# Performance Analysis of Throughput of Secondary Users and Packet Collision Probability of Primary user in Cognitive Radio Network

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***Abstract*** **– In this paper, we investigate a scenario in Cognitive Radio Network (CRN) where multiple unlicensed users (secondary users) sense multiple licensed frequency bands and try to access the idle frequency bands opportunistically without causing any harmful interference to the licensed users (primary users). To access a channel, each secondary user (SU) calculates access probability (AP) from its spectrum sensing results. The optimal AP values are derived for maximum throughput of a SU. The missed detection of a channel by SUs causes interference to the PUs which results in packet collision. Hence, the packet collision probability of a PU is also analyzed along with simulation results.**

***Index Terms – Cognitive radio, Throughput analysis, collision probability, Spectrum Sensing.***

## I. INTRODUCTION

The radio frequency bands are a limited natural resource and getting enabled day by day due to growing demand of the wireless communication applications. Many of the existing wireless communication technologies support Non-Line-of-Sight (NLOS) communication to provide better quality of service. Due to considerable amount of absorption of very high frequency signals (above 4 GHz) by water drops in the environment / rain, scattering and shadowing of the signal due to obstacles in the environment, lower frequency bands are preferred for NLOS communication. But the lower frequency bands already have been allotted by government agencies under static spectrum allocation policy. The static spectrum allocation policy allows licensed user (primary user) to use the licensed frequency band and bars unlicensed users (secondary users). A study by Federal Communication Commission (FCC) shows that most of the static licensed spectrums are underutilized varies from 15 % to 85%, which is function of geographical location and time [1].

Thus to overcome the spectrum scarcity and improve spectrum utilization efficiency, Joe Mitola and Gerald Maguire introduced a new communication technique known as cognitive radio network (CRN) in 1999 [2]. In CRN, when the allotted frequency band is not utilized by the primary user (PU) the unlicensed secondary users (SUs) can use the unutilized frequency band opportunistically without causing harmful interference to the PU [3, 4].

In this paper, we analyze throughput [5, 6] of a SU and derived optimal values of AP. We assume that all SUs acquire common state information on all channels via a suitable channel sensing such as the negotiation-based sensing [7]. In the random channel access policy [8, 9], each SU accesses a channel sensed as idle at each slot based on the common access probability (AP). If two or more SUs randomly select the same channel, it leads to a collision between them. In addition, if any SU selects a busy channel sensed as idle, the PU occupying the busy channel experiences interference [10], resulting in a collision between the PU and the SU. Clearly, a channel access policy that adapts the AP to the knowledge on idle channels so as to mitigate collisions and interference to PUs, is desirable. So we assume that the AP is adapted to the number of channels sensed as idle. Then it is important to determine the optimal AP values that optimize the performance of SUs while guaranteeing a given interference requirement for PUs, which is one of our objectives in this paper. We consider the throughput of an arbitrary SU as a performance metric [11] for SUs. We also consider the packet collision probability between SUs and an arbitrary PU, called the packet collision probability of a PU, as a performance metric for PUs. Moreover, we consider a requirement that the collision probability of a PU should not be greater than a given threshold. Under the requirement we formulate an optimization problem and obtain an explicit expression on the optimal AP values that maximize the throughput of a SU.

The organization of this paper is as follows. In Section II, we explain the system model. In Section III, we analyze the throughput of an SU and the packet collision probability of a PU. In Section IV, we provide numerical results to investigate the performance of throughput of SUs and collision probability requirement of PU. Conclusions are given in Section V.

## II. SYSTEM MODEL

### A. *Wireless Channel Occupancy Model*

We consider a time-slotted CR networks with N wireless channels where time is indexed by $t$ ($t = 0, 1, 2, \ldots$). Each channel is designated to each PU and the occupancy process of each channel is assumed to be a two-state Markov chain with state space $\{0, 1\}$ as in [9]. Here, state 0 (1,resp.) implies that the channel is occupied (unoccupied, resp.) by its designated PU. The transition probability matrix of the Markov chain is given by

$$Q = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}$$

Where $p$ is the transition probability from state 0 to state 1 and $q$ is the transition probability from state 1 to state 0. Let $\pi = (\pi_0, \pi_1)$ be the stationary probability vector of Q, i.e., $\pi Q = \pi$, $\pi_0 + \pi_1 = 1$. It then satisfies that

$$\pi_0 = \frac{q}{p+q}, \pi_1 = \frac{p}{p+q} \qquad (1)$$

We assume that channel state transitions occur at slot boundaries and the channel occupancy process is independent of each other.

