Generation at x)-(Rate of Heat Conduction at x+dx) +(Rate of Heat Generation inside the element)=(Rate of energy content of the element)

$$\dot{q}_x + \dot{q}_{x+dx} + \dot{G}_{element} = \frac{\Delta E_{element}}{\Delta_t} \quad ..(1)$$

$$\Delta E_{element} = E_{t+\Delta t} - E_t = mC_p(T_{t+\Delta t} - T_t) = \rho C_p A\Delta x(T_{t+\Delta t} - T_t) \quad ...(2)$$

$$\dot{G}_{element} = \dot{g}V_{element} = \dot{g}A\Delta x \quad ...(3)$$

$$\dot{q}_x + \dot{q}_{x+dx} + \dot{g}A\Delta x = \rho C_p A\Delta x \frac{(T_{t+\Delta t} - T_t)}{\Delta_t}$$

….(4)

Dividing by $A\Delta x$ gives:

$$-\frac{1}{A}\left[\frac{\dot{q}_{x+\Delta x} - \dot{q}_x}{\Delta x}\right] + \dot{g} = \rho C_p\left[\frac{T_{t+\Delta t} - T_t}{\Delta t}\right] \quad ...(5)$$

Taking the limits as $\Delta x \to 0, \Delta t \to 0$ and from Fourier's law we obtain finally:

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C_p \frac{\partial T}{\partial t} \quad ...(6)$$

III. FINITE VOLUME METHOD

The FV method is a Control volume based technique of numerical discretisation. It offers several advantages (including its conservativeness and robustness) that make it particularly attractive for use in both academic and commercial CFD applications. It is also well suited for investigating diffusion problems. It implements the following computational steps: (a)Mesh generation entailing the division of the domain of interest into discrete control volumes, (b) Integration of the governing equations on the individual control volumes to construct algebraic equations for the discrete variables (the unknowns) such as pressure, velocity, temperature, and conserved scalars. (c) Linearization of the discretised equations and solution of the resulting linear system of equations to yield updated values of the dependent variables (Njiofor, 2009[7] , Eymard et. al., 2000[8] and Versteeg and Malalasekara[9], 2007). The datas of different materials was extracted from Thermo Mechanical Properties of Materials: by A .V. Marchenko[11]

The basic governing equation is the Laplace's equation(G.E.): $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ ….(7)

The G.E for Heat conduction thus becomes:

$$\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{q} \quad ….(8)$$

The governing partial equations are converted to discretized algebraic equations using FVM.

$$\iiint_{\Delta V} \rho C_p \left(\frac{\partial T}{\partial t}\right) dV = \iiint_{\Delta V} k\nabla^2 T dV + \iiint_{\Delta V} \bar{q}\, dV$$

….(9)

The Finite Volume (Explicit) Discretized Equation is:

$$\Sigma_{f=e,w,n,s}\left(\frac{\partial T}{\partial n}\right)_f \Delta s_f = \left(\frac{T_P - T_w}{\delta s_w}\right)\Delta y - \left(\frac{T_e - T_p}{\delta s_e}\right)\Delta y + \left(\frac{T_P - T_S}{\delta s_s}\right)\Delta x - \left(\frac{T_N - T_P}{\delta s_n}\right)\Delta x = 0$$

…(10)

In this paper we have used 2-D Heat conduction: Solution Algorithm for Explicit method as it is relatively simple to set up and program.

3.1Solution Algorithm :

1. Enter the inputs: material properties ,geometric parameters (*l1*& *l2*) and maximum number of cvs in the x and y directions, b.cs input and convergence criteria's.

2. Grid generation: calculate all the geometric parameters of all the cvs

3. Set the initialization for t, and dirichlet b.c for temperature ,$t_{i,j}$[i.e.equation variable arrays].
4. Assign cell-wise conductivity.
5. Assign cell-wise density
6. Assign cell-wise specific heat
7. After choosing the generic boundary condition function we calculate error function.

