An Inventory Model for Deteriorating Items with Power Pattern Demand and Partial Backlogging with Time Dependent Holding Cost

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Abstract

This paper deals with an inventory model for deteriorating items with power pattern demand, partial backlogging in the form of two parameters weibull function. Holding cost is considered as time dependent.

Keywords

Power pattern demand, Deterioration, Partial backlogging, Holding cost.

Introduction

A large number of authors developed inventory models for deteriorating items. Deterioration is assumed to begin after a time interval which may be called life period of the items in inventory. When the items or commodities are kept in stock to meet the future demand then there may be deterioration of items in inventory due to many factors like weather, insects, rodents etc., so decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity of time factor into consideration.

In classical inventory models the demand rate is assumed to be constant. In reality, demand for physical goods may be time dependent, stock dependent, price dependent etc. Power pattern demand is a time dependent demand. The time factor is controlled by pattern index n, where $0<n<\infty$. In some inventory systems the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore the backlogging rate is variable and dependent on the length of the waiting time for the next replenishment. In a real life system time depending holding cost is increased or decreased with time. The constant partial backlogging rate during the shortage period in inventory was made by Park[1992]. Various types of inventory models for items deteriorating at a constant rate was discussed by Roy Chowdhury and Chaudhuri[1983]. An order level inventory model for deteriorating item with finite rate of replenishment was discussed by Dutta and Pal[1988]. EOQ models for deteriorating items under stock dependent selling rate were developed by Padmanabhan and Vrat[1995]. Chang and Dye[1999] developed an EOQ model with power demand and partial backlogging. An inventory model for deteriorating items with shortages and partial backlogging was developed by Wang[2002]. Time proportional partial backlogging rate was discussed by Teng and Yang[2004], Wu et al[2006] and Dye et al[2007]. An optimal inventory model for variable rate of deterioration and alternating production rates with exponentially declining demand was discussed by Sharma and Goyal[2008]. Optimum ordering interval with selling price dependent demand rate for items with random deterioration and shortages was considered by Sharma and Aggarwal[2009]. EOQ model for perishable items with power demand and partial backlogging was developed by Singh and Singh[2009].

In this model demand is taken as power pattern demand with two parameter Weibull function. When this type of demand occurs, then stockiest can use different policies. The holding cost is time dependent.

Assumptions and Notation

The following assumptions are made for developing model-

1. Shortages are allowed.
2. Lead time is zero.
3. T is the planning horizon.
4. No replenishment or repair of deteriorated items is made during a given cycle.
5. A single item is considered over the fixed time period T units of time, which is subject to variable deterioration rate.
6. The replenishment occurs instantaneously at an infinite rate.
7. Deterioration of the items is considered only after the life time of items.

The following notations are used-

$I(t)$	: Inventory level at any time t, $t \geq 0$, it is the initial inventory level
μ	: Life time of item
$\theta(t, \alpha)= \theta_0(\alpha)t$	: Deterioration rate where $0 < \theta_0(\alpha) << t, t > 0$ is assumed which is a special form of the two parameter Weibull distribution. The random variable 'α', which ranges over a space 'Γ' and probability density function p(α) is defined such that $\int_\Gamma p(\alpha)d\alpha = 1$
O_C	: Set up cost
C_d	: Deterioration cost per unit
C_S	: Shortage cost for backlogged items
$d(t) = \frac{dt^{\left(\frac{1}{n}-1\right)}}{nT^{\left(\frac{1}{n}\right)}}$	: Demand rate is given by 'd' which is fixed quantity, 'n' is the parameter of power demand pattern. Where the value of n may be any positive number
C_L	: Lost sales

$(H_c + at)$	:	Holding cost where 'H_c' and 'a' are positive constants
$\dfrac{d(t)}{1+\delta(T-t)}$	:	Backlogging rate where backlogging parameter is 'δ' it is positive constant and $0 < \delta << 1$

