# CERTAIN DOUBLE INTEGRAL RELATIONS INVOLVING THE PRODUCT OF SOME TRANSCEDENTAL FUNCTIONS

**BY**

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**Abstract**

The object of the present paper is to obtain few double integral relations involving the product of Fox's H-function, the multivariable H-function and the generalized polynomials. These integral relations are then applied to evaluate certain double integrals in terms of the multivariable H-function using Mellin Integral Transforms. Some interesting cases have also been discussed by proper choice of parameters.

**Key words and phrases:** Fox's H-Function, The multivariable H-Function, generalized polynomials, Mellin Integral Transform, G-function of several Complex variable, Lauricella's Function, Kampe de Feriet Function.

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## 1. Introduction

The general multivariable polynomials defined by Srivastava [10,p. 185,eq.(7)] represented in the following manner

$$S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}\left[x_1,\ldots,x_R\right]=\sum_{t_1=0}^{[N_1/M_1]}\sum_{t_R=0}^{[N_R/M_R]}\frac{(-N_1)_{M_1 t_1}}{t_1!}\cdots\frac{(-N_R)_{M_R t_R}}{t_R!}$$

$$.A\ \left[N_1,t_1;\ldots;N_R,t_R\right]\ x_1^{t_1}\ldots x_R^{t_R}, \qquad (1.1)$$

where $N_i=0,1,2,\ldots\ ;\qquad M_i\neq 0\quad \forall i\in\{1,\ldots,R\},$

$M_i$ is an arbitrary positive integer. The coefficients $A[N_1,t_1;\ldots;N_R,t_R]$ being arbitrary coefficients, real or complex.

The series representation of Fox's H- function is given by Braaksma

([1] and [8]) is defined as follows:

$$H^{M,N}_{P,Q}\left[z\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]=\sum_{G=0}^{\infty}\sum_{g=1}^{M}\frac{(-1)^G\,\Phi(\eta_G)\;z^{\eta\;G}}{G!\;F_g} \tag{1.2}$$

where

$$\Phi(\eta_G)=\frac{\prod\limits_{\substack{j=1\\j\neq g}}^{M}\Gamma\left(f_j-F_j\eta_G\right)\prod\limits_{j=1}^{N}\Gamma(1-e_j+E_j\eta_G)}{\prod\limits_{j=M+1}^{Q}\Gamma\left(1-f_j+F_j\eta_G\right)\prod\limits_{j=N+1}^{P}\Gamma(e_j-E_j\eta_G)}\quad, \tag{1.3}$$

and

$$\eta_G=\frac{f_g+G}{F_g} \tag{1.4}$$

For the H-function of several complex variables defined by Srivastava and Panda [(9) , p.251].

For the sake of brevity, we use the following notations through this paper:

$$\Omega=\frac{\pi}{2^{2\lambda+2}}\sum_{t_1=0}^{[N_1/M_1]}\cdots\sum_{t_R=0}^{[N_R/M_R]}\frac{(-N_1)_{M_1t_1}}{t_1!}\cdots\frac{(-N_R)_{M_Rt_R}}{t_R!}$$

$$A[N_1,t_1;\ldots;N_R,t_R]\;\;\mu_1^{t_1}\ldots\mu_R^{t_R}\sum_{g=1}^{M_1}\sum_{G=0}^{\infty}\frac{(-1)^G\,\Phi(\eta_G)\alpha_1^{\eta\;G}}{G!F_g\;2^{2\sigma\;\eta\;G}}, \tag{1.5}$$

$$\Psi=\frac{\pi\left(\xi\alpha_2^{\;\beta/\xi}\right)^{-1}}{2^{2\lambda+2}}\sum_{t_1=0}^{[N_1/M_1]}\cdots\sum_{t_R=0}^{[N_R/M_R]}\frac{(-N_1)_{M_1t_1}}{t_1!}\cdots\frac{(-N_R)_{M_Rt_R}}{t_R!}$$

$$A[N_1,t_1;\ldots;N_R,t_R]\mu_1^{t_1}\ldots\mu_R^{t_R}\sum_{g=1}^{M_1}\sum_{G=0}^{\infty}\frac{(-1)^G\,\Phi(\eta_G)\alpha_1^{\eta\;G}}{G!F_g\;2^{2\sigma\;\eta\;G}} \tag{1.6}$$

