Determination of Flow Stress Constants by Oxley’s Theory

Prakash Naik¹, Ankit Naik²

¹Mechanical Engineering Department, UTU, Bardoli.
²Mechanical Engineering Department, JHD, Palsana.

Abstract—Flow stress is an instantaneous yield stress needed to remove chip area and depends on strain, strain-rate, and temperature. The flow stress is an important input in metal forming and metal cutting processes. The relation of flow stress with strain, strain-rate, and temperature with some unknown constants is known as flow stress model or constitutive model of work material. Johnson and Cook (JC) flow stress model that considers the effect of strain, strain-rate, and temperature on material property is widely used nowadays in finite element method simulation and analytical modeling due to its simple form and easy to use. The constants of Johnson and Cook flow stress model can be obtained by two methods, which are direct and indirect. In the present study, orthogonal cutting in conjunction with an analytical-based computer code are used to determine flow stress data as a function of the high strains, strain rates and temperatures encountered in metal cutting. The automated technique for flow stress determination, developed in the present study, is easier and less expensive than other techniques such as the Hopkinson’s bar method. The constants of the JC flow stress model can be determine by utilizing an inverse solution of Oxley’s machining theory.

Keywords—Oxley’s theory, JC flows stress model, 0.38 % carbon steel, AL6061-T6, SHPB.

I. INTRODUCTION

Metal cutting is the process of producing a component with required size, shape and surface finish by removing a layer of unwanted material from a given work piece. In this process, a wedged shape sharp tool is constrained to move relative to work piece in such a way that layer of material is removed in the form of chip. The objective of metal cutting studies is to develop a model that would enable us to predict cutting performance such as chip formation, cutting forces, cutting temperature, tool wear and surface finish. For accurate modeling of metal cutting processes, number of inputs required such as cutting condition, tool geometry, workpiece material properties, and flow stress model of workpiece material. The flow stress model is one of the most important input for the accurate and reliable predictions of metal cutting models.

The well-known Oxley’s predictive machining theory [1], which is used by many other researchers have used power law equation ($\sigma = \sigma_1 \varepsilon^n$) for flow stress in which the constants $\sigma_1$ and $n$ of flow stress depend on velocity-modified temperature (Tmod) concept. The power law flow stress constants $\sigma_1$ and $n$ are expressed by a different order polynomial equation in the different ranges of Tmod and seventh-order polynomials for $\sigma_1$ and n were used in some range of Tmod for required accuracy.

Nowadays, Johnson and Cook (JC) flow stress model that considers the effect of strain, strain-rate, and temperature on material property is widely used in finite element method simulation and analytical modeling of metal cutting processes due to its simple form and easy to use. The JC flow stress model, also called material mode, is given below.

$$\sigma = (A + Be^n) \left(1 + C \ln \left(\frac{\dot{\varepsilon}}{\varepsilon_o}\right) \left(1 - \left(\frac{T - T_W}{T_m - T_W}\right)^m\right)\right)$$

The first term in parenthesis in Johnson and Cook (JC) equation represents strain hardening. Second term in parenthesis shows that flow stress increases when material are loaded with high strain-rate. The third term represents the well-known fact that as temperature increases the flow stress of material decreases.

The constants of JC flow stress model can be obtained by two methods which are direct and indirect method. In the direct method, usually constants are found by costly experimental Split Hopkinson Pressure Bar test. In the indirect method, constants can be found out using orthogonal cutting test with finite element method or analytical modeling of metal cutting process.

In this paper an attempt has been made to use Oxley’s predictive machining theory [1] to determine the constants of JC flow stress model. A computer program in MATLAB is written for the same.

II. METHODOLOGY

Oxley’s predictive machining theory [1] is extended for Johnson-Cook flow stress model and it is used to find flow stress constants using orthogonal cutting tests data. In Oxley’s theory the shear plane is considered as a thick plane extending on both sides of the shear plane center.
AB, which was considered as a thin shear plane in the theory of Merchant.

\[ AB = \frac{l}{\Delta s^2} \]

\[ Co = \frac{\text{shear plane length } AB}{\text{thickness of the primary shear zone } \Delta s^2}, \quad \delta = \frac{\text{thickness of secondary shear zone}}{t_2} \]

The basis of the theory is to analyze the stress distributions along the line AB, which is the center of the primary deformation zone, and along the tool–chip interface. At the early stages of developing the theory, two experimentally determined constants were required: Co and δ. Co is chosen when the normal stress at tool–chip interface, \( \sigma_n \) (calculated from resultant force at AB) equals \( \sigma_n' \), which is calculated using stress boundary condition at point B. To determine the value of \( \delta \), Oxley and Hastings proposed that it should satisfy the minimum work condition. That is, the value of \( \delta \) can be determined, as part of the solution, as the value that causes the cutting force to be a minimum. They showed that \( \delta \) predicted in this way agreed well with experimental results.