Let N(t) be the number of idle wireless channels at slot t. Then, N(t) is a Discrete Time Markov Chain (DTMC) with state space {0, 1, ….,N}. In addition, the steady state probabilities for N(t) satisfy

$$P\{N(t) = k\} = \binom{N}{k} \pi_1^k \pi_0^{N-k}, \; 0 \le k \le N \qquad (2)$$

Where $\pi_0$ and $\pi_1$ are given in equation (1).

### B. *Channel Sensing Error Model*

The CR network employs a channel sensing scheme such as the negotiation-based sensing [4] to obtain a common channel state information. The channel sensing is assumed to be imperfect and hence results in two types of errors [6]. The first type is the false alarm error that an idle channel is sensed as busy. The second type is the misdetection error that a busy channel is sensed as idle. Here, the false alarm error probability is denoted by $e_f$ and the misdetection error probability is denoted by $e_m$.

Let $A_e(t)$ be the number of idle channels sensed as idle at slot $t$ and $B_e(t)$ be the number of busy channels sensed as idle at slot $t$. By the definition, when the true number of idle channels $N(t)$, is $k$, we have

$$P\{A_e(t) = n, B_e(t) = l \mid N(t) = k\}$$

$$= \binom{k}{n}(1-e_f)^n {e_f}^{k-n} \times \binom{N-k}{l} {e_m}^l (1-e_m)^{N-k-l} \qquad (3)$$

## III. PERFORMANCE ANALYSIS

### A. *Throughput Analysis of an SU*

To analyze the throughput of an arbitrary SU, we assume that all SUs are saturated, i.e., they always have packets to transmit. We tag an arbitrary SU as the reference and denote by $c(t)$ the number of packets transmitted successfully by the tagged SU at slot $t$. Hence, $c(t) = 1$ if the tagged SU successfully transmits a packet at slot $t$, and $c(t) = 0$ otherwise.

Assuming that the network is in the steady state, the throughput of the tagged SU is given as follows:

$$E[c(t)] = \sum_{k=0}^{N} P\{N(t) = k\} E[c(t) \mid N(t) = k] \qquad (4)$$

Where $P\{N(t) = k\}$ is given in equation (2). Using $A_e(t)$ and $B_e(t)$ the conditional throughput $E[c(t)|N(t) = k]$ is further given by

$$E[c(t)|N(t) = k] =$$

$$\sum_{n=0}^{k} \sum_{l}^{N-k} P\{A_e(t) = n, B_e(t) = l \mid N(t) = k\} \times E[c(t) \mid A_e(t) = n, B_e(t) = l, N(t) = k\}] \qquad (5)$$

Where $P\{A_e(t) = n, B_e(t) = l \mid N(t) = k\}$ is given in (3). So it remains to compute the conditional throughput $E[c(t)|A_e(t) = n, B_e(t) = l, N(t) = k]$.

When $A_e(t) + B_e(t) = 0$, it is obvious that

$$E[c(t)|A_e(t) = n, B_e(t) = l, N(t) = k] = 0.$$

When $A_e(t) + B_e(t) > 0$, we obtain

$$E[c(t)|A_e(t) = n, B_e(t) = l, N(t) = k]$$

$$= a_{n+l}\binom{n}{n+l} \sum_{i=0}^{M-1} \binom{M-1}{i} {a_{n+l}}^i (1-a_{n+l})^{M-l-i}$$