## 2. The Main Integrals

(i) $$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{(u^2+v^2)^\lambda}\cos\left(2y\tan^{-1}v/u\right)S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{(u^2+v^2)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{(u^2+v^2)^{l_R}}\right]H_{P_1,Q_1}^{M_1,N_1}\left[\frac{\alpha_1 u^{2\sigma}}{(u^2+v^2)^{\sigma}}\middle|\begin{matrix}e_{P_1},E_{P_1}\\ f_{Q_1},F_{Q_1}\end{matrix}\right]$$

$$H_1\left[\begin{matrix}x_1u^{2\varepsilon}\left(u^2+v^2\right)^{k_1-\varepsilon}\\ x_2\left(u^2+v^2\right)^{k_2}\\ \\ x_r\left(u^2+v^2\right)^{k_r}\end{matrix}\right]f\left(u^2+v^2\right)\ dudv$$

$$=\Omega\int_0^\infty H_{A,C:\ [B'+1,D'+2]\ ;\ [B'',D'']\ ;\ldots;\ \left[B^{(r)},D^{(r)}\right]}^{0,0:\ (u',v'+1)\ ;\ (u'',v'')\ ;\ldots;\ \left(u^{(r)},v^{(r)}\right)}$$

$$\left[\begin{matrix}\left[(a):\theta',\ldots,\theta^{(r)}\right]:[-2\lambda-2\sigma\eta_G-\sum_{j=1}^{R}l_jt_j;2\varepsilon]\ ;\left[(b');\Phi'\right]\ ;\\ \left[(c):\Psi',\ldots,\Psi^{(r)}\right]:\left[(d'):\delta'\right]\ ;\ [-\lambda-\sigma\eta_G-\sum_{j=1}^{R}l_jt_j\pm y:\varepsilon]\ ;\end{matrix}\right.$$

$$\left.\begin{matrix}\left[(b''):\Phi''\right]\ ;\ldots:\ \left[\left(b^{(r)}\right):\Phi^{(r)}\right]\ ;\\ \left[(d''):\delta''\right]\ ;\ldots;\ \left[\left(d^{(r)}:\delta^{(r)}\right)\right]\ ;\end{matrix}\ x_1z^{k_1}4^{-\varepsilon},x_2z^{k_2},\ldots,x_rz^{k_r}\right]$$

$$f(z)dz, \tag{2.1}$$

(ii) $$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{(u^2+v^2)^\lambda}\cos\left(2y\tan^{-1}v/u\right)S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{(u^2+v^2)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{(u^2+v^2)^{l_R}}\right] H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1 u^{2\sigma}}{(u^2+v^2)^{\sigma}}\middle|\begin{matrix}(e_{P_1},E_{P_1})\\(f_{Q_1},F_{Q_2})\end{matrix}\right]$$

$$H_1\begin{bmatrix} x_1 u^{-2\varepsilon}(u^2+v^2)^{k_1+\varepsilon} \\ x_2 (u^2+v^2)^{k_2} \\ x_r (u^2+v^2)^{k_r}\end{bmatrix} f(u^2+v^2)\,du\,dv$$

$$=\Omega\int_0^\infty H^{0,0:(u'+1,v')\ ;(u'',v'');\ldots;(u^{(r)},v^{(r)})}_{A,C[B'+2,D'+1]\ ;[B'',D''];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b')\ ;\Phi']\ \ ;[1+\lambda+\sigma\eta_G+\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon]\ \ ; \\ [(c):\Psi',\ldots,\Psi^{(r)}]:[1+2\lambda+2\sigma\eta_G+\sum_{j=1}^{R} l_j t_j :\varepsilon];[(d'):\delta']\ \ ;\end{matrix}\right.$$

