Contact Stress Analysis and Optimization of Bevel Gear Pairs by Theoretical and FEA

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***Abstract*— The gears are used to transmit the motion and power from one shaft to another shaft by physical contact between gear teeth. The bevel gear is used to transmit motion and power between the axis of intersection and the non-intersecting axis. Gears are generally subjected to loads due to these loads, tooth bending stress and contact stress will be developed on the gear tooth. Much research on the action of the gear has confirmed that contact stresses also influence the formation of pits on the surface of the tooth. Gear tooth is the most important element in a gear system and has been focused on the current study. An analysis has been carried out for the three different types of materials used to make the bevel gear. The failure of gears due to contact stress is high compared to bending stress. Stress analysis has been a key area of research to minimize failure of the gear and optimize the design. The study of contact stress developed between the mating gears are very important for the gear design. The current goal is that the finite element analysis of the bevel gear is performed to determine the maximum contact stress by ABAQUS as the solver and also the theoretical maximum contact stress is calculated by the Hertz equation. In this study finite element analysis results were validated with theoretical results.**

***Keywords*— Bevel gear; contact stress; tooth bending stress; finite element method; optimization.**

I. INTRODUCTION

Each shaft includes gears which are connected to each other through the teeth. Gears are used to transmit power and motion from one shaft to another shaft through direct contact. Bevel gear is used to transmit the motion and power between two intersecting or non-intersecting shafts. Bevel gears are gears which intersect the axes of the two shafts and tooth bearing faces of the gears themselves are conical in shape. Bevel gears are most often mounted on shafts that are 90 degrees, but can also be designed in different angles. There are wide ranges of bevel gears that are used by industries, but each of these gears has the same use, which is to transfer the motion from one shaft to another. Gear can be classified as spur and helical gears, which have parallel shaft, while the other one are non-parallel shaft which are spiral gears and bevel gears [4]. This study mainly focuses on the gears tooth analysis. Bevel gears are widely used in automobiles, aircrafts, elevator's and heavy engineering machines [13]. They can be classified as,1) Spiral bevel gears, 2) Straight bevel gears, 3) Hypoid gears and 4) Zerol bevel gears. Two important concepts in gearing are pitch angle and pitch surface. For ordinary gear the pitch surface is the shape of a cylinder. The gear pitch angle is the angle between the face of the pitch surface and the axis of the gear. The best known types of bevel gears have pitch angles less than 90 degrees and thus are cone-shaped. This type of bevel gear is called as external gears because the gear teeth point outward. Bevel gears that have pitch angles of greater than 90 degrees have teeth that point inward and are called internal bevel gears [2]. If bevel gears have equal numbers of teeth and with axes at right angles, then this type of gears are called as mitre bevel gears.

II. PROBLEM DESCRIPTION

A very important parameter in the design of a pair of gears is the maximum surface contact stress that exists between two of the gear teeth in the mesh, because it affects surface fatigue (namely, pitting and wear) along with gear mesh losses. Much research on the action of the gear has confirmed that contact stresses also influence the formation of pits on the surface of the tooth. Hence the contact stresses are the causes for pitting failure of the gears.

III. AIM AND OBJECTIVES

1) AIM - Aim is to reduce the contact stress developed between a pair of bevel gears which in turn reduces the noise and vibration.

2) OBJECTIVES - The main focus of this research is to optimize the design of the bevel gear and reduce the contact stress.

1) Many iterations will be carried by changing the shape and profile of the gears.
2) Many iterations will be carried by changing the different materials and comparing all the results.
3) Static investigation for the all models is done by calculating the stress, displacement and contact stresses.
4) Contact stress and tooth bending stress are calculated theoretically and compared with the FEA results.

IV. METHODOLOGY

A. Geometric Modeling

The CAD model of the gear tooth is created by using NX CAD-8 modeling software.

