A BACKSTEPPING CONTROLLER FOR MAGNETIC LEVITATED BALL SISO SYSTEM AND INVERTED PENDULUM MIMO SYSTEM USING KALMAN FILTER APPROACH

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Abstract— In this paper, the nonlinear backstepping design controller with a velocity observer is proposed for trajectory tracking control of magnetic ball and stabilized the inverted pendulum, where only the position measurement is available for control. Therefore, a k-filter, a velocity observer is employed to compensate the external disturbance and model mismatch and estimate the unknown state variables of the system. The inverted pendulum is a highly nonlinear and open-loop unstable system. This means that standard linear techniques cannot model the nonlinear dynamics of the system therefore backstepping technique is used to control the inverted pendulum. The design uses the backstepping nonlinear control to make the ball able to track an output signal \( u \), a vertical position from some reference point, to a continuous twice differentiable positive reference signal \( x_r \), asymptotically. Furthermore, some simulation results are given to illustrate the excellent performance of the backstepping control design scheme applied to SISO and MIMO systems.

Index Terms— Magnetic Levitation System, Inverted Pendulum, Kalman Filter (K-filter), Backstepping Control Design

I. INTRODUCTION

Magnetic levitation (Maglev) systems suspend objects without mechanical contact. They have been widely used in various applications such as in magnetic bearings, high-speed trains, aerospace shuttles, and levitation of wind power generation [1,3]. Maglev systems are inherently unstable and uncertain nonlinear dynamical systems. Therefore, it is very challenging in order to construct the high performance feedback controllers to regulate the position of the levitation ball rapidly and exactly [4,5]. Recently, Extensive work has been reported for the nonlinear control schemes for a Maglev system. And some studies have shown that nonlinear control has provided a better transient response than the linear control. It is found that the performance of the SMC is better than that of the conventional controllers [2].

II. RESEARCH METHODOLOGY

Case: 1 Magnetic Levitated ball SISO System

A. Dynamic Model Analysis

The magnetic levitation system experiment is a magnetic ball suspension system which is used to levitate a steel ball on air by the electromagnetic force generated by an electromagnet. The ball position can be controlled by adjusting the current through the electromagnet.

The model of the magnetic levitation system given as

\[
M \ddot{x} = Mg - \frac{c}{x^2} f^2
\]  

Fig.1 Magnetic Levitation System

B. Backstepping Design

Considering the magnetic ball nonlinear model (1), taking input \( u = f^2 \) and \( x_r \) as the output of the reference model, we define the ball position error as \( e = x - x_r \) and take the following coordinates:

\[
x_1 = x
\]
\[
x_2 = \dot{x}_1 = \dot{x}
\]

Hence (1) can be written as:

\[
\dot{x}_1 = x_2
\]
\[
\dot{x}_2 = g - \frac{c}{Mx_2^2} u
\]

\[
y = x_1
\]

Step 1: Let us study the following subsystem

\[
\dot{z}_1 = x_2
\]

the coordinates:

\[
z_1 = e = x_1 - x_r
\]
\[
z_2 = x_2 - \Theta (z_1, \dot{x}_r)
\]

Where the function \( \Theta \) is referred to as intermediate control function which will be designed later using an appropriate Lyapunov function, while \( z_t \) is just course tracking error.
In light of (6.1), equation (5) becomes:
\[ \dot{z}_2 = x_2 - \dot{x}_r \]  
(7)
where \( x_2 \) is taken as a virtual control input.

The first step of backstepping is the definition of a Lyapunov function candidate. Consider a Lyapunov function candidate \( \psi_2 \)
\[ \psi_1 (z_1) = \frac{1}{2} z_1^2 \]  
(8)
Let the intermediate control function \( \psi \) be
\[ \varpi (z_1, \dot{x}_r) = \dot{x}_r - k_2 z_1 \]  
(9)
where the design constant \( k_2 \) will be chosen later.
\[ \psi_1 (z_1) = z_2 z_2' - k_1 z_1^2 \]  
(10)
the stability of \( \psi_2 \) depends on \( z_2 \) will be dealt at the second step.

