Flow and Heat Transfer of a Non-Newtonian Power-Law Fluid over a Non-Linearly Stretching Vertical Surface with Heat Flux and Thermal Radiation

Priya Mathur[1], Abhay Kumar Jha[2], Anil Sharma[3]

[1]Poornima Institute of Engineering & Technology, Jaipur,

[2]Department of Mathematics, JECRC University, Jaipur.

[3]Department of Mathematics, University of Rajasthan, Jaipur (Rajasthan)

***Abstract*- The present paper is focused on the study for the effects of constant heat generation and thermal buoyancy on the steady two-dimensional flow and heat transfer of a non-Newtonian power-law fluid over a non-linearly stretching vertical surface. Highly nonlinear momentum and thermal boundary layer equations which governing the flow and heat transfer are reduced to a set of nonlinear ordinary differential equations by appropriate transformation.The resulting ODEs are successfully solved numerically with the help of fourth order Runge–Kutta method coupled with the shooting technique. The effects of various parameters like the buoyancy(mixed convection) parameter, the radiation parameter, power-law index parameter and the local Prandtl number on the flow and temperature profiles as well as on the local skin-friction coefficient and the local Nusselt number are presented and discussed. Favorable comparisons of numerical results with previously published work on various special cases of the problem are obtained.**

***Keywords*: Power-law fluids, Buoyancy parameter, Non-uniform heat generation/absorption Viscous dissipation.**

I. INTRODUCTION

The study of flow and heat transfer of a non Newtonian fluid over stretching surface has received considerable attention due to their wide application in various engineering fields. The problem of flow and heat transfer over a stretching/shrinking sheet is relatively a new consideration in the laminar boundary layer flow. The important concept of boundary layer was applied to power law fluids by Schowalter [1].numerical solution for forced convection of power law fluid about a right angle wedge with isothermal surface has been investigated by Lee and Ames [2].The surface velocity on the boundary, the study of the boundary layer magnetohydrodynamic (MHD) flow towards a shrinking/stretching sheet has gained considerable attention of many researchers because of its frequent occurrence in Industrial technology, geothermal application and high temperature plasmas applicable to nuclear fusion energy conversion and MHD power generation systems. Recently several attempts [3-10] have been made on the study of shrinking phenomena Muhamin and Khamis [11] studied the effects of heat and mass transfer on the non-linear MHD viscous fluid flow over a shrinking sheet in the presence of suction. They explored the industrial sheet had a substantial effect on the flow fields. Mostafa A.A. Mahmaud [12] studied the non-uniform heat generation effect on heat transfer of a non-Newtonian power-law fluid over a non-linear stretching sheet. However, all the studies mentioned above are restricted in linear shrinking/stretching sheet with constant viscosities.The flow over a non-linear stretching sheet has widely studied [13-15]. In many practical situations the continuous stretching/shrinking surface is assumed to have a power-law velocity. It is well known that the physical properties of fluid flow may change with temperature especially for the variable fluid viscosity and the thermal conductivity. Prasad et al. [16] investigated the effects of variable viscosity and variable thermal conductivity on the hyderomagnetic flow and heat transfer over a non-linear stretching sheet. Nadeem and Hussain [7]examined the MHD flow of a viscous fluid on a non-linear porous shrinking sheet with the hemotopy analysis. They made an observation in the existence of the shrinking sheet solution and found that the solution might exist if either the magnetic field or the stagnation point flow is taken into account. However, the effect of thermal radiation and viscous dissipation were not considered in their studies. All of the above studies were done for the case that the effect of non-uniform heat generation/absorption on heat transfer is not taken into consideration. The study of heat generation or absorption effects on heat transfer is important in many physical problems dealing with chemical reactions and those concerned with dissociating fluids. The effect of non-uniform heat generation/absorption on heat transfer on Newtonian and non-Newtonian fluids over stretching/shrinking surface has been studied by many authors [17-21] under various physical situations. Recently, Abel et al. [22] investigated the effects of non-uniform heat source (sink) on the flow and heat transfer of a power-law fluid on a linearly stretching/shrinking sheet in the presence of magnetic field and variable thermal conductivity. Xa and Lia [23] presented a theoretical analysis for the linear boundary-layer flow and heat transfer of a non-Newtonian power-law fluids on a non-linearly stretching/shrinking sheet with variable wall temperature. Motivated by the above investigations, in this work, we deal with the problem of flow and heat transfer of a non-Newtonian power-law fluids past a non-linearly stretching shrinking sheet in the presence of viscous dissipation and non-uniform heat generation/absorption. It can be seen that the problem considered in Xu and Liao [24] is a specific case of the present work. Recently, Aman and Ishak [26] investigated the problem of mixed convection boundary layer flow adjacent to a stretching vertical sheet in an incompressible electrically conducting fluid in the presence of a transverse magnetic field. Recently, Elbashbeshy et al. [27] investigated the effects of thermal radiation and magnetic field on an unsteady mixed convection flow and heat transfer over an exponentially stretching permeable surface in the presence of internal heat generation/absorption. To the best of the author knowledge, there seems to be no existing document about heat transfer characteristics of a non-Newtonian power law fluid flow over a nonlinearly stretching vertical sheet, taking into account the effect of thermal buoyancy, thermal radiation and constant heat flux condition. Therefore the purpose of the present paper is to examine the heat transfer aspects in a non-Newtonian power law fluid flow driven by a non-linearly impermeable stretching vertical sheet in the presence of thermal radiation and constant heat flux.Using the similarity transformations, the set of governing P.D.E. and the boundary conditions are reduced to a system of non-linear O.D.E. which solved numerically using a fourth order Runge-Kutta scheme coupled with the shooting method for various parameters.