$$\times \binom{n+l-1}{n+l}^i$$

$$= a_{n+l}\binom{n}{n+l}[a_{n+l} \, {}^{n+l-1}_{\;n+l} + 1 - a_{n+l}]^{M-1}$$

$$= a_{n+l}\binom{n}{n+l}[1 - \frac{a_{n+l}}{n+l}]^{M-1} \qquad (6)$$

On the right hand side (RHS) of the first equation, $a_{n+l}$ is the probability that the tagged SU becomes active, $\frac{n}{n+l}$ is the probability that the tagged SU selects a true idle channel, $\binom{M-1}{i} {a_{n+l}}^i (1-a_{n+l})^{M-l-i}$ is the probability that there are *i*nactive untagged SUs, and $\binom{n+l-1}{n+l}^i$ is the probability that all active untagged SUs do not select the same idle channel that the tagged SU selects. By substituting $x$ for $n+l$ in (6) and combining (4), (5), and (6), the throughput of the tagged SU, $E[c(t)]$, is given by

$$E[c(t)] = \sum_{k=1}^{N} \binom{N}{k} \pi_1^k \pi_0^{N-k} \sum_{n=1}^{k} \sum_{l=0}^{N-k} \binom{k}{n} \left(1 - e_f\right)^n {e_f}^{k-n}$$

$$\times \binom{N-k}{l} {e_m}^{l}(1-e_m)^{N-k-l} \quad \times$$

$$a_{n+l}\binom{n}{n+l}\,[1-\frac{a_{n+l}}{n+l}]^{M-1}$$

$$= \sum_{x=1}^{N}\frac{a_x}{x}(1-\frac{a_x}{x})^{M-1}\sum_{k=1}^{N}\binom{N}{k}\pi_1^k\pi_0^{N-k}$$

$$\times \sum_{n=\max(1,x+k-N)}^{\min(k,x)} n\binom{k}{n}(1-e_f)^n {e_f}^{k-n}$$

$$\times \binom{N-k}{x-n}{e_m}^{x-n}(1-e_m)^{N-k-x+n}$$

$$= \sum_{x=1}^{N} f_x(a_x)\mathrm{F(x)} \qquad (7)$$

Where

$$f_x(a_x) = \frac{a_x}{x}(1-\frac{a_x}{x})^{M-1}$$

$$\mathrm{F(x)} = \sum_{k=1}^{N}\binom{N}{k}\pi_1^k\pi_0^{N-k}$$

$$\times \sum_{n=\max(1,x+k-N)}^{\min(k,x)} n\binom{k}{n}(1-)^n {e_f}^{k-n}$$

$$\times \binom{N-k}{x-n}{e_m}^{x-n}(1-e_m)^{N-k-x+n}$$

We investigate the characteristics of the throughput of the tagged SU for later use. Noting that F(x) for each x is nonnegative, if we maximize $f_x(a_x)$ for each x, then the throughput E[c(t)] is maximized. Differentiating $f_x(a_x)$ with respect to $a_x$, we obtain

$$\frac{d}{da_x}f_x(a_x) = \frac{1}{x}(1-\frac{a_x}{x})^{M-1}(1-\frac{M}{x}a_x)$$

By checking the condition $\frac{d}{da_x}f_x(a_x) > 0$ we see that, we see that $f_x(a_x)$ strictly increases for $0 \le a_x \le \min\left(\frac{x}{M},1\right)$ and strictly decreases for $\min\left(\frac{x}{M},1\right) \le a_x \le 1$. Therefore, the APs that maximize the throughput of the tagged SU, denoted by $\{\tilde{a}_0, \tilde{a}_1, \cdots, \tilde{a}_N\}$, are given in the following theorem.

**Theorem 1.** When $N_S(t) = (A_e(t)+B_e(t)) = x$, $0 \le x \le N$, the APs that maximize the throughput of the tagged SU, $\tilde{a}_x$, satisfy

$$\tilde{a_x} = \min\left(\frac{x}{M},1\right), 0 \le x \le N$$

Note that the APs $\tilde{a}_x$ depend only on the sum of $A_e(t)$ and $B_e(t)$ (which is known to all SUs) and do not depend on N(t). We next derive the maximum throughput of the tagged SU corresponding to the APs $\{\tilde{a}_0, \tilde{a}_1, \cdots, \tilde{a}_N\}$. For simplicity, we assume $N \le M$. By substituting $\tilde{a}_x$ for $a_x$ in (7) we obtain

$$\mathrm{E}[\tilde{c}(t)] = \sum_{x=1}^{N} f(\tilde{a_x})\mathrm{F(x)}$$

$$= \sum_{x=1}^{N}\frac{1}{M}(1-\frac{1}{M})^{M-1}\mathrm{F(x)}$$

$$\mathrm{E}[\tilde{c}(t)] = \frac{1}{M}(1-\frac{1}{M})^{M-1}N\pi_1(1-e_f) \qquad (8)$$