$$\left.\begin{matrix}\ldots;[(b^{(r)}):\Phi^{(r)}]\\ \ldots;[(d^{(r)}):\delta^{(r)}]\end{matrix}\ x_1 z^{k_1} 4^{\varepsilon}, x_2 z^{k_2},\ldots,x_r z^{k_r}\right] f(z)\,dz, \tag{2.2}$$

(iii) $$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{(u^2+v^2)^{\lambda}}\cos\left(2y\tan^{-1} v/u\right) S^{M_1,\ldots,M_R}_{N_1,\ldots,N_R}\left[\frac{\mu_1 u^{2l_1}}{(u^2+v^2)^{l_1}},\ldots,\right.$$

$$\left.\frac{\mu_R u^{2l_R}}{(u^2+v^2)^{l_R}}\right] H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1 u^{2\sigma}}{(u^2+v^2)^{\sigma}}\middle|\begin{matrix}e_{P_1},E_{P_1}\\ f_{Q_1},F_{Q_1}\end{matrix}\right]$$

$$H_1\begin{bmatrix} x_1 u^{2\varepsilon_1}(u^2+v^2)^{k_1-\varepsilon_1} \\ \\ x_r u^{2\varepsilon_2}(u^2+v^2)^{k_r-\varepsilon_r}\end{bmatrix} f(u^2+v^2)\ \ du\,dv$$

$$= \Omega . \int_0^\infty H^{0,1\,:\,(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+1\,[B',D'];\ldots;[B^{(r)},D^{(r)}]} \left[ \begin{matrix} [-2\lambda - 2\sigma\eta_G - 2\sum_{j=1}^{R} l_j t_j : \\ [-\lambda - \sigma\eta_G - \sum_{j=1}^{R} l_j t_j \pm y : \end{matrix} \right.$$

$$2\varepsilon_{1,\ldots,}2\varepsilon_r]; [(a):\theta',\ldots,\theta^{(r)}]; [(b'):\Phi'];\ldots;[(b^{(r)}):\Phi^{(r)}];$$
$$\varepsilon_{1,\ldots,}\varepsilon_r]; \; [(c):\Psi',\ldots,\Psi^{(r)}];[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];$$

$$\left. \begin{matrix} x_1 z^{k_1} 4^{-\varepsilon_1} \\ x_r z^{k_r} 4^{-\varepsilon_r} \end{matrix} \right] f(z) dz, \tag{2.3}$$

where

$$y = 0,1,2,\ldots; \;\; \lambda > 0$$

$$l_j > 0, \;\; u_j > 0 \;\; \forall j = 1,\ldots,R \;\; ; \;\; \alpha_1 > 0 \;\; ; \;\; \sigma > 0 \;\; ; \;\; k_i > 0 \;\; \forall i = 1,\ldots,r;$$

i >0 and j >0,where we have chosen f(z) such that the integrals (2.1) through (2.3) exist.

**3. Proof**

To prove(2.1) through (2.3), we use the same technique as used by Chaurasia and Olkha [4,p.82-84].

**4. Application**

If we set

$$f(z) = z^{\beta-1} H^{M_2,0}_{P_2,Q_2} \left[ \alpha_2 z^{\xi} \middle| \begin{matrix} e_{P_2}, E_{P_2} \\ f_{Q_2}, F_{Q_2} \end{matrix} \right], \tag{4.1}$$

in the integrals (2.1) through (2.3) and evaluate the z-integral by means of the known result [7,p.122 (4.2)] we have the following double integral relations:

