Deteriorating Items Inventory Model with Stock Dependent Demand under Shortages and Variable Selling Price

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Abstract- **An inventory model for Weibull deteriorating items with stock dependent demand and variable selling price is developed. Holding cost is linear function of time. Shortages are allowed and completely backlogged. Numerical example is considered and sensitivity analysis is also carried out for parameters.**

Key Words: Stock dependent demand, Weibull Deterioration, Variable selling price, Shortages, Inventory

I. INTRODUCTION

Research work related to inventory models for deteriorating items has been reported very much in recent years. Within [20] first developed inventory models for deteriorating items. Ghare and Schrader [3] considered inventory problem under constant demand and constant deterioration. An order level inventory model for items deteriorating at a constant rate was studies by Shah and Jaiswal [13]. Aggarwal [1] discussed an order level inventory model with constant rate of deterioration. Burewell [2] developed economic lot size model for price dependent demand under quantity and freight discounts. Mukhopadhyay et al. [7] developed an inventory model for deteriorating items with a price-dependent demand rate. Teng and Chang [16] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Other research work related to deteriorating items can be found in, for instance (Raafat [9], Goyal and Giri [4], Ruxian et al. [10]).

Wee [19] developed EOQ model to allow deterioration and an exponential demand pattern. Shortages were considered and completely backlogged. Sarkar et al. [12] developed a lot size inventory model with backorders, inventory level dependent demand and deterioration. A review of different inventory models with shortages of different types for deteriorating items with different demand patterns and proposed future need of research was investigated by Karmakar and Choudhury [5]. Wang et al. [18] considered the problem of determining the optimal replenishment policy for deteriorating items with variable selling price under stock dependent demand. Patra et al. [8] developed a deterministic inventory model when deterioration rate was time proportional. Demand rate was taken as a nonlinear function of selling price, deterioration rate, inventory holding cost and ordering cost were all functions of time. Tripathy and Mishra [17] dealt with development of an inventory model when the deterioration rate follows Weibull two parameter distribution, demand rate is a function of selling price and holding cost is time dependent. Sana [11] studied inventory model with demand rate dependent on selling price of the item. A deterministic inventory model has been developed for deteriorating items with two parameter Weibull distribution with power pattern demand, shortages and time dependent holding cost by Sharma et al. [14]. Sharma and Chaudhary [15] studied an inventory model for deteriorating items, where rate of deterioration follow two parameter Weibull distribution and shortages. Mathew [6] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time.

In this paper we have developed an inventory model for stock dependent demand, time varying holding cost and variable selling price. Shortages are allowed and completely backlogged. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

$$D(t) = \begin{cases} a+bI(t), & I(t) > 0 \\ a, & I(t) \le 0, \end{cases}$$

A : Ordering cost per order
c : Unit purchasing cost per item
c_2 : Shortage cost per unit
HC : Holding cost per unit time is a linear function of time t ($x+yt$, $x>0$, $0<y<1$)
SC : Shortage cost
DC : deterioration cost
MC : Manufacturing cost
SR : Sales Revenue
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q_1 : Inventory level initially
Q_2 : Shortage of inventory
Q : Order quantity
T : Cycle length
α : Scale parametrs ($0 < \alpha < 1$)
β : Shape parameter ($\beta > 0$)
π : Total profit

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a function of inventory level I(t).
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
- The deterioration of the items follows a Weibull deterioration with parameter α and β.
- The variable selling price S(t) is a function of demand,

  i.e. $S(t) = S_0 - \rho\left(D(t)\right)$

  where S_0, ρ, a and b are positive constants.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

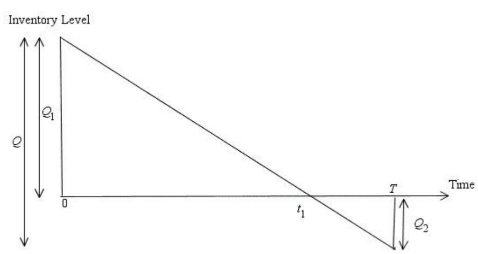


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a + bI(t)), \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -a, \qquad t_1 \le t \le T \qquad (2)$$

with boundary conditions $I(0) = Q_1$, $I(t_1)=0$ and $I(T) = -Q_2$.