A simplified illustration of the plastic deformation for the formation of a continuous chip when machining a ductile material is given in Fig. 1. There are two deformation zones in this simplified model – a primary zone and a secondary zone. It is commonly recognized that the primary plastic deformation takes place in a finite-sized shear zone. The work material begins to deform when it enters the primary zone from lower boundary CD, and it continues to deform as the material streamlines follow smooth curves until it passes the upper boundary EF. Oxley and coworkers assumed that the primary zone is a parallel-sided shear zone. There is also a secondary deformation zone adjacent to the tool–chip interface that is caused by the intense contact pressure and frictional force. After exiting from the primary deformation zone, some material experiences further plastic deformation in the secondary deformation zone. After exiting from the primary deformation zone, some material experiences further plastic deformation in the secondary deformation zone. Using the quick-stop method to experimentally measure the flow field, Oxley proposed a slip-line field similar to the one shown in Fig. 1. Initially, Oxley and coworkers assumed that the secondary zone is a constant thickness shear zone. In this study, we assume that the secondary deformation zone is triangular shape and the maximum thickness is proportional to the chip thickness, i.e., \( \delta t_2 \).

The assumptions made by Oxley are as follows: (1) plane-strain and steady state conditions are assumed with sharp tool, (2) primary shear zone is assumed to be parallel-sided and secondary shear zone (tool–chip interface) is assumed to be of constant thickness for simplifying the analysis, (3) shear strain at AB is uniform and equal to one-half of the strain in the primary shear zone, (4) temperature and strain are uniform along AB, (5) line AB is a straight slip line during chip formation near the center of slip line field, considered as shear plane in the shear-plane model of chip formation (Ernst and Merchant, 1941), (6) both AB and tool–chip interface are assumed to be the directions of maximum shear stress and maximum shear strain-rate, (7) \( Co \) and \( \delta \) are strain-rate constants for finding strain-rate at the shear zone and tool–chip interface zone respectively. \( Co \) is the ratio of shear plane length AB to thickness of the primary shear zone \( l/\Delta s^2 \) and \( \delta \) is the ratio of the thickness of secondary shear zone to chip thickness \( \Delta s_1 l/2 \).

The basis of tuning the model is to fix the value of shear angle \( \phi \), strain-rate constant Co at shear zone and strain-rate constant \( \delta \) at tool–chip interface zone using iteration.
The basic model of chip formation considers a continuous chip with no built-up edge. This then assumes that the chip formation is in a steady state process. Also the relatively simple case of orthogonal machining in which the cutting edge is set normal to the cutting velocity is considered. If the thickness of the layer to be removed is small compared to its width then deformation occurs in approximately plane strain conditions. The model of chip formation is shown in Fig.1 where the tool in contact with the work piece is assumed to be perfectly sharp. The model was developed from the slip-line field analysis of experimental flow fields of Palmer and Oxley and Stevenson and Oxley. The plane AB, near the centre of the zone in which the chip is formed and the tool-chip interface are both assumed to be directions of maximum shear stress and maximum shear strain-rate. It should be noted that the plane AB is found from the geometric construction as used in defining the shear plane in the Merchant shear plane model of chip formation. The basis of the theory is to analyze the stress distributions along AB and the tool-chip interface in terms of the shear angle \( \phi \), work material properties, etc., and then to select \( \phi \) so that the resultant forces transmitted by AB and the interface are in equilibrium. The tool is assumed to be perfectly sharp. Once \( \phi \) is known then the chip thickness \( t_1 \) and the various components of force can be determined from the following geometric relations:

\[
\begin{align*}
F_c &= R \cos(\lambda - \alpha); \\
F_t &= R \sin(\lambda - \alpha); \\
F &= R \sin \lambda; \\
N &= R \cos \lambda; \\
R &= F_s / \cos \theta;
\end{align*}
\]  

(1)

By starting at the free surface just ahead of A and applying the appropriate stress equilibrium equation along AB it can be shown that for \( 0 < \phi < \pi/4 \) the angle made by the resultant \( R \) with AB is given by

\[
\tan \theta = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - C_0 n
\]  

(2)

From the geometry of Fig.1 the angle \( \theta \) can also be expressed in terms of other angles by the equation

\[
\theta = \phi + \lambda - \alpha
\]  

(3)

Oxley and co-workers utilized a modified Boothroyd’s temperature model and used it in their analysis. In this model, the temperature rise at the primary shear zone is given by

\[
T_{AB} = T_w + \psi \Delta T_{SZ}
\]  

(4)

The work carried out in the shear zone is \( F_s V_s \) and a mass of chip per unit time, \( m_{chip} \), is given by

\[
\Delta T_{SZ} = \frac{(1 - \beta) F_s V_s}{m_{chip} C_p}
\]