II. FORMULATION OF THE PROBLEM

Consider a steady, two-dimensional MHD flow and heat transfer of an incompressible non-Newtonian fluid obeying the power-law model past a stretching/shrinking porous sheet, the origin is located at the slit, through which the sheet is drawn through the fluid medium. The x-axis is chosen along the sheet and y-axis is taken normal to it. The continuous stretching/shrinking sheet is assumed to have power-law velocity $u = c\,x^m$; c (> 0) is a constant and m is an exponent. The steady two-dimensional boundary-layer equation for non-Newtonian fluids taking into account the viscous dissipation and internal heat generation effects in the energy equation are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}=\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-g\beta(T-T_\infty) \tag{2}$$

$$\rho C_p\left(u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}\right)=k\,\frac{\partial^2 T}{\partial y^2}+K\left|\frac{\partial u}{\partial y}\right|^{n-1}+q''' \tag{3}$$

Where u and v are the velocity component in the x and y directions, respectively.ρ and κ yare the fluid density and the thermal conductivity , respectively. N is the power low index. If n<1 the fluid is said to be pseudo plastic, if n>1 it is known as dilatants and when n=1, it becomes Newtonian fluid. g is the acceleration due to gravity,β is the thermal expansion coefficient, T is the temperature of the fluid , c_p is specific heat flux.

We assumewe assume that the temperature difference within the flow are small such that T^4 may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher-orderterms, we have :

$$T^4 \cong 4T_\infty^3 T-3T_\infty^4 \tag{4}$$

With the boundary condition

$$U= U,\ v= 0, -k\left(\frac{\partial T}{\partial y}\right)=q_w \text{ at } y=0 \tag{5}$$

$$u\to 0, T\to T_\infty \text{ as } \quad y\to\infty \tag{6}$$

$$\eta=y\left(\frac{c^{\frac{2-n}{n+1}}\,x^{\frac{m(2-n)-1}{n+1}}}{\left(\frac{\mu}{\rho}\right)^{1/n+1}}\right) \tag{7}$$

$$\psi(\eta)=\left(\frac{K}{\rho}\right)^{\frac{1}{n+1}} c^{\frac{2n-1}{n+1}}\, x^{\frac{m(2n-1)+1}{n+1}}\, f(\eta) \tag{8}$$

$$\theta(\eta)=T-T_\infty/T_w-T_\infty$$

the internal heat generation or absorption term q^m is modeled as:

$$q'''=\frac{\rho k\,u_w}{K\,x}[A(T_w-T_\infty)e^{-\eta}+B(T-T_\infty)], \tag{9}$$

where A and B are the coefficient of space and temperature dependent heat generation/absorption respectively. Note that the case A > 0, B > 0 corresponds to internal heat generation, that A < 0, B < 0, corresponds to internal heat absorption f(η) is the dimensionless stream function, θ is the dimensionless temperature of the fluid in the boundary layer region and ψ is the stream function that satisfies the continuity equation (1) and is defined by

$$u=\frac{\partial\psi}{\partial y}:v=-\frac{\partial\psi}{\partial x} \tag{10}$$