The solutions of equations (1) and (2) using boundary conditions are given by:

$$\begin{aligned} I(t) = & -a\left(t + \frac{\alpha}{(\beta+1)}t^{\beta+1} + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right) \\ & - \alpha t^{\beta}\left(-a\left(t + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{1}{2}bt_1^2\right)\right) \\ & - bt\left(-a\left(t + \frac{\alpha}{(\beta+1)}t^{\beta+1} + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right)\right) \end{aligned}$$

$$0 \le t \le t_1 \qquad (3)$$

$$I(t) = a(t_1 - t), \qquad t_1 \le t \le T \qquad (4)$$

(by neglecting higher powers of α and β)

Putting t=0 in equation (3) we get

$$Q_1 = a\left(t_1 + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right). \qquad (5)$$

Putting t = T in equation (4) we get

$$Q_2 = -a(t_1 - T), \qquad (6)$$

And the order quantity is

$$Q = a\left(\frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2 + T\right). \qquad (7)$$

Based on the assumptions and descriptions of the model, the total annual relevant costs TC_i, include the following elements:

(i) Ordering cost (OC) = A (8)

(ii) $HC = \int_0^{t_1}(x+yt)I(t)dt$

$$= \int_0^{t_1}(x+yt)\begin{pmatrix} -a\left(t + \frac{\alpha}{(\beta+1)}t^{\beta+1} + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right) \\ -\alpha t^{\beta}\left(-a\left(t + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{1}{2}bt_1^2\right)\right) \\ -bt\left(-a\left(t + \frac{\alpha}{(\beta+1)}t^{\beta+1} + \frac{1}{2}bt^2\right) + a\left(t_1 + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right)\right) \end{pmatrix}dt$$

$$= -\frac{1}{8}\frac{at_1^2}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}$$

$$\left(\begin{aligned} & -8\left(\begin{aligned} & (-2yt_1 - 2x)\beta^2 \\ & + \left(-14x + ybt_1^2 - 11yt_1\right)\beta - 24x + 3ybt_1^2 - 12yt_1 \end{aligned}\right)\alpha t_1^{\beta} \\ & + (\beta+2)(\beta+3)(\beta+4)\left(\begin{aligned} & 4\alpha t_1^{\beta}\left(\frac{2}{3}ybt_1^2 + \left(-y + xb\right)t_1 - 2x\right) \\ & + (\beta+1)\left(\begin{aligned} & \frac{8}{15}yb^2t_1^3 + \left(-\frac{1}{3}y + xb\right)bt_1^2 \\ & + \left(-\frac{4}{3}y - \frac{4}{3}xb\right)t_1 - 4x \end{aligned}\right) \end{aligned}\right) \end{aligned}\right) \qquad (9)$$

(iii) Deterioration cost:

$$DC = c\left[Q_1 - \int_0^{t_1} D(t)dt\right] = c\left[Q_1 - \int_0^{t_1}(a + bI(t))dt\right]$$

$$= \frac{1}{8}\frac{ac}{(\beta+1)(\beta+2)(\beta+3)}$$

$$\left(\left(\begin{aligned} & -8b^2\alpha(\beta+2)t_1^{\beta+3} - 8\alpha(\beta+2)(\beta+3)(bt_1-1)t_1^{\beta+1} \\ & +b\left(\begin{aligned} & 8\alpha t_1^{\beta+2}\left((bt_1+1)\beta + 2bt_1 + 3\right) \\ & + (\beta+3)\left(8\alpha t_1^{\beta+2} + \left(4\alpha t_1^{\beta} + (\beta+1)(bt_1 - \frac{4}{3})\right)b(\beta+2)t_1^3\right) \end{aligned}\right) \end{aligned}\right)\right) \qquad (10)$$

(iv) Manufacturing cost is given by

$$MC = cQ = ca\left(T + \frac{\alpha}{(\beta+1)}t_1^{\beta+1} + \frac{1}{2}bt_1^2\right) \qquad (11)$$