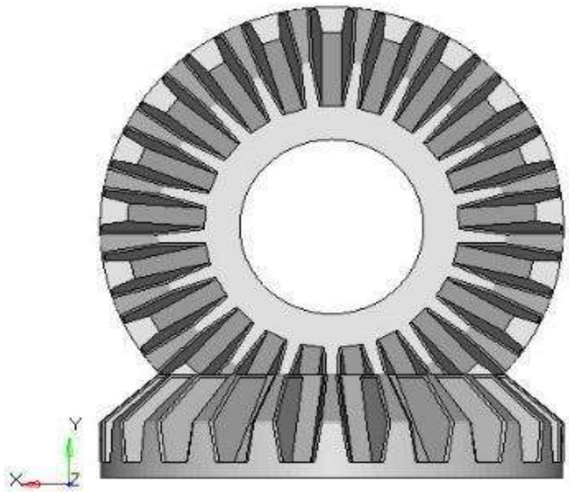

Fig 1. Bevel gear front view

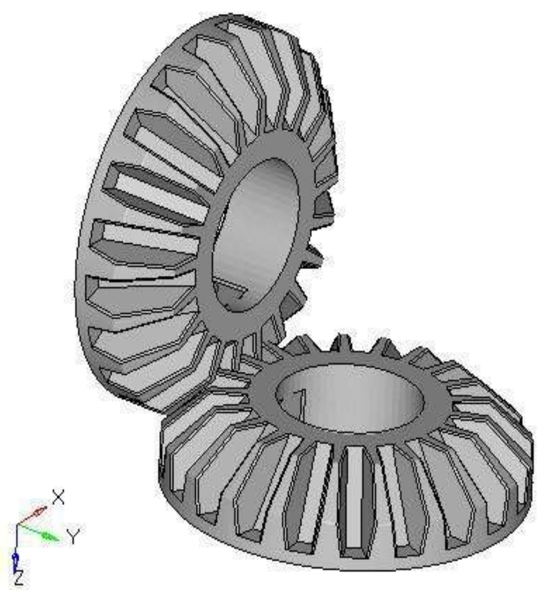

Fig 2. Bevel gear isometric view

TABLE I
BEVEL GEAR SPECIFICATION

Sl No	Parameters of the Gear	Value
1	Number of teeth (Z)	21
2	Pitch diameter (mm)	80
3	Circular Pitch (mm)	11.96
4	Module (mm)	4
5	Pressure Angle (α)	20^0
6	Diametral Pitch $\left(\frac{1}{mm}\right)$	0.2625
7	Addendum(mm)	89.65
8	Dedendum(mm)	68.27
9	Clearance (mm)	1
10	Dedendum angle	5.05^0
11	Gear ratio	1
12	Pitch Angle	45^0
13	Root Angle	39.95^0
14	Face Angle	49.05^0
15	Face width (mm)	10
16	Cone distances (mm)	59.39
17	Teeth thickness (mm)	6.28
18	Addendum angle	4.04^0

B. Modified Bevel Gear

Contact stress of the bevel gear can be reduced by slightly changing the standard teeth dimension and modification of the gear geometry [7]. The bevel gear micro geometry modification is done for the tooth profile at the tip relief. This modification is very important for the proper gear mesh and engagement process, especially when the assembly deflection is significant. For mating pair of teeth under load, it is not possible to have the next tip enter contact in the pure involute position, because there would be sudden interference corresponding to the elastic deflection.

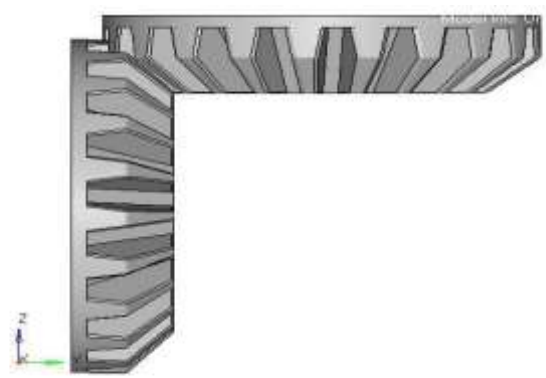

Fig 3. Bevel gear modified design

C. Materials and Their Properties

TABLE II
DIFFERENT MATERIALS AND PROPERTIES

Materials Names	Density (ρ) Ton/mm^3	Young's Modulus (E)N/mm^2	Poisson's Ratio
AlSl 9310 Steel	$8.03*10^{-9}$	206842	0.30
AlSl 1018 Carbon	$7.80*10^{-9}$	205000	0.29
AlSl 9310H	$7.85*10^{-9}$	200000	0.30

D. FE Modeling of Bevel Gear

Element C3D4 is used for 3-D modeling of solid structure. Elements that have nodes only at their corners, such as the 4-node tetra element use linear interpolation in each direction and are often called as linear elements or first order elements. The element is defined with four nodes having three degrees of freedom at each node such as translation in the nodal X, Y, and Z directions. A four noded tetra element is as shown in figure 4.

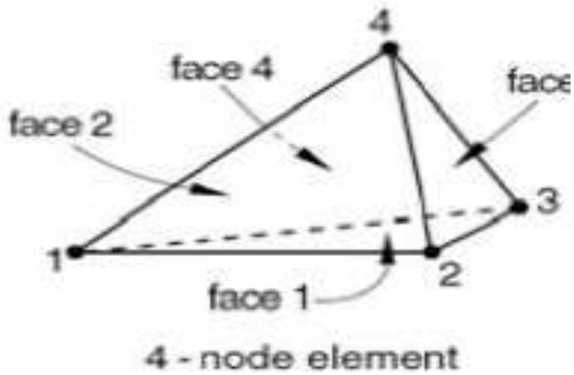

Fig 4. Nodes and face number of the element.

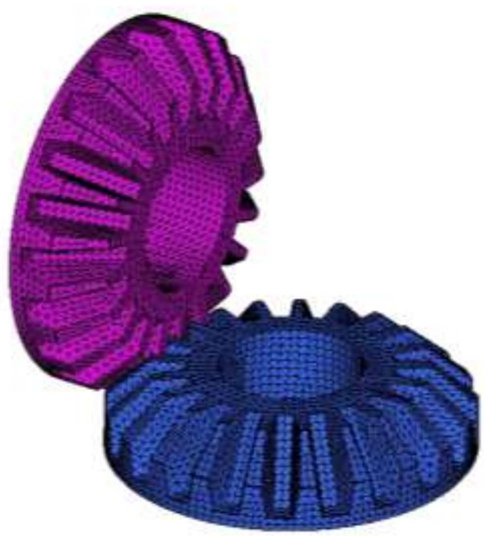

Fig 5. Tetra meshing of gear.

E. Loading and Boundary Conditions

The proper specification of boundary conditions is a very important step involved in the analysis. The improper specification of the boundary conditions leads to incorrect answers. Suitable boundary conditions can be applied based upon the actual state of the system. One gear is constrained in all degree of freedom (blue color gear) and another gear (pink color gear) is constrained in x, y, z, Rx, Ry and Torque of 38 N-m is applied at gear center in Rz direction and the loading and boundary conditions are shown in figure 6.