\(
\beta = 0.5 - 0.35 \log_{10} (R_t \tan \phi) \) for \( 0.04 \leq R_t \tan \phi \leq 10 \)

\[
\beta = 0.3 - 0.15 \log_{10} (R_t \tan \phi) \) for \( R_t \tan \phi \geq 10 \)

(5)

Where \( R_t \) is non-dimensional thermal number given by

\[
R_t = \frac{\rho C_p V_t}{K}
\]  

(6)

The shear strain at AB is given by

\[
\gamma_{AB} = \frac{1}{2} \left( \frac{\cos a}{\sin \phi \cos (\beta - \phi)} \right)
\]  

(7)

Shear strain rate along AB is given by,

\[
\dot{\gamma}_{AB} = \frac{C_\phi V_s}{l}
\]

(8)

The average temperature at the tool-chip interface (T\textsubscript{int}) from which the average shear flow stress at the interface is determined is given by

\[
T_{int} = T_w + \Delta T_{SZ} + \psi \Delta T_M
\]  

(9)

\( \Delta T_M \) is the maximum temperature rise in the chip and the factor \( \psi \) (0 \leq \psi \leq 1) allows for \( T_{int} \) being an average value. Using numerical methods Boothroyd has calculated \( \Delta T_M \) by assuming a rectangular plastic zone (heat source) at the tool-chip interface and has shown that his results agree well with experimentally measured temperatures. If the thickness of the plastic zone is taken as \( \delta t_2 \), where \( \delta \) is the ratio of this thickness to the chip thickness \( t_2 \), then Boothroyd’s results can be represented by the equation

\[
\log \left( \frac{\Delta T_M}{\Delta T_C} \right) = 0.06 - 0.195 \delta \left( \frac{R_t t_2}{h} \right)^2 + 0.5 \log \left( \frac{R_t t_2}{h} \right)
\]  

(10)

\( \Delta T_C \) is given by equation,

\[
\Delta T_C = \frac{F V_c}{m_{chip} C_p}
\]  

(11)

'\( h \)' is the tool-chip contact length which can be calculated from the equation
\[ h = \frac{t_1 \sin \theta}{\cos \lambda \sin \theta} \left( 1 + \frac{C_0 n_{eq}}{3 \left( 1 + 2 \frac{\pi}{4} - \theta - C_0 n_{eq} \right)} \right) \]  
\[ \text{Eq. 12} \]

The maximum shear strain-rate at the tool-chip interface, which is also needed in determining the shear flow stress, is found from the equation

\[ \dot{\gamma}_{int} = \frac{v_c}{\delta t_2} \]  
\[ \text{Eq. 13} \]

Shear stress at tool chip interface is given by,

\[ \tau_{int} = \frac{F}{h_w} \]  
\[ \text{Eq. 14} \]

To determine \( C_{eq} \), Oxley and Hastings considered the stress boundary condition at the cutting edge in Fig.1 which had previously been neglected. For a uniform normal stress distribution at the interface the average normal stress is given by

\[ \sigma_N = \frac{N}{h_w} \]  
\[ \text{Eq. 15} \]

If \( AB \) turns through the angle \( \phi-a \) (in negligible distance) to meet the interface at right angles, as it must do if the interface is assumed to be a direction of maximum shear stress, then it can be shown that

\[ \sigma_N' = k_{AB} \left( 1 + \frac{\pi}{2} - 2a - 2C_{0n_{eq}} \right) \]  
\[ \text{Eq. 16} \]

The maximum shear strain at the tool–chip interface is calculated as

\[ \dot{\gamma}_{int} = 2Y_{AB} + 0.5Y_M \]  
\[ \text{Eq. 17} \]

\( Y_M \) is the total maximum shear strain occurring at the tool–chip interface and is given by

\[ Y_M = \frac{h}{\delta t_2} \]  
\[ \text{Eq. 18} \]

Therefore, equivalent strain at the tool–chip interface is

\[ \varepsilon_{int} = \left( \frac{\dot{\gamma}_{int}}{\sqrt{3}} \right) \left( 2Y_{AB} + 0.5Y_M \right) \]  
\[ \text{Eq. 19} \]

Once the strain and strain-rate at the tool–chip interface are found, the flow stress at the tool–chip interface can be calculated as

\[ k_{chip} = \frac{1}{\sqrt{3}} \left( A + B \varepsilon_{int}^n \right) \left( 1 + C \ln \frac{\dot{\gamma}_{int}}{\varepsilon_0} \right) \left( 1 - \frac{T_{int}-T_W}{T_m-T_W} \right)^m \]  
\[ \text{Eq. 20} \]

Work Material Properties of Steel [5]