(v) Shortage cost is given by

$$SC = -c_2\left[\int_{t_1}^{T} I(t)dt\right] = -c_2\left[\int_{t_1}^{T} a(t_1 - t)dt\right]$$

$$= -c_2\left[-\frac{1}{2}a(T^2 - t_1^2) + at_1(T - t_1)\right] \qquad (12)$$

(vi) $$SR = \int_0^T S(t)D(t)dt = \int_0^{t_1} S(t)D(t)dt + \int_{t_1}^{T} S(t)D(t)dt$$

$$= \int_0^{t_1}\left(S_0 - \rho(a+bI(t))\right)(a + bI(t))dt + \int_{t_1}^{T} a\left(S_0 - \rho a\right)dt$$

$$= -\frac{a}{(\beta+1)^2(\beta+2)(\beta+3)(\beta+4)}\left(\begin{array}{l}\frac{1}{3}\rho b^2\left(3-3bt_1+b^2t_1^2\right)\alpha^2(\beta+2)(\beta+3)(\beta+4)t_1^{2\beta+3}a \\ +\frac{2}{15}b\alpha t_1^{\beta+2} \\ \left(\begin{array}{l}-15\rho b\alpha\left(\begin{array}{l}(2-2t_1b)\beta^2 \\ +\left(14-11bt_1+b^2t_1^2\right)\beta \\ +24-12bt_1+3b^2t_1^2\end{array}\right)at_1^{\beta+1} \\ +(\beta+1)(\beta+2)(\beta+3)(\beta+4) \\ \left(-\frac{15}{2}S_0+15\rho a+\frac{15}{4}bt_1S_0-\frac{5}{2}\rho ab^3t_1^3+\rho ab^4t_1^4\right)\end{array}\right)\end{array}\right)$$

$$-\frac{1}{4}\frac{a^2\rho b^2\alpha^2t_1^{2\beta+3}}{\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2(\beta+2)}$$

$$\left(\begin{array}{l}\left(5-2bt_1+b^2t_1^2\right)\beta^3 \\ +\left(\frac{51}{2}-\frac{15}{2}bt_1+5b^2t_1^2\right)\beta^2 \\ +\left(\frac{77}{2}-\frac{25}{2}bt_1+\frac{15}{2}b^2t_1^2\right)\beta+15+3b^2t_1^2\end{array}\right)$$

$$-\frac{2ab\alpha t_1^{2\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}$$

$$\left(\begin{array}{l}(-2a\rho + S_0)\beta^4+\left(\begin{array}{l}-36a\rho - 3a\rho bt_1 - a\rho b^2t_1^2 \\ +3a\rho b^3t_1^3 + a\rho b^4t_1^4+18S_0\end{array}\right)\beta^3 \\ +\left(\begin{array}{l}-45a\rho bt_1 - 238a\rho - 18a\rho b^2t_1^2 \\ +31a\rho b^3t_1^3 + 8a\rho b^4t_1^4 + 119S_0\end{array}\right)\beta^2 \\ +\left(\begin{array}{l}-684a\rho - 222a\rho bt_1 - 107a\rho b^2t_1^2 \\ +86a\rho b^3t_1^3 + 15a\rho b^4t_1^4+342S_0\end{array}\right)\beta \\ -720a\rho - 360a\rho bt_1 - 210a\rho b^2t_1^2 + 48a\rho b^3t_1^3 + 360S_0\end{array}\right)$$

$$-a\left(\begin{array}{l}\frac{2}{105}a\rho b^6t_1^7 - \frac{1}{30}a\rho b^5t_1^6 - \frac{1}{15}a\rho b^4t_1^5 + \frac{1}{8}S_0b^3t_1^4 \\ +\frac{2}{3}\left(a\rho - \frac{1}{4}S_0\right)b^3t_1^3 + \left(a\rho - \frac{1}{2}S_0\right)bt_1^2 + T\left(a\rho - S_0\right)\end{array}\right) \qquad (13)$$

The total profit per unit time is given by

$$\pi(t_1,T) = \frac{SR - A - MC - DC - HC - SC}{T} \qquad (14)$$

Putting values from equations (8) to (13) in equation (14), we get the average profit.

The optimal value of $t_1 = t_1^*$ and $T = T^*$ (say), which maximizes profit $\pi(t_1,T)$ can be obtained by differentiating equation (14) with respect to t_1 and T and equate it to zero

i.e. $$\frac{\partial\pi(t_1,T)}{\partial T} = 0,\ \frac{\partial\pi(t_1,T)}{\partial t_1} = 0, \qquad (15)$$

provided it satisfies the condition

$$\frac{\partial^2\pi(t_1,T)}{\partial T^2} < 0,\ \frac{\partial^2\pi(t_1,T)}{\partial t_1^2} < 0 \text{ and}$$