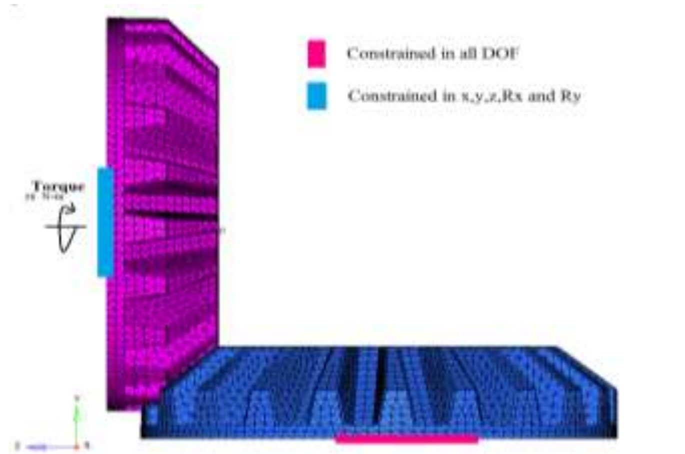

Fig 6. Loading and boundary conditions

V. RESULTS AND DISCUSSIONS

A. Base Design

Figures 7 to 15 show the tooth bending stress, equivalent plastic strain and contact stresses for torque of 38 N-m is applied at gear center.

1) Material (AISI9310 Steel)

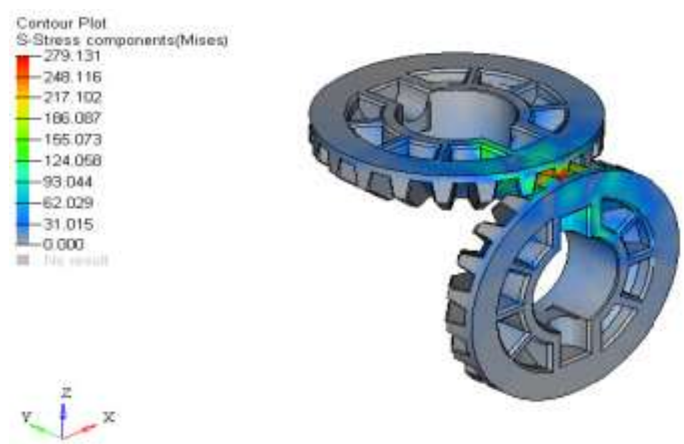

Fig 7. Tooth bending stress distribution before modification.

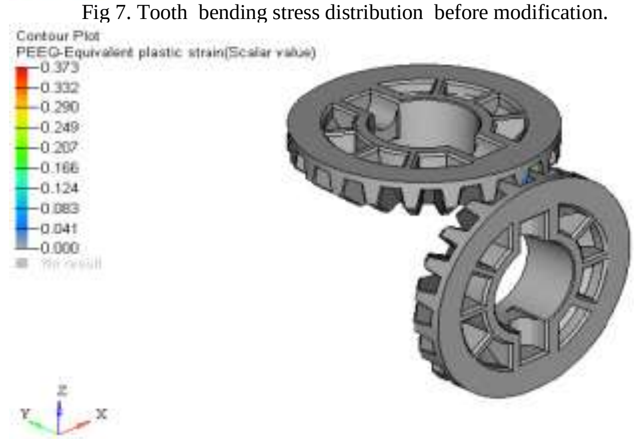

Fig 8. PEEQ before modification

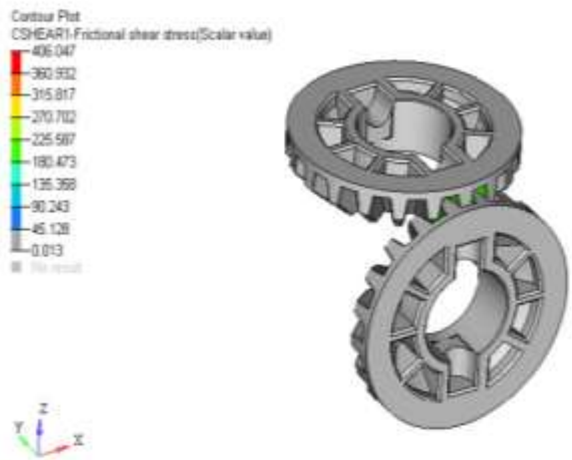

Fig 9. Contact stress before modification

2) Material (AISI 1018 CARBON STEEL)

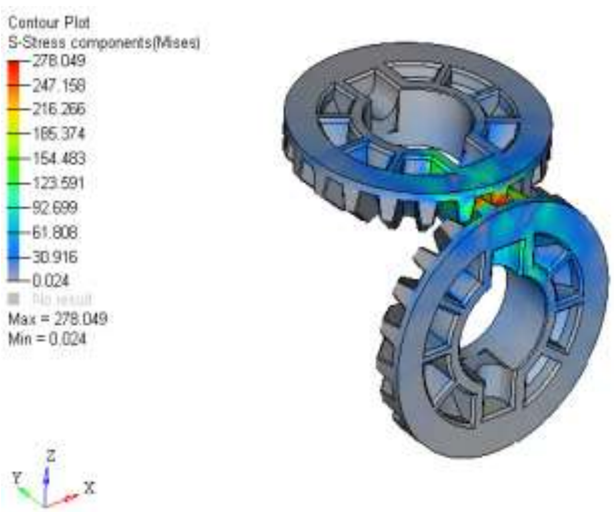

Fig 10. Tooth bending stress distribution before modification.