(i) $$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda-\beta+1}} \cos\left(2y\tan^{-1} v/u\right) \; S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}\left[\frac{\mu_1 u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right]$$

$$H_{P_1,Q_1}^{M_1,N_1}\left[\frac{\alpha_1 u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\middle|\begin{matrix}(e_{P_1},E_{P_1})\\(f_{Q_1},F_{Q_1})\end{matrix}\right] \quad H_{P_2,Q_2}^{M_2,0}\left[\alpha_2\left(u^2+v^2\right)^{\xi}\middle|\begin{matrix}(e'_{P_2},E'_{P_2})\\(f'_{Q_2},F'_{Q_2})\end{matrix}\right]$$

$$H_1\begin{bmatrix} x_1 u^{2\varepsilon}\left(u^2+v^2\right)^{k_1-\varepsilon}\\ x_2\left(u^2+v^2\right)^{k_2}\\ \\ x_r\left(u^2+v^2\right)^{k_r}\end{bmatrix} du\,dv$$

$$=\Psi\cdot H_{A+Q_2,C+P_2:[B'+1,D'+2]\;;\;[B'',D'']\;;\ldots;\left[B^{(r)},D^{(r)}\right]}^{0,M_2:(u',v'+1)\;;\;(u'',v'');\ldots;\left(u^{(r)},v^{(r)}\right)}$$

$$\left[\begin{matrix}[1-f_j'-F_j'\,\beta/\xi : F_j'\,k_1/\xi,\ldots F_j'\,k_r/\xi]_{1,Q_2} : [(a):\theta',\ldots\theta^{(r)}]:\\ [1-e_j'-E_j'\,\beta/\xi : E_j'\,k_1/\xi,\ldots E_j'\,k_r/\xi]_{1,P_2} : [(c):\Psi',\ldots\Psi^{(r)}]:\end{matrix}\right.$$

$$[-2\lambda-2\sigma\eta_G-2\sum_{j=1}^{R} l_j t_j : 2\varepsilon];[(b'):\Phi'];\ldots;[(b^{(r)}):\Phi^{(r)}];$$

$$[(d'):\delta'];[-\lambda-\sigma\eta_G-\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon];\ldots;[(d^{(r)}):\delta^{(r)}];$$

$$\left. x_1\alpha_2^{-k_1/\xi}4^{-\varepsilon}, x_2\alpha_2^{-k_2/\xi},\ldots x_r\alpha_2^{-k_r/\xi}\right], \tag{4.2}$$

valid under the same conditions obtainable from (2.1)

(ii) $$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda-\beta+1}} \cos\left(2y\tan^{-1} v/u\right) S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right] H_{P_1,Q_1}^{M_1,N_1}\left[\frac{\alpha_1 u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\middle|\begin{matrix}(e_{P_1},E_{P_1})\\(f_{Q_1},F_{Q_1})\end{matrix}\right]$$

$$H_{P_2,Q_2}^{M_2,0}\left[\alpha_2\left(u^2+v^2\right)^{\xi}\left|\begin{matrix}(e'_{P_2},E'_{P_2})\\(f'_{Q_2},F'_{Q_2})\end{matrix}\right.\right]H_1\begin{bmatrix}x_1u^{-2\varepsilon}\left(u^2+v^2\right)^{k_1+\varepsilon}\\x_2\left(u^2+v^2\right)^{k_2}\\ \\x_r\left(u^2+v^2\right)^{k_r}\end{bmatrix}dudv$$

$$=\Psi\, H_{A+Q_2,C+P_2:\ [B'+1,D'+1];\ldots;[B^{(r)},D^{(r)}]}^{0,M_2:\ (u'+1,v');(u'',v'');\ldots;(u^{(r)},v^{(r)})}$$

$$\left[\begin{matrix}[1-f_j'-F_j'\,{}^{\beta}/_{\xi}:F_j'\,{}^{k_1}/_{\xi},\ldots F_j'\,{}^{k_r}/_{\xi}]_{1,Q_2}:[(a):\theta',\ldots\theta^{(r)}]:\\ [1-e_j'-E_j'\,{}^{\beta}/_{\xi}:E_j'\,{}^{k_1}/_{\xi},\ldots E_j'\,{}^{k_r}/_{\xi}]_{1,P_2}:[(c):\Psi',\ldots\Psi^{(r)}]:\end{matrix}\right.$$