Equation (2) and (3) reduce to

$$nf'''\,|f''|^{n-1}+\frac{1+m(2n-1)}{n+1}f\,f''-m\,f'^2+\lambda\theta=0 \tag{11}$$

$$\frac{1}{Pr}\theta''+\frac{1+m(2n-1)}{n+1}f\,\theta'-r\,f'\theta+Ec\,|f''|^{n+1}+\gamma^{*}e^{-\eta}+\gamma\theta=0 \tag{12}$$

With boundary condition

$$f=0,\ f'=1,\theta'=-1,\eta\to 0 \tag{13}$$

$$f'\to 0,\theta\to 0,\eta\to\infty \tag{14}$$

Where

$Pr=\frac{\rho C_p}{k}\left[x^{(3n-1)(n-1)}c^{3(n-1)}\left(\frac{\mu}{\rho}\right)^2\right]^{1/n+1}$ is local non Newtonian Prandtl number

$\gamma^{*}=\frac{kA}{K\,C_p}$ (space dependent heat generation (> 0) or absorption (< 0) parameter)

$\gamma=\frac{kB}{KC_p}$ (temperature-dependent heat generation (> 0) or absorption (< 0) parameter)

$Ec=\frac{u_w^2}{C_p(T-T_\infty)}=\frac{U^2\theta q_w x e_x^{\frac{-1}{n+1}}}{\kappa}$ (local Eckert number)

$\lambda=\frac{g\beta q_w c^n}{k\nu x^{n(m-1)-2}}$ is mixed convection parameter

III. NUMERICAL SOLUTIONS

In this section, an efficient Runge–Kutta fourth order method along with shooting technique has been employed to analyze the flow model for the above coupled ordinary differential Eqs. (11), (12) along with the boundary conditions (13), (14) for different values of the governing parameters. The system (11), (12) with the boundary conditions (13), (14), is integrated numerically by means of Runge–Kutta method with systematic estimate of $f''(0)$ and $\theta(0)$ with Newton–Raphson shooting technique until the boundary conditions at the infinity $f'(\eta_\infty) and \theta(\eta_\infty)$ decay exponentially to zero. At every position, the iteration process continues until the convergence criterion for all the variables, 10^{-6} , is achieved. In order to get a clear insight of physical problem, numerical results are displayed with the help of graphical illustrations. Also, to asses the accuracy of the numerical method, comparison with those obtained by Ahmed M. Megahed [28] are shown in Table 1. From this comparison and without any doubt, from this table, we can claim that our results are in excellent agreement with this reference. Also, the obtained results demonstrate reliability and efficiency of the proposed method.

IV. RESULTS AND DISCUSSION

The system of non-linear O.D.E. (11) and (12) with the boundary condition (13-14) is solved numerically using the shooting technique with the fourth-order Runge-Kutta scheme. We guessing of $f''(0)$ and θ'(0) by showing technique until the boundary conditions at infinity are satisfied. The step size Δη= 0.001 is used while obtaining the numerical solution and accuracy up to seventh decimal place which very sufficient for converses.In this method, we choose suitable finite values which depended on the values of the parameters used. The computations where done by a programme which uses a symbolic and computational computer language MATLAB.

It is found that the velocity within the boundary layer is monotonically tends to zero as the distance increases from the boundary. The effect of the increasing values of the power law index n is to reduce the horizontal boundary layer thickness. That is, the thickness is much larger for shear thinning (pseudo plastic; 0\n\1) fluids than that of shear thickening (dilatants; 1\n\2) fluids. This behavior is noticeable in Fig. 1. The effect of the same parameter on thetemperature distribution hðgÞ in the boundary layer is shown in Fig. 2. It is observed from this figure that,increasing the values of the power law index n leads to thicken the thermal boundary layer thickness. Also, itis seen that the temperature distribution hðgÞ asymptotically tends to zero in the free stream region.