Note that $\mathrm{E}[\tilde{c}(t)]$ is the maximum throughput when all SUs use the optimal values of APs $\{\tilde{a}_0, \tilde{a}_1, \cdots, \tilde{a}_N\}$ without considering the packet collision probability of a PU. In this we see from (8) that the misdetection error does not play any role in the throughput maximization problem because $\mathrm{E}[\tilde{c}(t)]$ is not a function of the misdetection error probability $e_m$.

### B. *Packet Collision Probability of a PU*

In this section we analyze the packet collision probability of a PU which is defined by the probability that a PU's packet transmission is interfered by one or multiple SUs. From now on, the term "packet collision" denotes the packet collision between a PU and SUs due to imperfect channel sensing. We consider the packet collision probability as a metric to measure the protection level for PUs. We tag an arbitrary channel and its designated PU as the reference and assume that the tagged channel has the packet collision probability requirement ŋ. The ŋ is the maximum allowable packet collision probability for the tagged PU. Let $P_C$ be the conditional probability that there occurs a packet collision on the tagged channel, given that the tagged channel is occupied by its designated PU. We call $P_C$ the collision probability of the tagged PU for simplicity.

Let $I_C(t)$ be a random variable that is defined by 1 if the tagged PU's packet transmitted on the tagged channel collides with any of SU's packets at slot *t*, and by 0 otherwise. Then by the definition of the collision probability of the tagged PU, $P_C$ is given by

$$P_C := P\{I_C(t) = 1 | T_{PU}(t)\}$$

Where $T_{PU}(t)$ denotes the event that the tagged PU transmits a packet at slot $t$.

Using the conditional probability approach, $P_C$ is computed as follows :

$P_C := P\{I_C(t) = 1 | T_{PU}(t)\} = \sum_{k=0}^{N-1} P\{I_C(t) = 1 | \mathrm{TPUt}, \mathrm{Nt}=k\} \times P\{\mathrm{Nt}=k | T_{PU}(t)\}$
(9)

Where $\tilde{N}(t)$ is the true number of *untagged* idle channels. Note that the maximum number of $\tilde{N}(t)$ isN-1, $P\{\tilde{N}(t) = k | T_{PU}(t)\}$ is given by, for $0 \le k \le N-1$,

$$P\{\tilde{N}(t) = k | T_{PU}(t)\} = \binom{N-1}{k}\pi_1^k\pi_0^{N-1-k} \qquad (10)$$

because PUs behave independently. Using $A_e(t)$ and $B_e(t)$, the conditional probability $P\{I_C(t) = 1 | T_{PU}(t), \tilde{N}(t) = k\}$ is further given by

$$P\{I_C(t) = 1 | T_{PU}(t), \tilde{N}(t) = k\}$$

$= \sum_{n=0}^{k} \sum_{l=0}^{N-k} \mathrm{P}\{A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l} | \}$

$\times P\{I_C(t) = 1 | A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l}, T_{PU}(t), \tilde{N}(t) = k\}$ (11)

Where $\mathrm{P}\{A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l} \mid T_{PU}(t), \tilde{N}(t) = k\}$ *is given by*

$\mathrm{P}\{A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l} \mid T_{PU}(t), \tilde{N}(t) = k\}$

$$= \binom{k}{n}(1 - e_f)^n {e_f}^{k-n} \times \binom{N-k}{l} {e_m}^{l} (1 - e_m)^{N-k-l} \quad (12)$$

If we compute $\mathrm{P}\{\mathrm{I_C}(t) = 1 | \mathrm{A_e}(t) = \mathrm{n}, \mathrm{B_e}(t) = \mathrm{l}, \mathrm{T_{PU}}(t), \tilde{\mathrm{N}}(t) = \mathrm{k}\}$, then by computing (9), (10), (11) and (12) together we obtain the collision probability $P_C$. So our next step is to compute $\mathrm{P}\{\mathrm{I_C}(t) = 1 | \mathrm{A_e}(t) = \mathrm{n}, \mathrm{B_e}(t) = \mathrm{l}, \mathrm{T_{PU}}(t), \tilde{\mathrm{N}}(t) = \mathrm{k}\}$.