$$[(b'):\Phi'];[1+\lambda+\sigma\eta_G+\sum_{j=1}^{R}l_jt_j\pm y:\varepsilon];\ldots;[(b^{(r)}):\Phi^{(r)}];$$

$$[1+2\lambda+2\sigma\eta_G+2\sum_{j=1}^{R}l_jt_j:\varepsilon];[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];$$

$$\left.x_1\alpha_2^{-k_1/\xi}4^{\varepsilon},x_2\alpha_2^{-k_2/\xi},\ldots,x_r\alpha_2^{-k_r/\xi}\right], \qquad (4.3)$$

which holds under the same conditions obtainable from (2.2).

(iii) $$\int_0^{\infty}\int_0^{\infty}\frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda-\beta+1}}\cos\left(2y\tan^{-1}v/u\right)S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}$$

$$\left[\frac{\mu_1u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_Ru^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right]H_{P_1,Q_1}^{M_1,N_1}\left[\frac{\alpha_1u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\left|\begin{matrix}(e_{P_1},E_{P_1})\\(f_{Q_1},F_{Q_1})\end{matrix}\right.\right]$$

$$H_{P_2,Q_2}^{M_2,0}\left[\alpha_2\left(u^2+v^2\right)^{\xi}\left|\begin{matrix}(e'_{P_2},E'_{P_2})\\(f'_{Q_2},F'_{Q_2})\end{matrix}\right.\right]$$

$$H_1\begin{bmatrix} x_1 u^{2\varepsilon_1}\left(u^2+v^2\right)^{k_1-\varepsilon} \\ \\ x_r u^{2\varepsilon_r}\left(u^2+v^2\right)^{k_r-\varepsilon_r} \end{bmatrix} dudv$$

$$=\Psi H^{0,M_2+1:\ (u',v');\ldots;(u^{(r)},v^{(r)})}_{A+Q_2+1,C+P_2+2:\ [B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left[\begin{array}{l} [1-f_j'-F_j'\,{}^{\beta}/_{\xi}:F_j'\,{}^{k_1}/_{\xi},\ldots,F_j'\,{}^{k_r}/_{\xi}]_{1,Q_2}\ :[-2\lambda-2\sigma\eta_G \\ \quad [1-e_j'-E_j'\,{}^{\beta}/_{\xi}:E_j'\,{}^{k_1}/_{\xi},\ldots,E_j'\,{}^{k_r}/_{\xi}]_{1,P_2}\ :[-\lambda-\sigma\eta_G \\ \quad -2\sum_{j=1}^{R} l_j t_j\ :2\varepsilon_1,\ldots,2\varepsilon_r];\ \ [(a):\theta',\ldots,\theta^{(r)}]:[(b'):\Phi'];\ldots; \\ -\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon_1,\ldots,\varepsilon_r];\ \ [(c):\Psi',\ldots,\Psi^{(r)}]:\ \ [(d^{(r)}):\delta^{(r)}];\ldots; \\ [(b)^{(r)}:\Phi^{(r)}];x_1 4^{-\varepsilon_1}\alpha_2^{-k_1/\xi} \\ [(d^{(r)}):\delta^{(r)}];x_r 4^{-\varepsilon_r}\alpha_2^{-k_2/\xi} \end{array}\right], \qquad (4.4)$$

which holds under the same conditions those required for (2.3)