Step 2: In light of (6.2), the time derivative of the second backstepping variable \( z_2 \) is defined by
\[ \dot{z}_2 = g - \frac{c}{Mz_2^2} u - \dot{x}_r + c_1 (z_2 - c_1 z_1) \]  
(11)
Construct the second Lyapunov function \( \psi_2 \)
\[ \psi_2 (z_1, z_2) = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \]  
(12)
In order to make \( \psi_2 \leq 0 \), we design the controller
\[ u(x) = \frac{Mz_2^2}{c} (g + (k_1 + k_2) (x_2 - \dot{x}_r) - \dot{x}_r + (1 + k_2 k_1) (x_1 - \dot{x}_r)) \]  
(13)

**Case2: Inverted Pendulum MIMO system**

After neglecting the air resistance and all kinds of friction, the linear inverted pendulum system can be abstracted to the system which is composed of the car and the homogeneous pendulum. It is shown in Fig 2. Regarding the pendulum as rigid body, the parameters of the linear inverted pendulum system are as follows: \( M \)—the quality of car, \( m \)—the quality of the pendulum, \( l \)—the length from the gravity center of pendulum to the hinge, \( g \)—acceleration of gravity, \( x \)—the position of car, \( \theta \)—the angle of pendulum, \( F \)—the driving forces to the car.

![Mechanical Schematic Diagram of the Inverted Pendulum System](image)

For the convenience of design by simplifying two equations, the mathematical model of linear inverted pendulum system is described as below:
\[ \begin{align*}
\dot{\theta} &= \frac{(M + m) g \sin \theta - (F + ml \theta^2 \sin \theta) \cos \theta}{\frac{4}{3} (M + m) l - ml \cos^2 \theta} \\
\dot{x} &= \frac{F + ml (\theta^2 \sin \theta - \theta \cos \theta)}{M + m}
\end{align*} \]  
(14)

**B Backstepping Controller Design**

In view of the above inverted pendulum system (To be) \( x_1 = \theta \), \( x_2 = \dot{\theta} \), \( x_3 = x \), \( x_4 = \dot{x} \), \( M=2.0 \text{kg} \), \( m=8.0 \text{kg} \), \( l=0.5 \text{m} \), \( g=9.8 \text{m/s}^2 \). Firstly the dynamic model of pendulum subsystem is studied.

![Mechanical Schematic Diagram of the Inverted Pendulum System](image)

The backstepping is one of the most important results, which provides a powerful design tool, for nonlinear system in the pure feedback and strict feedback form. Unfortunately, its application fails for systems which do not appear (or not transformable in either of the above two forms). The backstepping technique cannot application for the system (15) because this system is not appearing in either of the above two forms. To overcome the fact that the system cannot be rewritten in such a triangular form we can convert it into vector form, then rewrite in triangular form.

Let \( X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \) and \( X_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \), and
\[ a = 20 - 12 \cos^2 x_3 \quad b = 3 \cos x_3 \quad c = 294 \sin x_3 - 6x_3^2 \sin(2x_3) \]  
then from system (15) given as:
\[ \begin{align*}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= f(X_1, X_2) + g(X_1, X_2) u
\end{align*} \]  
(16)
Where \( f \), \( g= f(X_1, X_2) = \left[ \begin{array}{c}
\left[ c/a; 0.4x_3^2 \sin x_3 - 0.4 \cos x_3 (c/a) \right]
\end{array} \right] \\
g=[a-b/a 0; 0 2]
\]

(1) Coal The stabilization of \( z_1 \) steps
a) To be \( z_1 = X_1, \dot{z}_1 = X_1 \)
b) Constructing lyapunov function.

\[ \dot{V}_1(z_1) = (1/2)z_1^2, \quad \text{then} \quad \dot{V}_1(z_1) = z_1 \dot{z}_1 = z_1 X_2 \]

\[ \dot{V}_1(z_1, z_2) = (1/2)z_2^2, \quad \text{then} \quad \dot{V}_1(z_1, z_2) = \dot{z}_2^2 + z_2 \dot{z}_2 \]

c) Taking \( p(X_t) = -k_1 z_1, k_1 > 0, k_1 > 0 \) is a constant to design.