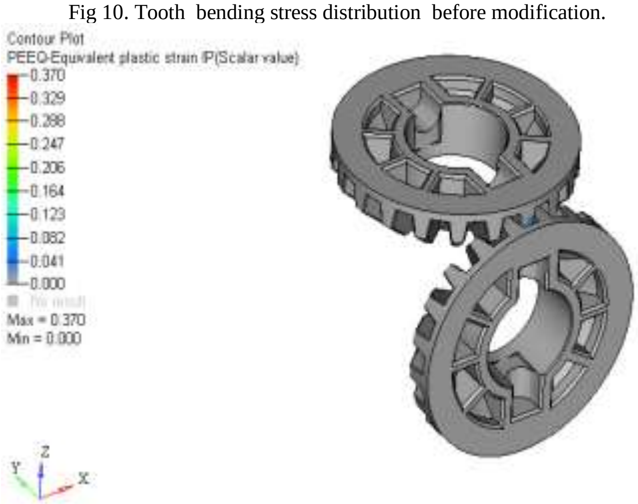

Fig 11. PEEQ before modification

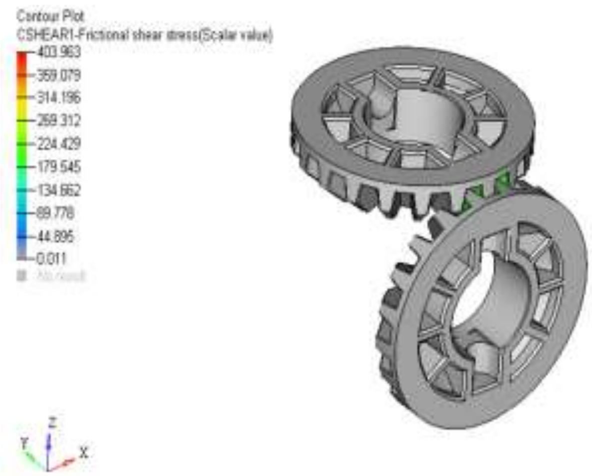

Fig 12. Contact stress before modification

3) Material (AISI E9310H)

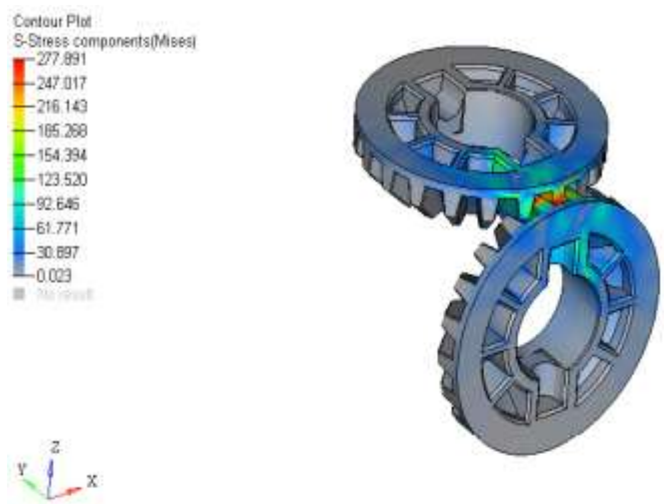

Fig 13. Tooth bending stress distribution before modification.

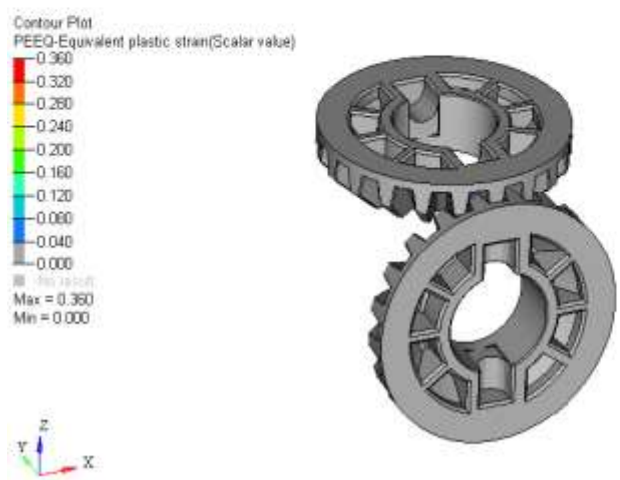

Fig 14. PEEQ before modification

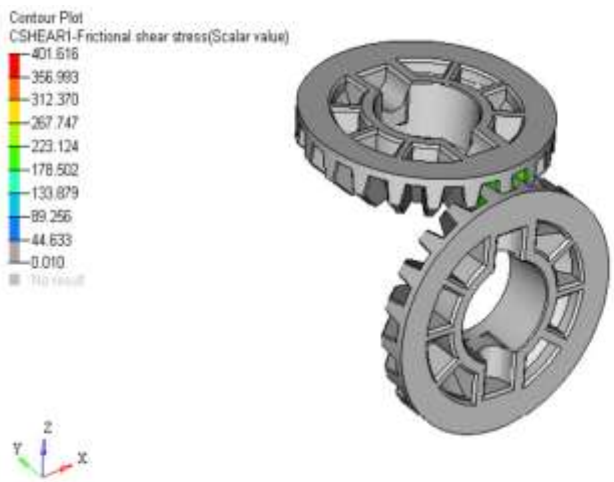

Fig 15. Contact stress before modification

Results for all three materials are shown in the table III.