Figure-1The behavior of the velocity distribution for various values of n

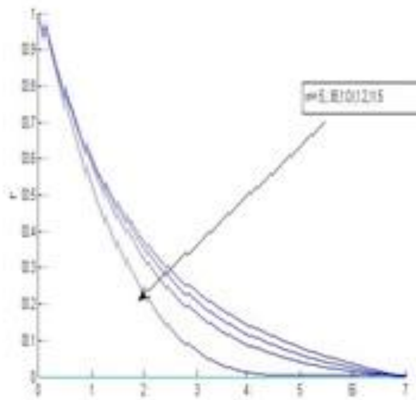


Figure-2The behavior of the temperature distribution for various values of n

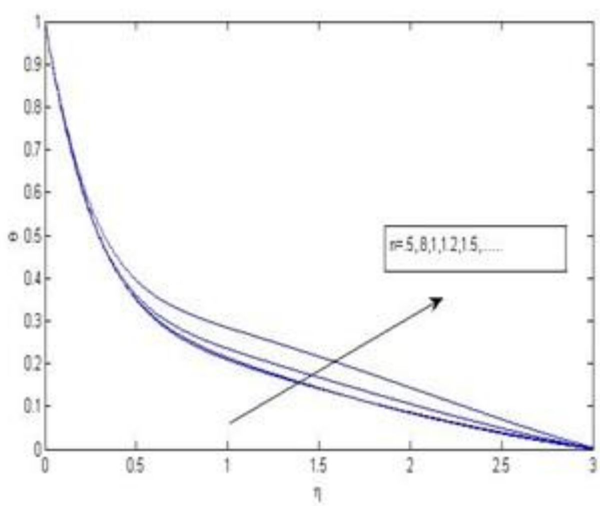


Fig. 3 display the effect of temperature dependent heat absorption γ(< 0) or generation γ (> 0) parameter on the dimensionless temperature. It is shown that the effect of heat absorption causes a drap in the temperature as the heat following from the wall is absorbed. When γ> 0, the heat generation brings about a temperature increase throughout the entire boundary layer for the case of heat absorption γ (< 0) one sees that the thermal boundary layer thickness as the absolute value of γ increases.

Figure 3Temperature profiles for various values of **γ,** with n=0.8, R=0.1, Ec=0.1, Pr=10, **γ**=0

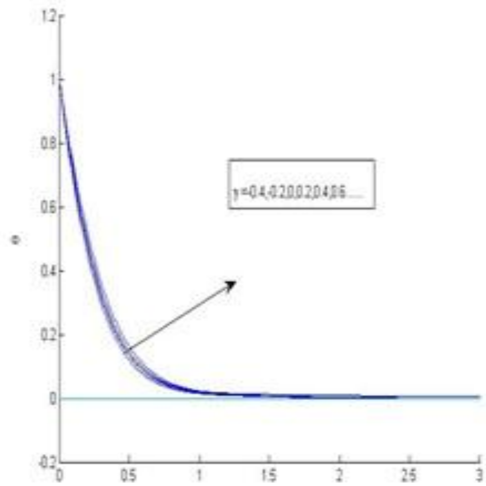
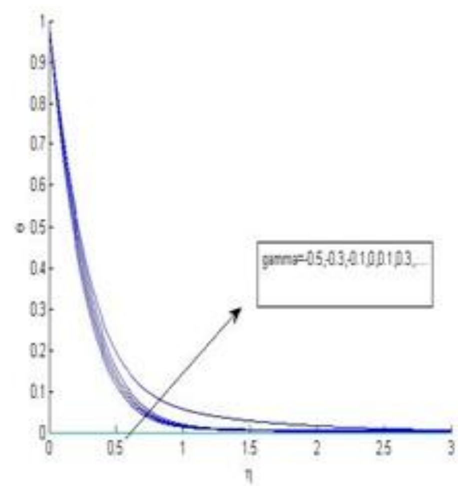


Figure 4 Temperature profiles for various values of **γ,** with n=1.2, R=0.1, Ec=0.1, Pr=10, **γ**=0



The effect of the Prandtl number Pr on the dimensionless temperature profiles is illustrated in figure 5 and figure 6. It can be seen that the dimensionless temperature decreases with increasing Pr for both n < 1 and n > 1. Physically, a higher value of Pr is equivalent to decreasing thermal conductivity which reduces conduction and thereby increases the wall heat transfer