d) Introducing error variance \( z_2 \) to be

\[ z_2 = X_2 - \varphi(X_t) \]

e) Then, \( \dot{z}_1 = -k_1 z_1 + z_2 \)

\[ \dot{z}_1 = -k_1 z_1^2 + z_2 \]  \( \text{(18)} \)

\[ \dot{z}_2 = f(X_1, X_2) + g(X_1, X_2) u + k_1 (-k_1 z_1 + z_2) \]  \( \text{(20)} \)

a) Constructing function

\[ \dot{V}_1(z_1, z_2) = (1/2)z_1^2 + (1/2)z_2^2, \quad \text{then} \quad \dot{V}_1(z_1, z_2) = -k_1 z_1^2 + z_2 \dot{z}_2 \]

d) To be

\[ z_1 + \frac{f(X_1, X_2) + g(X_1, X_2) u + k_1 (-k_1 z_1 + z_2)}{k_2} = -k_2 z_2 \]  \( \text{(22)} \)

In which \( k_2 > 0 \), and it is a design constant. The control input of the system is obtained by formula (22).

\[ u(x) = g^{-1}(-k_2 z_2 - z_1 - f(X_1, X_2) - k_1 (-k_1 z_1 + z_2)) \]  \( \text{(23)} \)

c) Being substituted to formula (21), then

\[ \dot{V}_1(z_1, z_2) = -k_1 z_1^2 - k_2 z_2^2 < 0 \]

Consequently that subsystem \( z_1 \) and \( z_2 \) are stabilized.

Substituting \( z_1 = X_1 \) and \( z_2 = X_2 + k_2 X_1 \) into formula (23), getting the control input of the system (115)

\[ u(x) = g^{-1}(-f(X_1, X_2) - (k_1 + k_2) X_2 - (1 + k_2 k_1) X_1) \]  \( \text{(24)} \)

The controller can control the balance of the pendulum while tracking the location of car.

C) KF-Based Estimation of Model Parameters

Recently, researchers are focusing on the sequential estimation and its applications on active modeling and model-reference control [8]. In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete data linear filtering problem [10]. The Kalman filter [9] is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance—when some presumed conditions are met.

**Standard K-Filter Approach**

Consider a discrete-time nonlinear dynamics system:

\[ x_k = A x_{k-1} + B u_k + w_{k-1} \]

\[ z_k = H x_k + v_k \]

Where \( x_k \in \mathbb{R}^n \) is system state vector; \( z_k \in \mathbb{R}^m \) is the output vector; \( H \) is the input vector. The random variables \( w_k \) and \( v_k \) represent the process and measurement noise (respectively).

Definition of standard K-Filter:

a) Initialization:

\[ \hat{x}_0 = E x_0 \]

\[ P_0 = E (x_0 - \hat{x}_0) (x_0 - \hat{x}_0)^T \]  \( \text{(26)} \)

b) Time update:

(1) Project the state ahead

\[ \hat{x}_k = A \hat{x}_{k-1} + B u_k \]

(2) Project the error covariance ahead

\[ P_k^- = A P_{k-1} A^T + Q \]

Where \( Q \) is process noise covariance and \( P_k \) is error covariance.

c) Measurement update:

(1) Compute the Kalman gain

\[ K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \]

(2) Update estimate with measurement \( z_k \)

\[ \hat{x}_k = \hat{x}_{k-1} + K_k (z_k - H \hat{x}_k) \]

(3) Update the error covariance

\[ P_k = (I - K_k H) P_k^- \]  \( \text{(28)} \)

Where \( R \) is measurement noise covariance and the difference \( (z_k - \hat{x}_k^{-1}) \) in equation (28) is called the measurement innovation, or the residual. The matrix \( n \times m \) \( K \) in equation (25) is chosen to be the gain or blending factor that minimizes the error covariance.

**IV Simulation Results**
Fig. 1 (a) The true, measured and estimated position of the ball.

Fig. 1 (b) True and estimated velocity of the ball.

Fig. 2 The true, measured and estimated angular position of pendulum.

Fig. 3 (a) System output response of magnetic levitated ball position with reference ball position.

Fig. 3 (b) The error between measured & desired response

Fig. 4 Output response with observer based controller for magnetic levitated ball

Fig. 5 Controller output response for pendulum and cart position

Fig. 6 Observer based controller output response for pendulum and cart position
The controller outputs are shown in above figures for both systems. The systems outputs are controlled after applying backstepping controller for both systems.

IV CONCLUSION

The backstepping technique design method has been successfully applied to a wide variety of nonlinear and linear systems. The feature of backstepping designs is that they do not force the designed system to appear linear, which can avoid cancellations of useful nonlinearities. In practice, all state variables are rarely available for direct on-line measurement. In most cases, there is a substantial need for a reliable estimation of the unmeasurable state variables.

REFERENCES