TABLE III
RESULTS OF THREE DIFFERENET MATERIALS

Material	AISI 1018 Carbon Steel	AISI 9310 Steel	AISI E9310H
Stress (N/mm^2)	278.049	279.131	277.891
PEEQ	0.370	0.373	0.360
Contact Stress (N/mm^2)	403.963	406.047	401.619

From the above table III, it can be concluded that for the base design of the bevel gear, the stress of the tooth yields approximately the same value for all the three materials, But Equivalent Plastic Strain (PEEQ) for material AISI 9310H is less when compared with other two different materials viz., AISI 1018Carbon, AISI 9310 Steel.

B. Modified Design

1) Material (AISI 9310 STEEL)

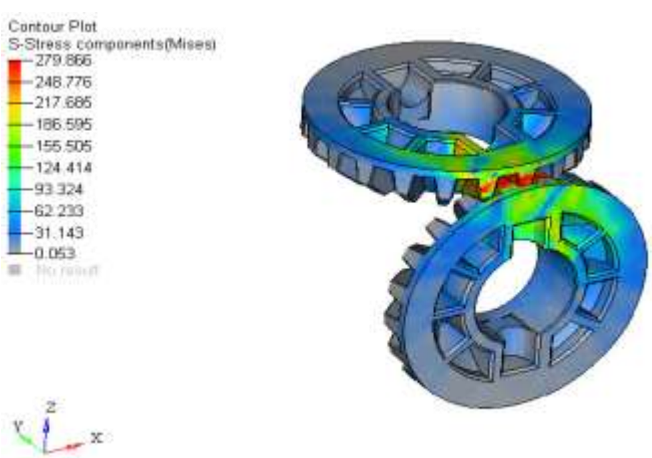

Fig 16. Tooth bending stress distribution of bevel gear after modification

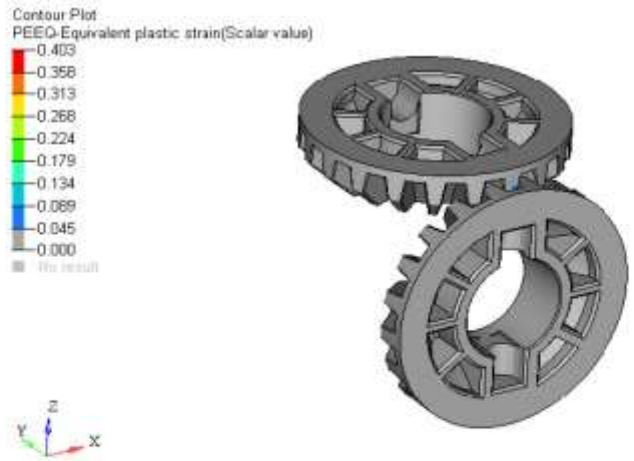

Fig 17. PEEQ of bevel gear after modification

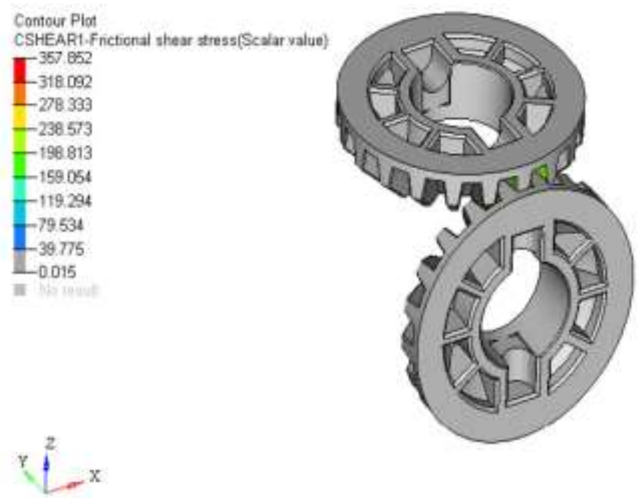

Fig 18. Contact stress distribution of bevel gear after modification

2) Material (AISI 1018 CARBON STEEL)