Figure 5 Temperature profiles for various values of **Pr,** with n=0.8, R=0.03, Ec=0.1, **γ*=.03**, **γ**=0.1

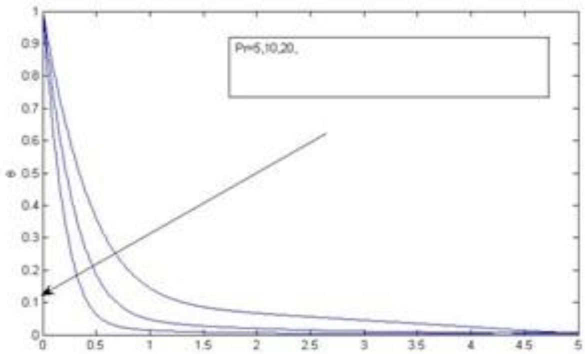
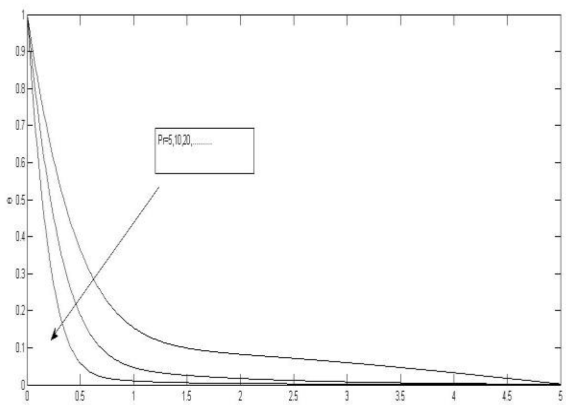


Figure 6Temperature profiles for various values of **Pr,** with n=0.8, λ=0.01, Ec=0.1, **γ*=.03**, γ=0.1



The dimensionless temperature distribution within the boundary layer region for various values of Eckert number Ec are illustrated in figure7 and figure 8. As compared to the case for no viscous dissipation, it can be seen that the dimensionless temperature increases as Ec increases. The increase in the fluid temperature due to frictional heating is observed to be more pronounced for higher value of Ec as expected.

Figure 7 Temperature profiles for various values of Ec with fixed values of n=0.8,γ*=.01,γ=0.1,Pr=10

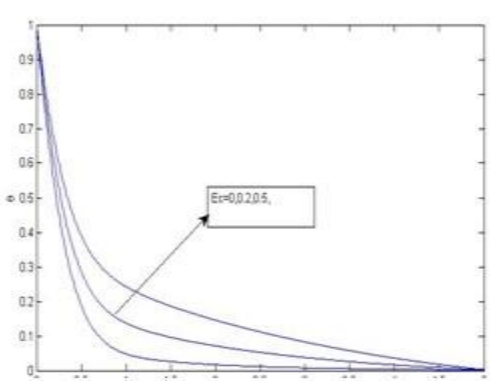


Figure 8 Temperature profiles for various values of Ec with fixed values of n=1.2,γ*=.01,γ=.1,Pr=10