When $B_e(t) = 0$, *i.e.*, there occur no misdetection errors at slot $t$, it is obvious that

$P\{I_C(t) = 1 | A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l}, T_{PU}(t), \tilde{N}(t) = k\} = 0.$

When $B_e(t) > 0$, we obtain

$P\{I_C(t) = 1 | A_e(t) = \mathrm{n}, B_e(t) = \mathrm{l}, T_{PU}(t), \tilde{N}(t) = k\}$

$$= \frac{l}{N-k} \sum_{i=1}^{M} \binom{M}{i} {a_{n+l}}^{i} (1 - a_{n+l})^{M-i}$$

$$\times \left(1 - \left(\frac{n+l-1}{n+l}\right)^{i}\right)$$

$$= \frac{l}{N-k}\left(1 - \left(1 - \frac{a_{n+l}}{n+l}\right)^{M}\right) \quad (13)$$

On the RHS of the first equation, $\frac{l}{N-k}$ is the probability

that the tagged channel is sensed as idle by misdetection, $\binom{M}{i} {a_{n+l}}^{i} (1 - a_{n+l})^{M-i}$ is the

probability that there are $i$ active SUs, and $1 - \left(\frac{n+l-1}{n+l}\right)^{i}$ is the probability that at least one of the active SUs transmits a packet on the tagged channel.

By substituting $x$ for $n$+$l$ in (13), the collision probability $P_C$ is given by

$$P_C = \sum_{k=0}^{N-1} \binom{N-1}{k} \pi_1^k \pi_0^{N-k} \sum_{n=0}^{k} \sum_{l=1}^{N-k} \binom{k}{n} \times (1 - e_f)^n {e_f}^{k-n} \times \binom{N-k}{l} {e_m}^{l} (1 - e_m)^{N-k-l}$$

$$\times \frac{l}{N-k}\left(1 - \left(1 - \frac{a_{n+l}}{n+l}\right)^{M}\right)$$

$$= \sum_{x=1}^{N} \left(1 - \frac{a_x}{x}\right)^M \sum_{k=0}^{N-1} \binom{N-1}{k} \pi_1^k \pi_0^{N-k}$$

$$\times \sum_{n=\max(0,x+k-N)}^{\min(k,x-1)} \binom{k}{n}(1 - e_f)^n {e_f}^{k-n}$$

$$\times \binom{N-k}{x-n} {e_m}^{x-n} (1 - e_m)^{N-k-x+n}$$

$$= e_m - \sum_{x=1}^{N} \left(1 - \frac{a_x}{x}\right)^M \sum_{k=0}^{N-1} \binom{N-1}{k} \pi_1^k \pi_0^{N-k}$$

$$\times \sum_{n=\max(0,x+k-N)}^{\min(k,x-1)} \binom{k}{n}(1 - e_f)^n {e_f}^{k-n}$$

$$\times \binom{N-k}{x-n} {e_m}^{x-n} (1 - e_m)^{N-k-x+n} \quad (14)$$

$$= e_m - \sum_{x=1}^{N} g_x(a_x) \mathrm{G}(\mathrm{x})$$

Where

$g_x(a_x) = \left(1 - \frac{a_x}{x}\right)^M$

$\mathrm{G(x)} = \sum_{k=0}^{N-1} \binom{N-1}{k} \pi_1^k \pi_0^{N-k}$

$\times \sum_{n=\max(0,x+k-N)}^{\min(k,x-1)} \binom{k}{n}(1 - e_f)^n {e_f}^{k-n} \times \binom{N-k}{x-n} {e_m}^{x-n} (1 - e_m)^{N-k-x+n}$

Since $g_x(a_x)$ and $G(x)$ are nonnegative for each $x$, the collision probability $P_C$ is always less than or equal to the misdetection error probability $e_m$. In addition, $P_C$ strictly increases in $a_x$ because $g_x(a_x)$ strictly decreases in $a_x$.