8. Modify the other (non-dirichlet) boundary conditions(if necessary).

5. Set *told(i,j)= t(i,j)* for all cvs.

6. For each faces, calculate heat fluxes.

7.Check the harmonic mean of the properties of interface (See Fig 2B)

7. For each "interior" and "border" cv, calculate $Q_{conduction}$ and finally,

8. Check for convergence.

$$rmsT = \sqrt{\frac{\Sigma_{i,j}(T_{i,j} - Told_{i,j})^2}{(i\max - 2)X(j\max - 2)}}; rmsT \le \varepsilon$$

…(11)

9. If not, go to step 3 and continue till convergence is achieved.
The geometric parameters of grid are:

$\Delta x = \frac{L_1}{(i\max - 2)}, \Delta y = \frac{L_2}{(j\max - 2)}$ .surface area

Heat transfer related parameters obtained for different test cases of 2D heat Conduction				
Parameter	Case A	Case B	Case C	Case D
Heat transfer from left	1.594e+02	0.000e+00	-1.500e+03	1.073e+04
Heat transfer from right	-1.500e+03	-1.500e+03	0.000e+00	1.577e+04
Heat transfer from top	1.341e+03	9.333+03	1.696e+04	0.000e+00
Heat transfer from bottom	0.000e+00	5.667e+02	9.538e+03	-1.500e+03
Total heat gain	1.500e+03	1.500e+03	2.650e+04	2.650e+04
Total heat loss	1.500e+03	1.500e+03	2.650e+04	2.650e+04
Energy balance	-7.568e-04	-9.058e-04	-9.026e-04	-8.475e-04

$\Delta S_e = \Delta S_w = \Delta y; \Delta S_n = \Delta S_s = \Delta x$.

Volume= $\Delta V = \Delta x.\Delta y$

Note1:FVM UNSTEADY TERM:I Level of Approximation Volume Averaging of rate of change of velocity/temperature in a CV as a value at the centroid of CV i.e. cell center. (II Order).

$$\int_{\Delta V_P} \frac{\partial T}{\partial t} dV = (\frac{\partial T}{\partial t})_{av} \Delta V_P \approx (\frac{\partial T}{\partial t})_P \Delta V_P \quad \ldots(12)$$

Note2: •II Level of Approximation Discrete Representation of the rate of change (I order Forward Finite Difference) : $(\frac{\partial T}{\partial t})_P \approx \frac{T_P^{\,n+1} - T_P^{\,n}}{\Delta t}$

…(13)

IV. BOUNDARY CONDITIONS

In our problem there are four types of Boundary conditions. These equations must be accompanied by boundary conditions appropriate to the particular problem. For transient problems, an initial condition is required, and in all problems boundary values for all variables are needed. In heat transfer, three types of condition on T are encountered:

- specified temperature: $T = T_w$ at a boundary, where the wall temperature T_w is known; this is called a dirichlet condition.

- specified heat flux: $-\lambda(\partial T/\partial n)_w = q_w$, where n is normal to the surface, and the wall heat flux q_w is known; this is called a Neumann condition;

- the specified heat transfer coefficient α: $=\lambda(\partial T/\partial n)_w = \alpha(T_w - T_i)$; this is called a mixed condition

Here we will be dealing with both dirichlet condition & non- dirichlet condition BC’s: **Table B**:

V. RESULTS & DISCUSSIONS:

Fig 3(Case A):

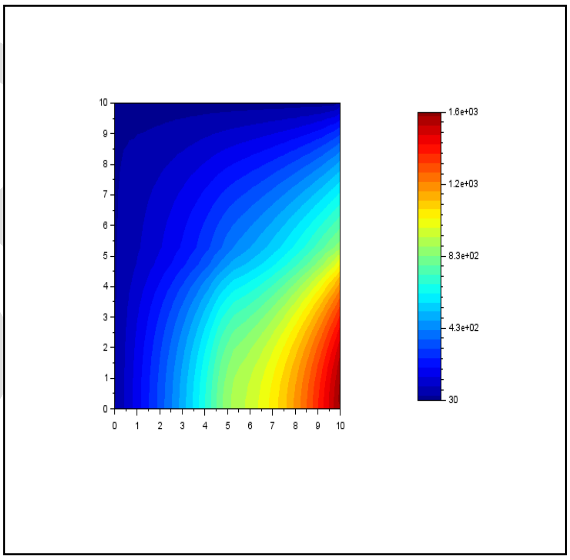


Fig 4(Case B):