Mathematical Model

The differential equation describing the behaviour of the system is given by-

$$\frac{dI(t)}{dt} = -d(t) \qquad 0 \le x \le \mu \qquad \text{...(1)}$$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -d(t) \qquad \mu \le t \le t_1 \qquad \text{...(2)}$$

$$\frac{dI(t)}{dt} = \frac{-d(t)}{1+\delta(T-t)} \qquad t_1 \le t \le T \qquad \text{...(3)}$$

The boundary conditions are

$$\begin{aligned} I(t) &= I_s, \quad \textit{when } t = 0 \\ I(t) &= 0, \quad \textit{when } t = t_1 \end{aligned} \qquad \text{...(4)}$$

The solutions of the equations are-

$$I(t) = I_s - \frac{dt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \qquad \text{...(5)}$$

$$I(t) = \frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\theta_0(\alpha)}{2(2n+1)}\left(t_1^{\frac{1}{n}+2} - t^{\frac{1}{n}+2}\right)\right]e^{-\theta_0(\alpha)\frac{t^2}{2}} \qquad \text{...(6)}$$

$$I(t) = \frac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}) + \frac{\delta}{n+1}(t_1^{\frac{1}{n}+1} - t^{\frac{1}{n}+1})\right] \qquad \text{...(7)}$$

The value of initial inventory level (S) is given by

$$I_s = \frac{d\mu^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}} - \mu^{\frac{1}{n}}\right) + \frac{\theta_0(\alpha)}{2(2n+1)}\left(t_1^{\frac{1}{n}+2} - \mu^{\frac{1}{n}+2}\right)\right]e^{-\theta_0(\alpha)\frac{\mu^2}{2}}$$

...(8)

Deterioration units-

$$= \frac{C_d}{T}\int_{\mu}^{t_1}\theta_0(t,\alpha)I(t)dt \qquad ...(9)$$

Storage units-

$$= -\frac{C_s}{T}\int_{t_1}^{T} I(t)dt \qquad ...(10)$$

Holding cost-

$$= \frac{1}{T}\left[\int_{0}^{\mu}(H_c + at)I(t)dt + \int_{\mu}^{t_1}(H_c + at)I(t)dt\right] \qquad ...(11)$$

Lost Sales unit-

$$= \frac{C_L}{T}\int_{t_1}^{T}\left\{1 - \frac{1}{1+\delta(T-t)}\right\}d(t)dt \qquad ...(12)$$

Total average cost of the system per unit time is given by-

TC = Ordering Cost + Holding Cost +Deterioration Cost +Shortage Cost +Lost Sale ...(13)

To maximize total average cost per unit time (TC), the optimal value of t_1 can be obtained by solving the following equation: $\frac{dc}{dt_1} = 0$

Numerical Example

To illustrate the model numerically, the following parameter values are considered:

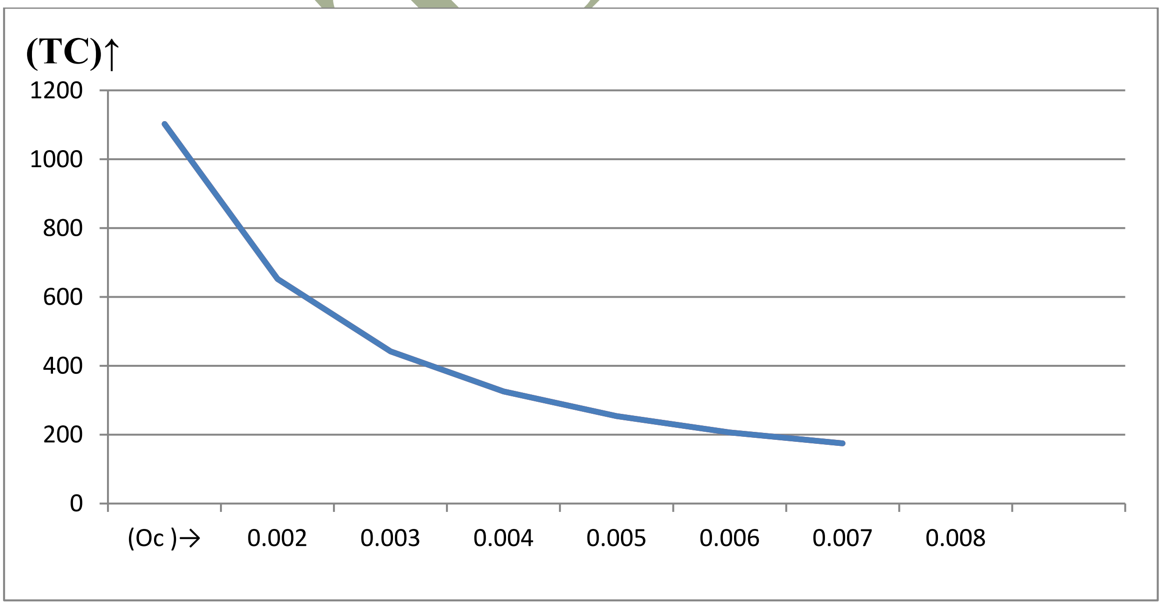
d=70 units	n=1 units	a=0.1
$C_L = 10$Rs. / unit,	T=1 year	C_s=20 Rs./ unit
O_c=100 Rs./ unit	μ=0.5 units	C_d=25 Rs./ unit
H_c=0.5		