5. **Particular cases**

(1) Reducing the H-function of several complex variables to the G-function by putting $\theta',\ldots,\theta^{(r)}=\Phi',\ldots,\Phi^{(r)}=\Psi',\ldots,\Psi^{(r)}=\delta',\ldots,\delta^{(r)}$ $k_1,\ldots,k_r=\rho_1,\ldots,\rho_r$

in (2.1) we have the following consequence of the main result

$$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda}}\cos\left(2y\tan^{-1}v/u\right)S^{M_1,\ldots M_R}_{N_1,\ldots,N_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right]H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1 u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\middle|\begin{matrix}(e_{P_1},E_{P_1})\\(f_{Q_1},F_{Q_1})\end{matrix}\right]$$

$$G_{A,C:\ [B',D']\ ;\ldots;\ [B^{(r)},D^{(r)}]}^{0,0:\ (u',v')\ ;\ldots;\ (u^{(r)},v^{(r)})}\left[\begin{array}{l}(a):(b'),\ldots,(b^{(r)})\ ;\\ (c):(d'),\ldots,(d^{(r)})\ ;\end{array}\right.$$

$$\left.\begin{array}{l}x_1^{1/\rho_1}u^{2\xi/\rho_1}\left(u^2+v^2\right)^{1-\xi/\rho_1}\\ x_2^{1/\rho_1}\left(u^2+v^2\right)\\ \\ x_r^{1/\rho_r}\left(u^2+v^2\right)\end{array}\right]f\left(u^2+v^2\right)\ dudv$$

$$=\Omega\int_0^\infty G_{A,C\ [B'+2,D'+2]\ ;\ [B'',D'']\ ;\ldots;\ [B^{(r)},D^{(r)}]}^{0,0\ :\ (u',v'+1);(u'',v'');\ldots;(u^{(r)},v^{(r)})}$$

$$\left[\begin{array}{l}(a)\ ;\ [-2\lambda-2\sigma\eta_G-2\sum_{j=1}^{R}l_jt_j:2\,\varepsilon/\rho_1];(b');\ldots;(b)^{(r)};\\ (f)\ ;\ (d')\ ;\ [-\lambda-\sigma\eta_G-\sum_{j=1}^{R}l_jt_j\pm y:\varepsilon/\rho_1];\ldots,(d^{(r)})\ ;\end{array}\right.$$

$$\left.x_1^{1/\rho_1}z4^{-\varepsilon/\rho_1},x_2^{1/\rho_2}z,\ldots,x_r^{1/\rho_r}z\right]f(z)dz, \qquad (5.1)$$

which holds under the same conditions those required for (2.1).

(2) Taking $\lambda=A,\ u^{(i)}=1,\ v^{(i)}=B^{(i)},\ D^{(i)}=D^{(i)}+1$ $\forall i=1,2,\ldots,r$ in mains double integral (2.1), we get the following result involving generalized Lauricella function [12].

$$\int_0^\infty\int_0^\infty\frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda}}\cos\left(2y\tan^{-1}v/u\right)S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}$$

$$\left[\frac{\mu_1u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_Ru^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right]H_{P_1,Q_1}^{M_1,N_1}\left[\frac{\alpha_1u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\middle|\begin{array}{l}(e_{P_1},E_{P_1})\\ (f_{Q_1},F_{Q1})\end{array}\right]$$

$$F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}\left[\begin{matrix}[1-(a):\theta;\ldots,\theta^{(r)}];[1-(b'):\Phi'] \\ [1-(c):\Psi;\ldots,\Psi^{(r)}];[1-(d'):\delta']\end{matrix}\right.$$

$$\left.\begin{matrix} & -x_1u^{-2\varepsilon}\left(u^2+v^2\right)^{k_1-\varepsilon} \\ ;\ldots;[1-(b^r):\Phi^r]; & -x_2\left(u^2+v^2\right)^{k_2} \\ ;\ldots;[1-(d'):\delta']; & \\ & -x_r\left(u^2+v^2\right)^{k_r}\end{matrix}\right] f\left(u^2+v^2\right)\, du dv$$