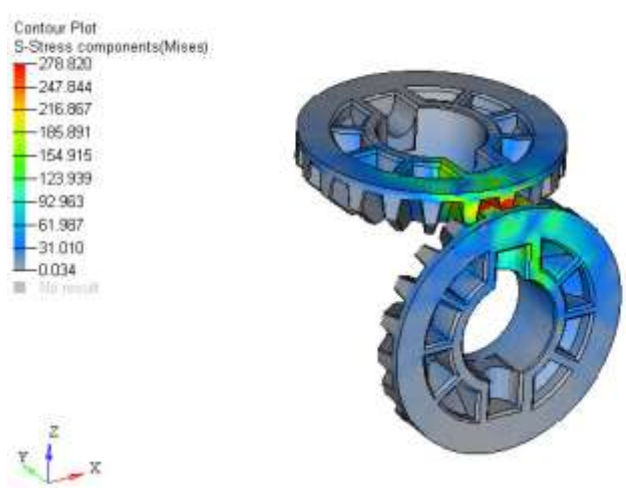

Fig 19. Tooth bending stress distribution of bevel gear after modification

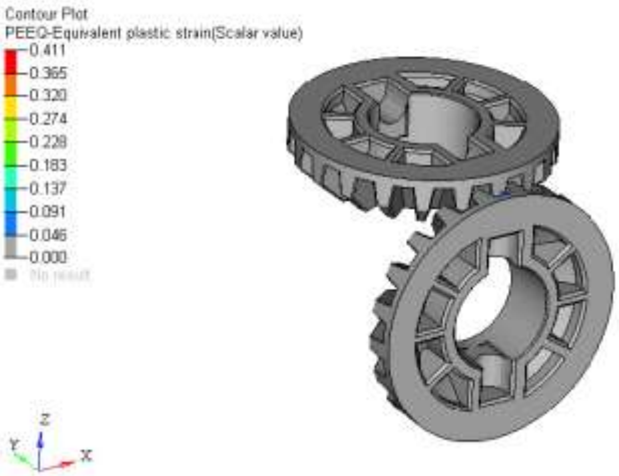

Fig 20. PEEQ of bevel gear after modification

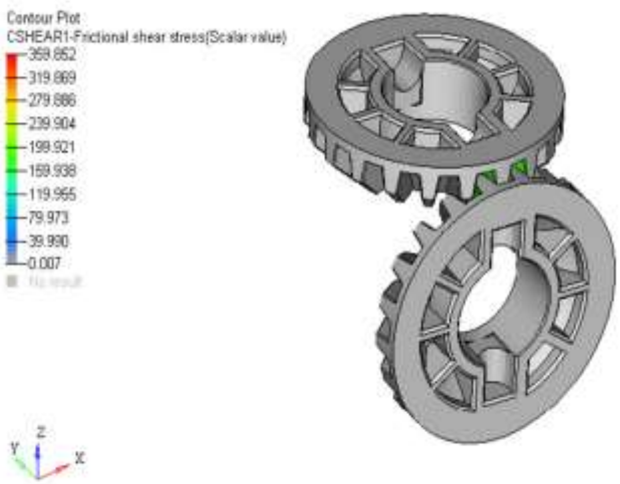

Fig 21. Contact stress distribution of bevel gear after modification

3) Material (AISI E9310H)

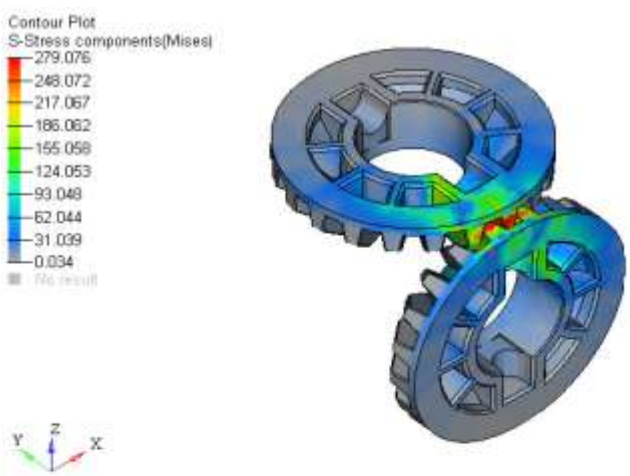

Fig 22. Tooth bending stress distribution of bevel gear after modification

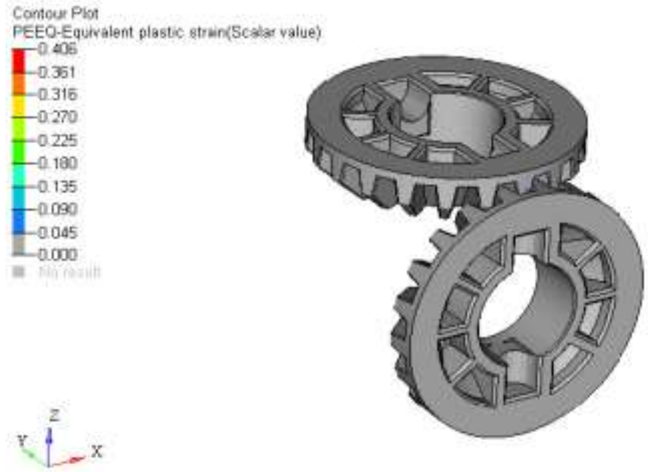

Fig 23. PEEQ of bevel gear after modification

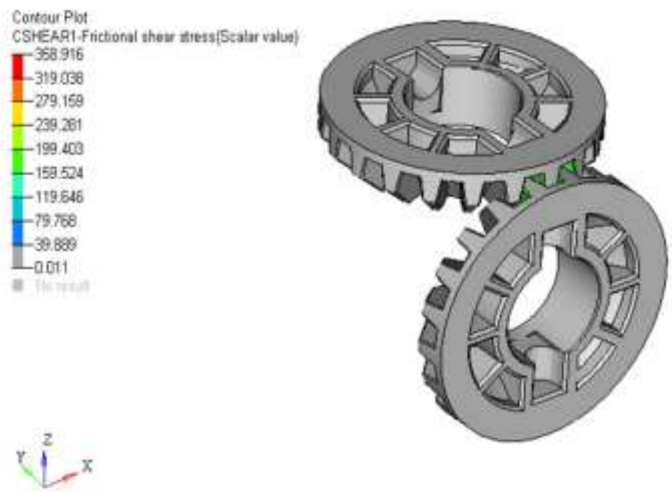

Fig 24. Contact stress distribution of bevel gear after modification

The below tables IV to VI show the results of all materials with base design and modified design

TABLE IV
RESULTS OF MATERIAL AISI 9310STEEL

Description	Base-Design	Modified Design
Stress N/mm^2	279.131	279.866
PEEQ	0.373	0.403
Contact stress (N/mm^2)	406.047	357.852

TABLE V
RESULTS OF MATERIAL AISI 1018 CARBON

Description	Base-Design	Modified Design
Stress (N/mm^2)	278.049	278.820
PEEQ	0.370	0.411
Contact Stress (N/mm^2)	403.963	359.852