## IV. RESULTS

We will use the following parameters in the numerical examples.

- The number of primary channels N=8.
- The transition probabilities between the states of a primary channel p=0.8 and q=0.35.
- The sensing error probabilities: $e_f$ = 0.2 and $e_m$ = 0.1.

We compute the throughput of an SU by using different sets of the APs that satisfy the collision probability requirement when $N$ = 3 and $M$ = 11. In this case, the optimal APs are {0.02, 0.04, 0.06} and the optimal throughput of an SU is 0.0273. Next, Fig. 1 provides the collision probability of a PU using the optimal random access policy with/without the collision probability requirement. We see that our analytical results are well matched with simulation results. In addition, we show that the optimal random access policy protects PUs, i.e., the collision probability of a PU does not exceed the given requirement $\eta$ = 0.02.

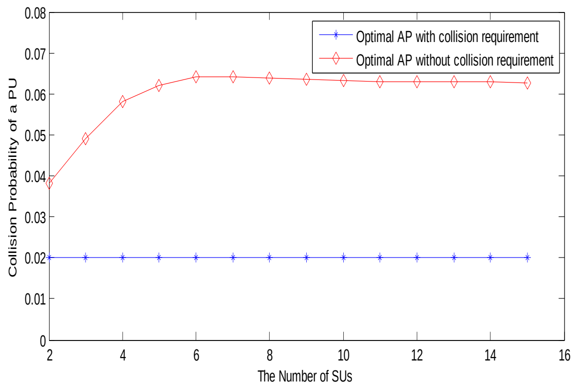


Fig. 1. The packet collision probability of a PU in CRN

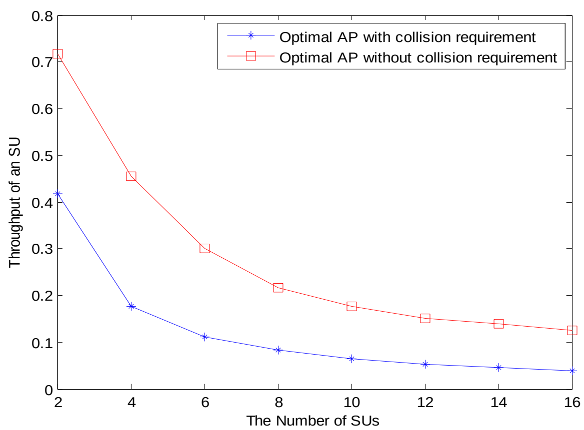


Fig. 2.The throughput (Packet/slot) of an SU in CRN with the packet collision probability requirement

In Fig. 2, we plot the throughput of an SU as a function of the number of SUs when each SU adopts the optimal random access policy. We see that our analytical results are well matched with simulation results as in Fig. 1. We observe that the throughput of an SU decreases as the number of SUs increases. We also observe that the consideration of the collision probability requirement reduces the optimal throughput of an SU. In addition, as the number of SUs increases, the gap between the optimal throughput of an SU with the requirement and that without the requirement becomes smaller.

## CONCLUSIONS

In this paper, we considered a random channel access policy in a cognitive radio network with an imperfect channel sensing scheme. We focused on a channel access policy where each secondary user (SU) stochastically determines to access a channel based on the AP. In the channel access policy, the AP is adapted to the network environment to improve the performance of SUs and to protect the transmission of PUs. To obtain the optimal APs that maximize the throughput of an SU while guaranteeing a given collision probability requirement for PUs, we analyzed the throughput of an SU and the collision probability of a PU under imperfect channel sensing. We obtained an explicit expression on the optimal APs from rigorous mathematical analysis. Numerical results based on our analysis and simulation results showed the validity of our analysis. In addition, we investigated the impact of system parameters such as the number of SUs, sensing error probabilities, and the threshold of the collision probability requirement on the throughput of an SU and the collision probability of a PU.

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