$$=\Omega\frac{\Gamma(1+2\lambda+2\sigma\eta_G+2\sum_{j=1}^{R}l_jt_j)}{\Gamma(1+\lambda+\sigma\eta_G+\sum_{j=1}^{R}l_jt_j)}\int_0^\infty F^{A:B'+1;B'';\ldots;B^{(r)}}_{C:D'+2;D'';\ldots;D^{(r)}}$$

$$\left[\begin{matrix}[1-(a):\theta';\ldots,\theta^{(r)}]:[1+2\lambda+2\sigma\eta_G+2\sum_{j=1}^{R}l_jt_j:2\varepsilon] \\ [1-(c):\Psi';\ldots,\Psi^{(r)}]:[1-(d'):\delta'],[1+\lambda+\sigma\eta_G+\sum_{j=1}^{R}l_jt_j\pm y:\varepsilon]\end{matrix}\right.$$

$$\left.\begin{matrix} & -x_1z^{k_1}4^{-\varepsilon} \\ [1-(b^r):\Phi'];\ldots;[1-(b^{(r)}):\Phi^{(r)}] & -x_2z^{k_2} \\ [1-(d''):\delta''];\ldots;[1-(d^{(r)}):\delta^{(r)}] & \\ & -x_rz^{k^r}\end{matrix}\right] f(z)dz \qquad (5.2)$$

valid under the same conditions those are required for (2.1)

(3) If we put $\lambda=A,\ u^{(i)}=1,\ v^{(i)}=B^{(i)}=D^{(i)}+1,$ $\forall\ i=1,2,\ldots$ the main double integral involving kampe de Feriet function [13]

$$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda}}\cos\left(2y\tan^{-1}v/u\right)S^{M_1,\ldots,M_R}_{N_1,\ldots,N_R}$$

$$\left[\frac{\mu_1u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_Ru^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right]H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\middle|\begin{matrix}(e_{P_1},E_{P_1}) \\ (f_{Q_1},F_{Q_1})\end{matrix}\right]$$

$$S^{A:B'+1;B''}_{C:D',D''}\left[\begin{matrix}[(a):\theta',\theta'']:[(b'):\Phi'];[(b''):\Phi'']; \\ [(c):\Psi',\Psi'']:[(d'):\delta'];[(d''):\delta''];\end{matrix}\right.$$

$$\left.\begin{matrix} x_1 u^{2\varepsilon}\left(u^2+v^2\right)^{k_1-\varepsilon} \\ x_2\left(u^2+v^2\right)^{k_2} \end{matrix}\right] f\left(u^2+v^2\right)\ dudv$$

$$=\Omega.\int_0^\infty S^{A:B'+1;B''}_{C:D'+2,D''}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\Psi',\ldots,\Psi^{(r)}]:[(d'):\delta']\end{matrix}\right.$$

$$\left.\begin{matrix}[1+2\lambda+2\sigma\eta_G+2\sum_{j=1}^{R} l_j t_j:2\varepsilon];[(b'):\Phi'];[(b''):\Phi''];\quad x_1 z^{k_1}4^{-\varepsilon} \\ [1+\lambda+\sigma\eta_G+\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon];[(d''):\delta'']\quad x_2 z^{k_2}\end{matrix}\right] f(z)dz, \qquad (5.3)$$

which hold under the same conditions obtainable form (2.1)

(4) On taking $r=2$ is (2.1) we get the following double integral involving H- function of two variables [6, p.117]

$$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{\left(u^2+v^2\right)^{\lambda}} \cos\left(2y\tan^{-1}v/u\right) S^{M_1,\ldots,M_R}_{N_1,\ldots,N_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{\left(u^2+v^2\right)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{\left(u^2+v^2\right)^{l_R}}\right] H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1 u^{2\sigma}}{\left(u^2+v^2\right)^{\sigma}}\right|$$

$$\left.\begin{matrix}(e_{P_1},E_{P_1}) \\ (f_{Q_1},F_{Q_1})\end{matrix}\right] H^{0,0:(u',v');(u'',v'');}_{A,C:[B',D'];[B'',D'']}$$