Then for the minimization of the total average cost, optimal policy can be obtained such as,

t_1=**1.81** units, I_s=**94.35335** units, TC= **325.07** per year

Variation in total cost (TC) with ordering cost (O_c)

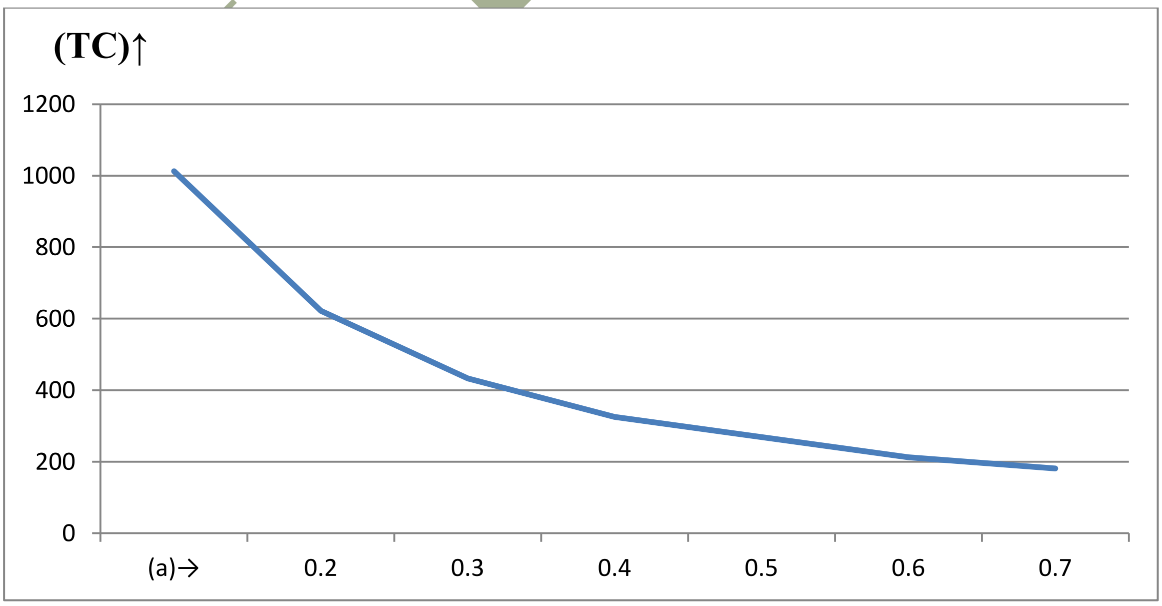
Ordering Cost (O_c)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
0.002	2.85	118.47	1103.17
0.003	2.33	107.09	652.10
0.004	2.02	99.71	442.02
0.005	1.81	94.35	325.07
0.006	1.65	90.21	253.52
0.007	1.53	86.86	206.71
0.008	1.43	84.07	174.85



Observation- Total cost (TC) decreases with increasing ordering cost (O_c).