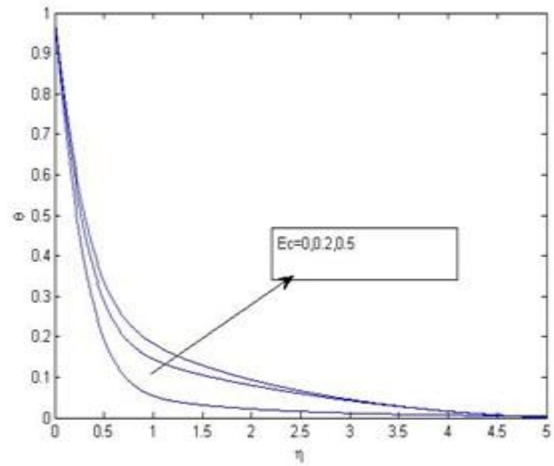


Figure 9 the behavior of the velocity profile for various values of λ

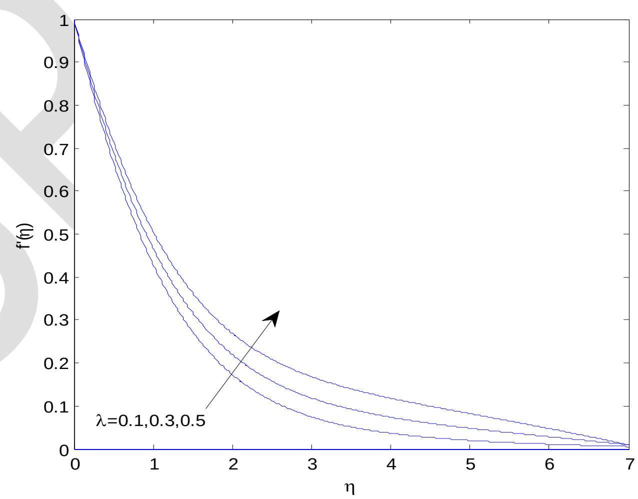
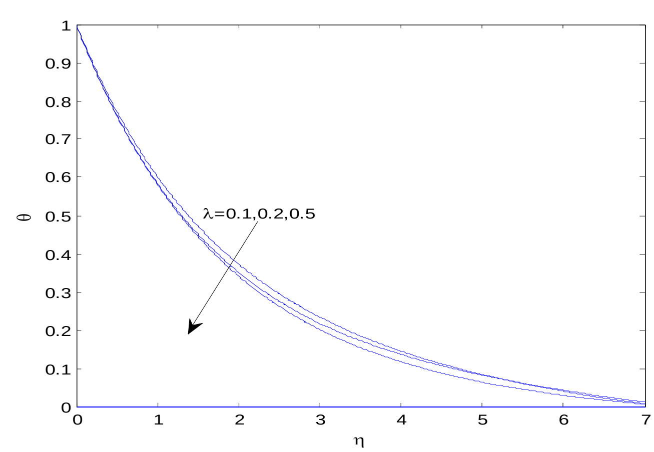


Figure 10 The behavior of the temperature profile for various values of λ



V. CONCLUSIONS

The problem of the MHD boundary layer flow and heat transfer of a non-Newtonian power-law fluids on a non-isothermal stretching porous surface in presence of non-uniform heat generation or absorption and viscous dissipation was studied. The governing equation describing the problem are transformed into a non-linear O.D.E. by using similarity transformation (when m=1/3, r=2/3) the transformed O.D.E. were solved numerically using fourth order Runge-Kutta method coupled with the shooting technique.

(i) It was found that the axial velocity across the stretching sheet decreases and the temperature increases with the increase in the value of n.

(ii) The temperature increases with increases γ ($n > 1$, $n < 1$) and Eckert number Ec ($n > 1$, $n < 1$) and decreases with increase the Prandtl number Pr ($n < 1$, $n > 1$).

(iii) The suction parameter R has significant reducing effects on the temperature profiles.

REFERENCES

[1]. SchowalterWR.The application of boundary layer theory to power-law pseudoplastic fluids: similar solution.AIChE J 6:24–28. (1960)

[2]. Kapur JN, SrivastavaRC.Similar solutions of the boundary layer equations for power-law fluids. ZAMP 14:383–389(1963)

[3]. Fang, T., Liang, W. and Lee, C.F. A new solution branch for the Blasius equation – a shrinking sheet problem. Computers and Mathematics with Applications, 56, 3088-3095 (2008).

[4]. Hayat, T., Javad, T. and Sajid, M. Analytic solution for MHD rotating flow of a second grade fluid over a shrinking surface. Physics Letters A, 372, 3264-3273 (1008).

[5]. Wang, C.Y. Stagnation flow towards a shrinking sheet. International Journal of Non-Linear Mechanics 43, 377-382 (2008).

[6]. Nadeem, S. and Awais, M. Thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity. Physics Letters A, 372, 4965-4972 (2008).

[7]. Nadeem, S. and Hussain, A. MHD flow of a visoucsluid on a non-linear porous shrinking sheet with homotopy analysis method. Applied Mathematics and Mechanics (English Edition), 30(12), 1569-1578 (2009) DOI 10.1007/s10483-009-1208-6.

[8]. Fang, T. Boundary layer flow over a shrinking sheet with power-law velocity. International Journal of Heat and Mass Transfer, 51, 5838-5843 (2008).

[9]. Fang, T. and Zhang,J. Closed-form exact solutions of MHD viscous flow over a shrinking sheet. Communications in Non-Linear Science and Numerical Simulation, 14, 2853-2857 (2009).