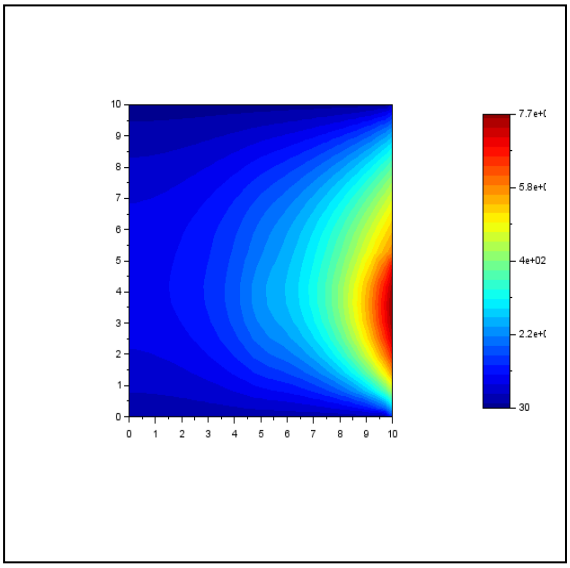


Fig 5(Case C):

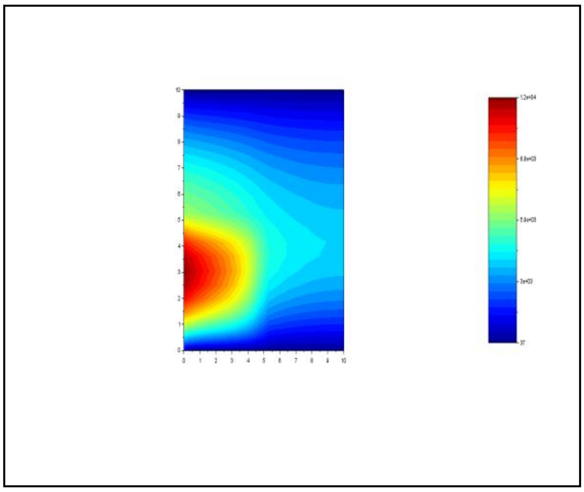
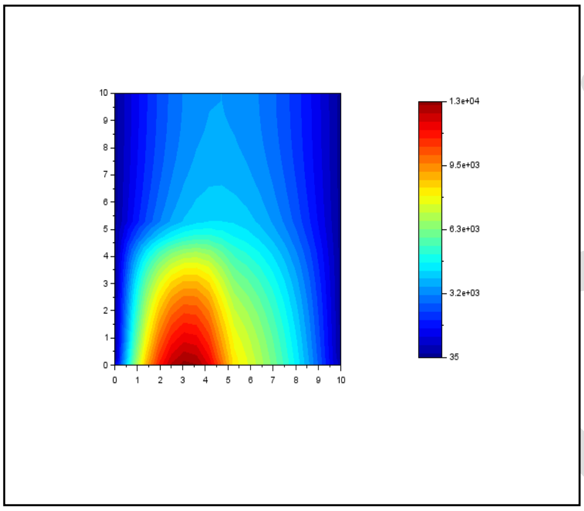


Fig 6(Case D):



Discussions

After analyzing the Case C & case D plots we come to the conclusion that on insulating the right and bottom for case C & D has resulted in more heat generation compared to case A or B . The temperature contour plots obviously tell us that in case of C & D higher temperature is achieved in plywood region whereas it is more in the case of gypsum for case of A & B . If we increasing the value of wall temperature it will result in more heat generation.

The net heat loss is least for case A and also lesser for case B compared to Case C & D. The Numerical error in energy balance is least for case B and even lesser when compared to case C.

VI. FINAL CONCLUSION & FUTURE WORK:

Choosing the material property condition for the above study holds the first key to results. The best optimisation can only be achieved by changing the nature of Boundary condition i.e. [Constant wall temperature, Insulated wall / Symmetry wall ,Convective heat loss from wall , Constant heat flux] and also the initial value of temperature.

All the problems below has been solved on a coarser grid size of imax=jmax=20. However on refining the grids to of imax=jmax=40 also does not refine our results much. Hence to adopt a Grid independent study we can choose to work with of imax=jmax=30 to save computational time and costs. We have adopted a steady state convergence criteria of 10^{-6}. **In future course** we can use any other Boundary conditions to get a better idea about the behavior & thermal properties of the building materials or any other material .

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