Variation in total cost (TC) with holding parameter (a)

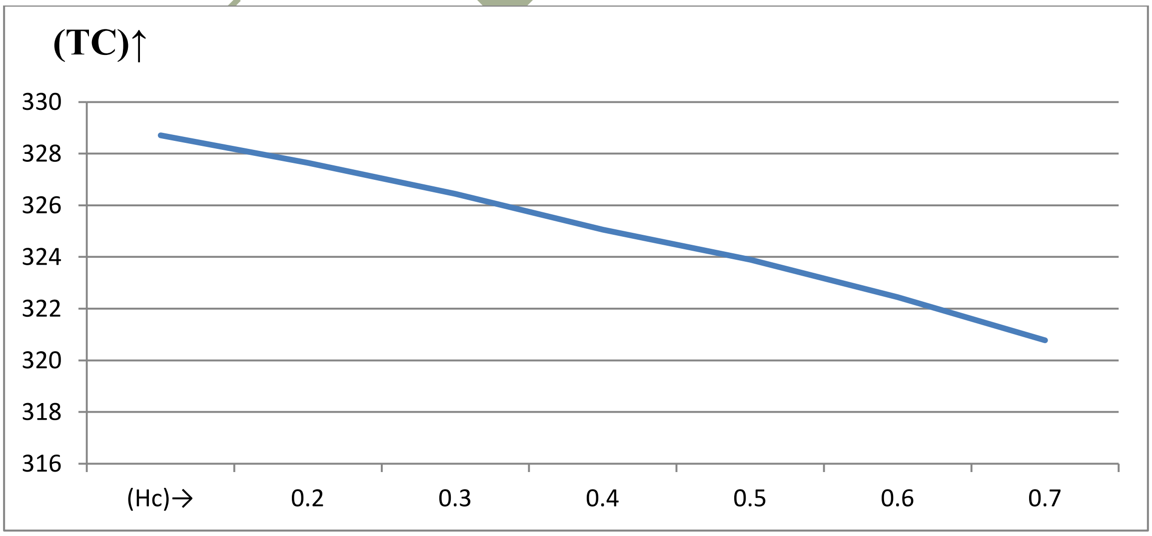
Holding parameter (a)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
0.04	2.75	116.59	1012.74
0.06	2.28	106.15	621.57
0.08	2.00	99.33	432.77
0.10	1.81	94.35	325.07
0.12	1.69	91.21	269.20
0.14	1.55	87.35	212.57
0.16	1.46	84.73	181.34



Observation- Total cost (TC) decreases with increasing holding parameter (a).

Variation in total cost (TC) with holding parameter (H_c)

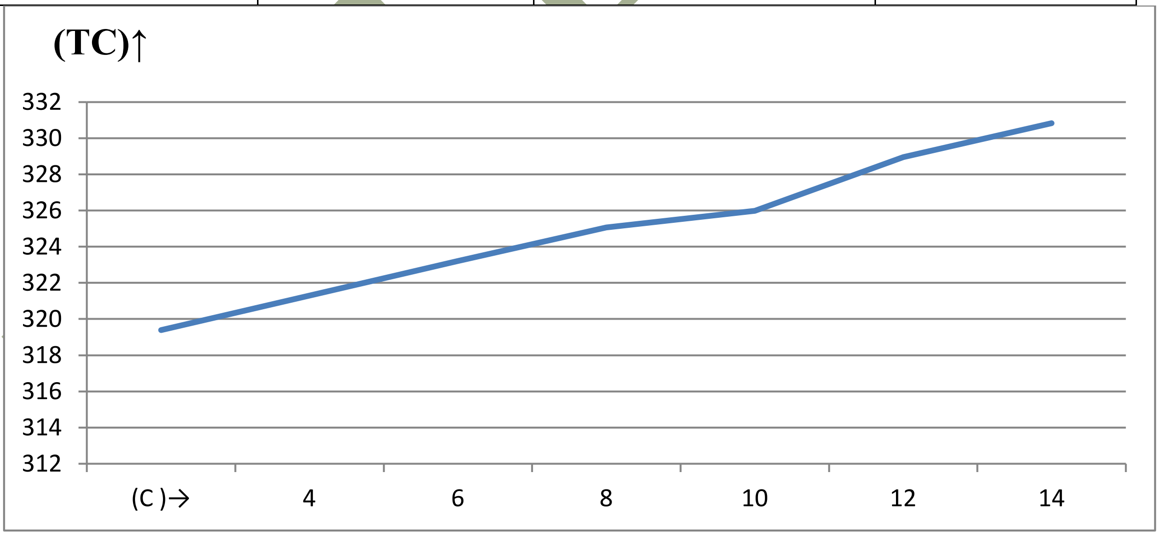
Holding parameter (H_c)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
0.2	1.85	95.42	328.71
0.3	1.83	95.07	327.65
0.4	1.82	94.71	326.45
0.5	1.81	94.35	325.06
0.6	1.79	93.99	323.90
0.7	1.78	93.26	322.45
0.8	1.77	92.88	320.77