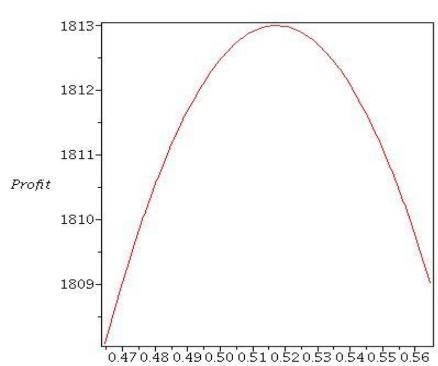
$$\left[\frac{\partial^2\pi(t_1,T)}{\partial T^2}\right]\left[\frac{\partial^2\pi(t_1,T)}{\partial t_1^2}\right] - \left[\frac{\partial^2\pi(t_1,T)}{\partial T\partial t_1}\right]^2 > 0. \qquad (16)$$

IV. NUMERICAL EXAMPLES:

Considering A= Rs.250, a = 600, b=0.05, c=Rs. 5, c_2 =Rs. 3, S_0= Rs. 15, α=0.01, β =2, x = Rs. 1.7, y=0.05, ρ=0.01, in appropriate units. The optimal values of $t_1^* = 0.5172$, $T^*=0.8433$, $Q^* = 510.2691$ and Profit π^* = Rs. 1813.0029

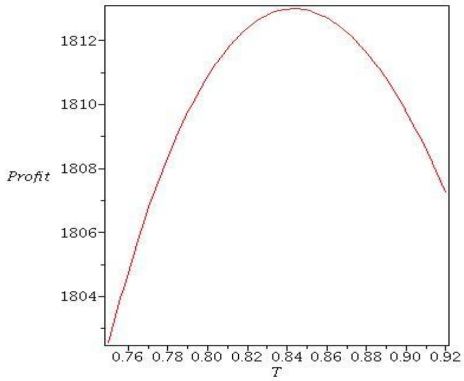
The second order conditions given in equation (16) are also satisfied. The graphical representation of the concavity of the cost functions for the three cases are also given.

t_1 and cost



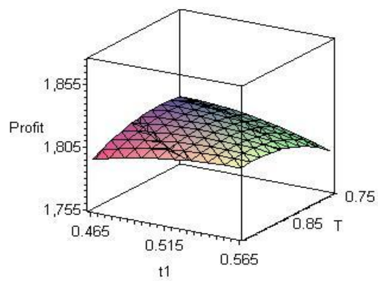
Graph 1

T and cost



Graph 2

t_1, T and cost



Graph 3

V. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Para-meter	%	t_1	T	Profit	Q
a	+20%	0.4541	0.7572	1361.1811	549.1204
	+10%	0.4835	0.7974	1622.6332	527.7499
	-10%	0.5561	0.8966	1932.4257	488.6483
	-20%	0.6017	0.9593	1981.0773	465.1570
x	+20%	0.4619	0.8046	1783.1648	486.1573
	+10%	0.4879	0.8227	1797.4117	496.3020
	-10%	0.5506	0.8672	1830.1507	525.2012
	-20%	0.5890	0.8950	1849.1178	542.6125
ρ	+20%	0.4959	0.8282	1081.7589	500.8526
	+10%	0.5063	0.8356	1447.2867	505.4646
	-10%	0.5287	0.8515	2178.9182	515.3884
	-20%	0.5407	0.8601	2545.0439	520.9431
α	+20%	0.5150	0.8415	1812.3284	509.2062
	+10%	0.5161	0.8424	1812.6647	509.7378
	-10%	0.5184	0.8443	1813.3429	510.8618
	-20%	0.5195	0.8452	1814.0927	511.3925
β	+20%	0.5196	0.8451	1814.0927	511.3002
	+10%	0.5185	0.8442	1813.6043	510.7818
	-10%	0.5159	0.8424	1812.2573	509.7681
	-20%	0.5145	0.8415	1811.3257	509.2806
A	+20%	0.5646	0.9222	1756.3654	558.4615
	+10%	0.5415	0.8837	1784.0524	534.9359
	-10%	0.4916	0.8009	1843.4117	484.4026
	-20%	0.4645	0.7558	1875.5295	456.9168

From the table we observe that as parameters a, A, x and ρ increases/ decreases, there is decrease/ increase in average total profit.

From the table we observe that with increase/ decrease in parameter α, there is corresponding small decrease/ increase in total profit.

From the table we observe that as parameter β increases/ decreases, there is very slight increase/ decrease in average total profit.

CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with linear demand, shortages under variable selling price. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding decrease/ increase in the value of profit.

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