TABLE VI
RESULTS OF MATERIAL AISI 9310H

Description	Base-Design	Modified Design
Stress (N/mm^2)	277.891	279.076
PEEQ	0.360	0.406
Contact. Stress (N/mm^2)	401.616	358.916

A. *Results with Graphical Representations*

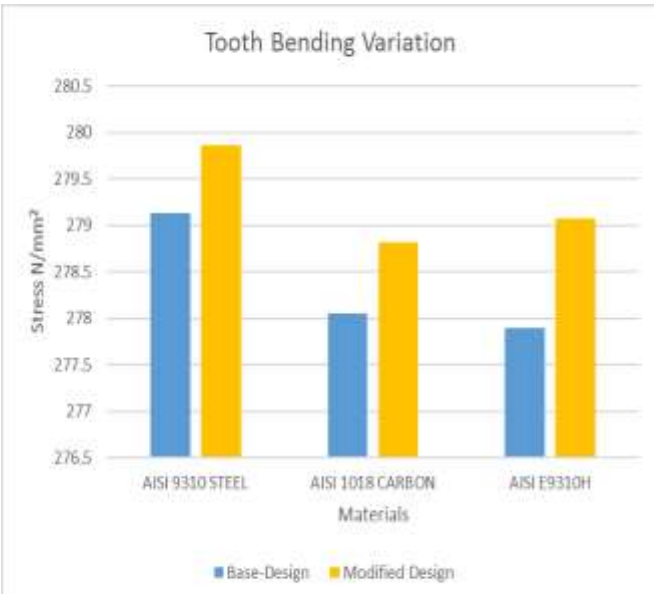

Fig 25. Tooth bending stress variation

Fig 26. PEEQ variation for base design

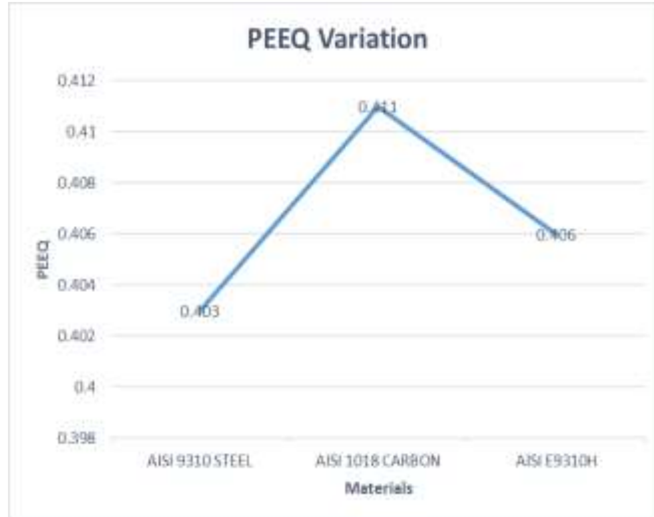

Fig 26. PEEQ variation for modified design

Fig 28. Contact stress variation for base design

Fig 29. Contact stress variation for modified design.

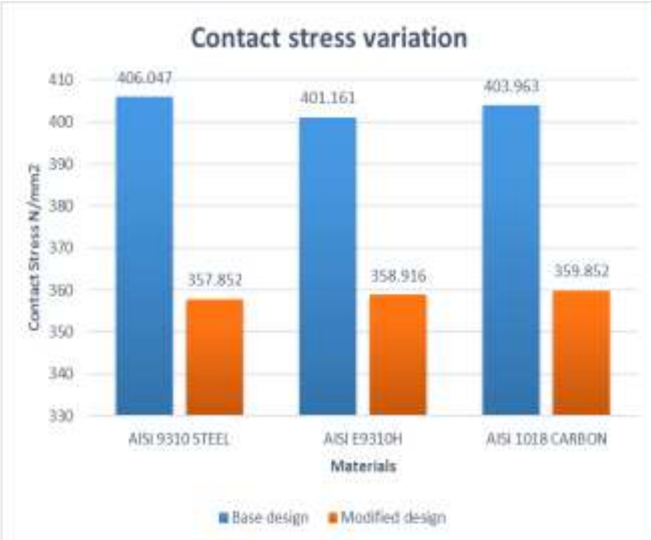

Fig 30. Contact stress variation

VI. THEORETICAL CALCULATIONS

A. Calculations of Contact Stress

Surface fatigue failure in the gear tooth occurs due to many repetitive contact stress at the time of power transmission. If a pair of teeth in contact are subjected to cyclic type of loading, then contact stress are induced on the gear tooth surface, which are higher than the fatigue strength of the gear. This causes the failure of the gear teeth.

In this research the maximum contact stress is calculated theoretically by using Hertz equation.