Observation- Total cost (TC) decreases with increasing holding cost parameter (H_c).

Variation in total cost (TC) with lost sale parameter (C_L)

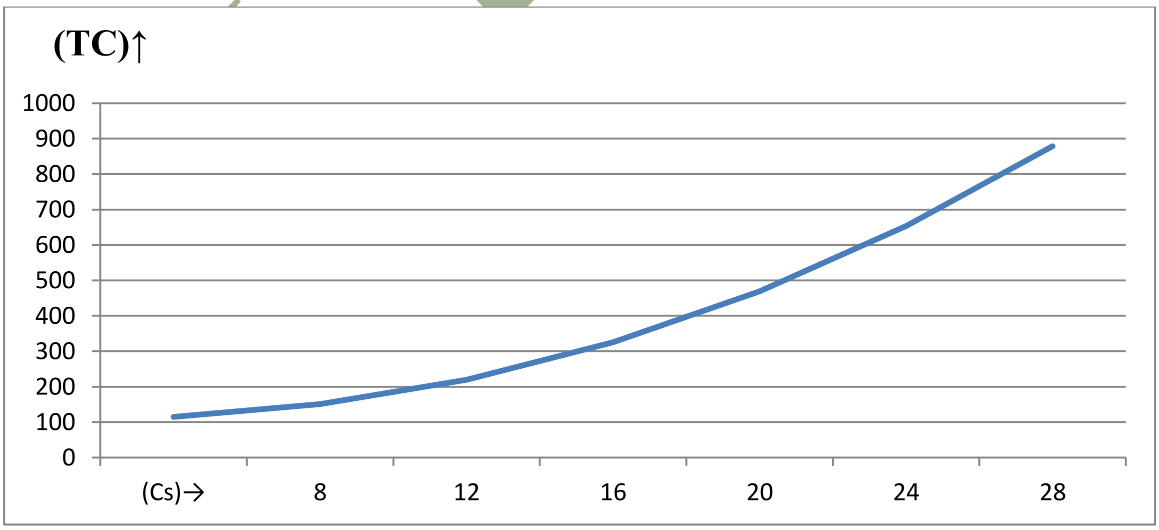
Lost Sale (C_L)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
4	1.81	94.31	319.38
6	1.81	94.32	321.29
8	1.81	94.34	323.20
10	1.81	94.35	325.07
12	1.81	94.36	325.98
14	1.81	94.38	328.96
16	1.81	94.39	330.83



Observation- Total cost (TC) increases with increasing lost sale parameter (C_L).