[10]. Nadeem, S., Hussain, A., Malik,M.Y. and Hayat, T. Series solutions for the stagnation flow of a second-grade fluid over a shrinking sheet. Applied Mathematics and Mechanics (English Edition), 30 (10), 1255-1262 (2008) DOI 10.1007/s10483-009-1005-6.

[11]. Muhaimin, R.K. and Khamis, A.B. Effects of heat and mass transfer on non-linear MHD boundary layer flow over a shrinking sheet in the presence of suction. Applied Mathematics and Mechanics (English Edition), 29(10), 1309-1317 (2008) DOI 10.1007/s10483-008-1006-z.

[12]. Mostafa, A.A., Mahmoud Ahmed M. Megahed. Non-uniform heat generation effect on heat transfer of a non-Newtonian power-law fluid over a non-linearly staretching sheet. Mecanica, 47: 1131-1139 (2012).

[13]. Cortell, R.Viscous flow and heat transfer over a non-linearly stretching sheet. Applied Mathematics and Computation, 184, 864-873 (2007).

[14]. Sajid, M., Hayat, T., Asghar, S. and Vajravelu, K. Analytical solution for axisymmetric flow over a non-linear stretching sheet. Archive of Applied Mechanics, 78, 127-134 (2008).

[15]. Prasad, K.V., Vajravelu, K. and Datti, P.S. The effect of variable fluid properties on the hydromagnetic flow and heat transfer over a non-linearly stretching sheet. International Journal of Thermal Science, 49, 603-610 (2010).

[16]. Prasad, K.V., Vajravelu, K. and Datti, P.S. Mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. International Journal of Non-Linear Mechanics, 45, 320-330 (2010).

[17]. Chamkha, A.J., Khaled, A.A. Similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. Heat Mass Transf. 37, 117-123 (2001).

[18]. Bataller, R.C. Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation. Int. J. Heat Mass Trans. 50, 3152-3162 (2007).

[19]. Nandeppanavar, M.M., Abel M.S., Tawade, J. (2010). Heat transfer in a Walter’s liquid B fluid over an incompressible stretching sheet with non-uniform heat source/sink and elastic deformation. Commun Nonlinear Sci. NumerSimul. 15, 1791-1802 (2010).

[20]. Pal, D. and Mondal, H. Effect of variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink. Commun Nonlinear Sci. NumerSimul 15, 1553-1564 (2010).

[21]. Abel, M.S., Siddheshwar, P.G. and Mahesha, N. Effects of thermal buoyancy and variable thermal conductivity on the MHD flow an,d heat transfer in a power-law fluid past a vertical stretching sheet in the presence of a non-uniform heat source. Int. J. Non-Linear Mech. 44, 1-12(2009).

[22]. Abel, M.S., Datti, P.S. and Mahesha, N. Flow and heat transfer in a power-law fluid over a stretching sheet with variable thermal conductivity and non-uniform heat source. Int. J. Heat Mass Transf. 52, 2902-2913 (2009).

[23]. Xu, H. and Liao, S.J. Laminar flow and heat transfer in the boundary-layer non-Newtonian fluids over a stretching flat sheet. Comput. Math. Appl. 57, 1425-1431 (2009).

[24]. Sakiadis, B.C. Boundary-layer behavior on continuous solid surfaces: I; Boundary-layer equations for two dimensional and axisymmetric flow. AIChE J.7, 26-28 (1961).

[25]. G.C. Shit and R. Halder. Effects of thermal radiation and MHD viscous fluid flow and heat transfer over non-linear shrinking porous sheet. Appl. Maths. Mech. Eng.Ed. 32(6), 677-688 (2011).

[26]. Aman F, IshakA Hydromagnetic flow and heattransfer adjacent to a stretching vertical sheet with prescribedsurface heat flux. Heat Mass Transfer 46:615–620(2010)

[27]. Elbashbeshy EMA, Emam TG, Abdelgaber KM. Effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorption. J Egypt Math Soc 20:215–222(2012)

[28]. Ahmed M. Megahed,Flow and heat transfer of a non-Newtonian power-law fluid over a non-linearly stretching vertical surface with heat flux and thermal radiation. Meccanica, DOI 10.1007/s11012-015-0114-3(2015)