Hertz equation: - $P_p = \left\{\sqrt{\frac{Ft}{bxd} x \frac{u+1}{u}}\right\}$ x Y_m x Y_p

Where P_p = Contact stress in N/mm^2

F_t = tangential force in N

b = Width of teeth in mm

d = Diameter of the gear in mm

u = Gear Ratio $= \frac{Z_1}{Z_2} = \frac{21}{21} = 1$

Y_m = Material coefficient

Y_p = Pitch point coefficient

For bevel gear $Ft = \frac{\text{Torque}}{(\frac{d}{2})}$

Where T = Torque in N-m,

d = Diameter of the gear

Material coefficient $Y_m = \sqrt{\frac{0.35*2E1E2}{E1+E2}}$

Where E_1 = Young's modulus of gear

E_2 = Young's modulus of pinion

Input Parameters:

Module=m=4mm

Gear ratio=u=1

Number of teeth=Z= 21

Material of Pinion and Gear = AISI 9310 Steel

$E_1=E_2$= 206842.718 N/mm^2

Material coefficient Y_m = 269.063 $\sqrt{N/mm^2}$

Y_p = Pitch point coefficient = 1

Torque (T) = 38N-m

Face width (b) = 10mm

Diameter of the gear (d) = 80mm,

Pressure angle (α) = 20^0

Tangential force $F_t = \frac{\text{Torque}}{(\frac{d}{2})} = \frac{38000}{(\frac{80}{2})}$ = 950 N

F_t = 950 N

Now contact stress $P_p = \left\{\sqrt{\frac{Ft}{b*d} * \frac{u+1}{u}}\right\} * Y_m * Y_p$

$= \left\{\sqrt{\frac{950}{10*80} * \frac{1+1}{1}}\right\} *269.063*1$

$= \{\sqrt{2.375}\}$*269.063*1

P_p = 1.54*269.063*1

Contact Stress P_p = 414.65 N/mm^2

B. Calculations of Tooth Bending Stress

The standard method for determining the bending stresses in bevel gears comes from the American Gear Manufacturers Association and is based on the equation below.

$$\sigma_b = \frac{2*T}{d} X \frac{P_d}{b*J} X \frac{K_a*K_m*K_s}{K_v*K_x}$$

Where

σ_b = Maximum bending stress in tooth (Compare to gear material strength to determine if the gear will break).

K_a = Application factor (Accounts for probability of greater than design load occurrences. This is not something we expect so neglect this and set K_a= 1).

K_v = Dynamic factor (Accounts for dynamic effects and velocity of tooth contact. For static loading, which is an assumption we will make since we are dealing with relatively low speeds, neglect this term and set K_v= 1).

P_d = Diametrical pitch. P = $\frac{Z}{D}$

b = Face width of the teeth.

T = Torque in N-m

K_s= Size factor (Accounts for unusually sized gears. Not applicable for normal gears. Therefore, Set K_s = 1).

K_m=Load distribution factor (Accounts for shaft misalignment and shaft bending. Build your gearbox carefully, so you can neglect this term and set K_m = 1.4).

J = Geometry factor (Similar to Lewi's equation used in spur gear. Obtained from chart based on number of teeth on gear and pinion).

Now input parameters for the calculations are;

Torque (T) = 38 N-m

Face width (b) = 10 mm

Number of teeth Z = 21

Pitch circle diameter =80 mm

$$\text{Diametral pitch} = P_d = \frac{21}{80} = 0.2625\ \frac{1}{mm}$$

From Dr.K. Lingaiah design data handbook [10] equation 23.115 Lewis form factor for 20^0 involute tooth profile

$$y = 0.154 - \frac{0.912}{Z\vartheta} \quad \text{Where } z\vartheta = \frac{Z2}{\cos\delta}$$

Where pitch cone angle $\delta = \tan^{-1}(i)$

$$i = \frac{Z2}{Z1} = \frac{21}{21} = 1$$

Pitch cone angle $\delta = \tan^{-1}(1)$

$$\delta = 45^0$$

$$z\vartheta = \frac{Z}{\cos\delta} = \frac{21}{\cos 45} = 29.698$$

$$z\vartheta = 29.698$$

$$\text{Lewis form factor } y = 0.154 - \frac{0.912}{z\vartheta}$$

$$y = 0.154 - \frac{0.912}{29.698} = \ y = 0.1232$$

$$\text{Tangential force } F_t = \frac{\text{Torque}}{(\frac{d}{2})} = \frac{38000}{(\frac{80}{2})} = 950 \text{ N}$$

$$F_t = 950 \text{ N}$$

Maximum tooth bending stress

$$\sigma_b = \frac{2*T}{d} X \frac{P_d}{b*J} X \frac{K_a*K_m*K_s}{K_v*K_x}$$

$$\sigma_b = \frac{2*38000}{80} X \frac{0.2625}{10*0.1232} X \frac{1*1.4*1}{1*1}$$

$$\sigma_b = 950 \text{ X } 0.213068 \text{ X } 1.4$$

$$\sigma_b = 283.38 \text{ N/mm}^2$$

The Maximum tooth bending stress σ_b = 283.386 N/mm^2

Similarly, for other two materials contact stress and tooth bending stress are calculated and listed in table VII.

TABLE VII

COMPARISION OF THEORETICAL RESULTS AND FEA VALUES

Materials	Stress (N/mm^2)	FEA	Theoretical	% Variation
AISI 9310 STEEL	Contact Stress	406.047	414.640	2%
	Tooth Bending Stress	279.131	283.386	1.5%
AISI 1018 CARBON	Contact Stress	403.963	412.561	2.1%
	Tooth Bending Stress	278.049	283.297	1.8%
AISI E9310H	Contact Stress	401.616	407.720	1.4%
	Tooth Bending Stress	277.891	283.312	1.9%

VII. CONCLUSION

Contact stress of the bevel gear can be reduced by slightly changing the standard teeth dimension and modification of the gear geometry. For the design changes made for the material AISI 9310 Steel, base design yields less stress when compared with the modified design. For the base design with AISI 9310H material yields less Equivalent Plastic Strain (PEEQ) when compared with the other two materials considered. Gear works without any noise at high speed because of its curvature and thereby reducing the surface pitting. There is fairly good agreement between theoretical and finite element results.

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