Variation in total cost (TC) with shortage cost parameter (C_s)

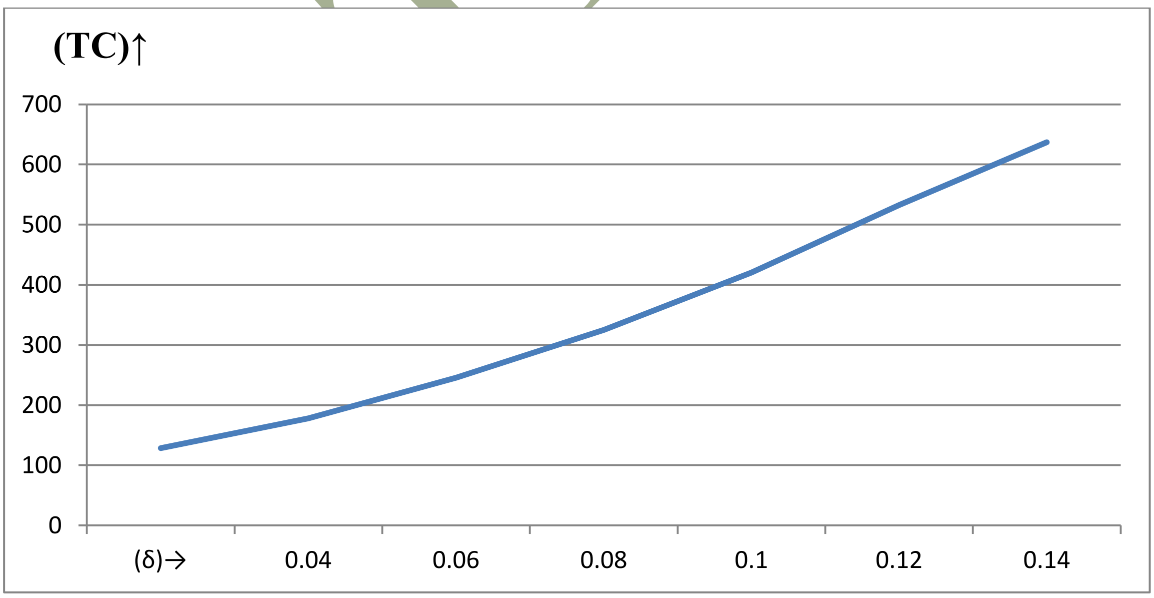
Shortage cost (C_S)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
8	**1.16**	**75.63**	**114.50**
12	**1.42**	**83.57**	**150.74**
16	**1.62**	**89.43**	**219.49**
20	**1.81**	**94.35**	**325.07**
24	**1.97**	**98.61**	**469.38**
28	**2.13**	**102.40**	**653.63**
32	**2.27**	**105.82**	**878.73**



Observation- Total cost (TC) increases with increasing shortage parameter (C_s).

Variation in total cost (TC) with demand parameter (δ)

Demand parameter (δ)	Total Time (t_1)	Initial Inventory (I_s)	Total Cost (TC)
0.04	1.25	78.39	128.23
0.06	1.45	84.48	177.95
0.08	1.64	89.75	245.82
0.10	1.81	94.35	325.07
0.12	1.97	98.42	420.30
0.14	2.11	102.08	532.70
0.16	2.25	105.41	637.07



Observation-Total cost (TC) increases with increasing demand parameter (δ).

Conclusion:

The optimal inventory control problem with a special form of the two parameter Weibull function is considered. Power pattern demand has been assumed with demand rate $\frac{dt^{\left(\frac{1}{n}-1\right)}}{nT^{\left(\frac{1}{n}\right)}}$, when 'n > 1', a large portion of the demand occurs at the beginning of the period and when 0 < n < 1, a large portion of the demand occurs at the end of the period. On substituting n=∞ in the power demand pattern formula, it shows that the demand is instantaneous in nature. For n=1, the demand pattern is uniform.

Deterioration of items is an important factor in inventory models and can not be avoided. Fruits, vegetables, fashionable commodities, drugs and pharmaceuticals, chemicals etc. are some examples of items in which deterioration occur with time. But at the starting, when an item is produced or purchased it is fresh and new, deterioration starts after a certain period. This certain period is called life time of that particular item. Total cost is controlled by variation in different parameters. In this model demand, shortage, lost sale parameters increases the total cost and we can control it by these parameters.

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