$$\left[\begin{matrix}[(a):\theta',\theta'']:[(b'):\Phi'];[(b''):\Phi''];x_1 u^{2\varepsilon}\left(u^2+v^2\right)^{k_1-\varepsilon} \\ [(c):\Psi',\Psi'']:[(d'):\delta'];[(d''):\delta''];x_2\left(u^2+v^2\right)^{k_2}\end{matrix}\right] f\left(u^2+v^2\right)\ dudv$$

$$= \Omega.\int_0^\infty H^{0,0:(u',v'+1);(u'',v'');}_{A,C:[B'+1,D'+2];[B'',D'']}\left[\begin{matrix}[(a):\theta',\theta'']: \\ [(c):\Psi',\Psi'']:\end{matrix}\right.$$

$$\left.\begin{matrix}[1-2\lambda-2\sigma\eta_G-\sum_{j=1}^{R} l_j t_j :2\varepsilon];[(b'):\Phi'];[(b''):\Phi'']; \\ [(d'):\delta'];[-\lambda-\sigma\eta_G-\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon];[(d''):\delta''];\end{matrix}\quad \begin{matrix}x_1 z^{k_1} 4^{-\varepsilon} \\ x_2 z^{k_2}\end{matrix}\right] f(z)dz, \qquad (5.4)$$

which holds under the same conditions those required for (2.1).

(5) By taking $\lambda = A = C = v'' = B'' = d'' = 0$ and $u'' = D'' = \delta'' = 1$ the multivariable H-function reduces to a relation obtained by Chaurasia [2,p.18,eq.(1.54)] and the double integral (5.4) reduces in the following form:

$$\int_0^\infty\int_0^\infty \frac{u^{2\lambda}}{(u^2+v^2)^{\lambda}} \cos(2y\tan^{-1} v/u)\, S^{M_1,\ldots,M_R}_{N_1,\ldots,N_R}$$

$$\left[\frac{\mu_1 u^{2l_1}}{(u^2+v^2)^{l_1}},\ldots,\frac{\mu_R u^{2l_R}}{(u^2+v^2)^{l_R}}\right] H^{M_1,N_1}_{P_1,Q_1}\left[\frac{\alpha_1 u^{2\sigma}}{(u^2+v^2)^{\sigma}}\left|\begin{matrix}(e_{P_1},E_{P_1}) \\ (f_{Q_1},F_{Q_1})\end{matrix}\right.\right]$$

$$H^{u',v'}_{B',D'}\left[\left.\begin{matrix}[(b'):\Phi'] \\ [(d'):\delta']\end{matrix}\right| x_1 u^{2\varepsilon}(u^2+v^2)^{k_1-\varepsilon}\right] f(u^2+v^2)\ dudv$$

$$= \Omega\int_0^\infty H^{u',v'}_{B',D'}\left[\begin{matrix}[-2\lambda-2\sigma\eta_G-2\sum_{j=1}^{R} l_j t_j :2\varepsilon];[(b');\Phi']; \\ [(d');\delta'];[-\lambda-\sigma\eta_G-\sum_{j=1}^{R} l_j t_j \pm y:\varepsilon];\end{matrix}\quad x_1 z^{k_1} 4^{-\varepsilon}\right] f(z)dz, \qquad (5.5)$$

valid under the same conditions obtain from (2.1)

(6) The special cases of (2.2) and (2.3) involving G-function, Lauricell's function and Kampe de Feriet function can be obtained on proceeding on similar ways.

(7) By taking R=1 in the double integral (2.1) through (2.3) reduce to the integrals obtained by Chaurasia and Gupta [3,p.76-77, eq.(2.1)-(2.3)].

(8) On taking R=2 in the result (2.1) through (2.3) reduce to the integrals obtained by Chaurasia and Singhal [5,p.235-244].

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