Work material properties are very important input for predictive theory. Since the predictive theory described above relies on work material properties, it is essential that the work material properties of the alloys being considered here be known. The effects of strain rate and temperature were combined into one parameter as introduced by MacGregor and Fisher called the velocity modified temperature, \( T_{mod} \). The relationship can be expressed as

\[ T_{mod} = T \left( 1 - \nu \left( \frac{\dot{\epsilon}}{\varepsilon_0} \right) \right) \]  
\[ \text{Eq. 21} \]

The values obtained from the plane strain machining tests are plotted as uniaxial flow stress and are related using the following relationships

\[ k_{AB} = \frac{\sigma_{\text{eq}}}{\sqrt{3}} \]  
\[ \text{Eq. 22} \]

\[ \varepsilon_{\text{eq}} = \frac{v_c}{\delta t_2} \]  
\[ \text{Eq. 23} \]

\[ \dot{\varepsilon}_{\text{eq}} = \frac{\dot{\gamma}_{\text{eq}}}{\sqrt{3}} \]  
\[ \text{Eq. 24} \]

When thermal properties are considered the influence of carbon content on specific heat is found to be small and the equation

\[ S = 420 + 0.5047^T \]  
\[ \text{Eq. 25} \]

There is a marked influence on thermal conductivity \( K \). \( K \) is allowed to vary with carbon content and other alloying elements on the basis of the experimental results of Woolman & Mottram. The equations obtained in this way give for example

\[ K = 54.17 - 0.02987^T \]  
\[ \text{Eq. 26} \]

Above equation used for a steel of chemical composition 0.02%\( C \), 0.15%\( Si \), 0.015%\( S \), 0.72%\( Mn \) and 0.015%\( Al \).

\[ K = 52.61 - 0.02817^T \]  
\[ \text{Eq. 27} \]

Above equation used for a steel of chemical composition 0.38%\( C \), 0.01%\( Si \), 0.77%\( Mn \), and 0.015%\( P \). In carrying out the calculations for temperatures it is found that the density can be taken as 7862 kg/m\(^3\) for all the steels considered.

Work material properties of aluminum alloy [5]

When the thermal properties are considered for the Aluminium alloys the following equations are used. These are based on a compilation of information from the
thermo physical Research Centre Handbook. The specific heat is given by

\[ S = 832.83 + 1.077T - 0.0021T^2 + 0.000027T^3 \]  

(28)

The thermal conductivity, \( K \), for pure aluminium is obtained from the following equation

\[ K = 237.89 + 0.0097T-0.00007T^2 \]  

(29)

The thermal conductivity for aluminium alloys reduces with the alloy content and to cater for this change a reduction factor is introduced. This factor is given by

\[ K_{rf} = Al \left[ 1 - \sum_{i=1}^{n} \text{other elements} \right] \]  

(30)

The parameters of the Johnson and Cook constitutive model can be computed in an iteration scheme by utilizing an inverse solution of Oxley’s machining theory.

Flow stress (\( \sigma \)) is calculated through the Johnson-Cook constitutive equation with the estimate parameters (A, B, n, C, m).

\[ \sigma = (A + Be^n) \left(1 + C \ln \left( \frac{\varepsilon}{\varepsilon_0} \right) \left(1 - \left( \frac{T-T_W}{T_m-T_W} \right)^m \right) \right) \]

Figure 2: Methodology to determine flow stress and obtain the parameters of the JC constitutive model

![Flow Chart](image-url)

Figure 3: Flow chart for the Oxley’s predictive theory applied to Johnson cook flow stress model [3]
III. RESULT AND DISCUSSION

Constants of JC flow stress model for 0.38% carbon steel and AL6061-T6 are determined by utilizing an inverse solution of Oxley’s machining theory and orthogonal cutting tests for various cutting conditions. A computer program in Matlab (2006) is developed to carry out the analysis.

3.1 0.38% Carbon steel

Orthogonal cutting tests data for 0.38% carbon steel are adopted from Oxley (1989).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat (J/KgK)</td>
<td>420 + 0.504T</td>
<td>52.61 - 0.0281T</td>
<td>8000</td>
</tr>
</tbody>
</table>

Orthogonal cutting conditions data for 0.38% carbon steel \((w = 4\text{mm}, \alpha = -5^\circ)\) (Oxley, 1989).

<table>
<thead>
<tr>
<th>Test</th>
<th>V(m/min)</th>
<th>(t_1)</th>
<th>(t_2)</th>
<th>(F_c)</th>
<th>(F_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.125</td>
<td>0.44</td>
<td>347</td>
<td>257</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>0.125</td>
<td>0.35</td>
<td>297</td>
<td>185</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>0.125</td>
<td>0.29</td>
<td>260</td>
<td>133</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>0.25</td>
<td>0.60</td>
<td>519</td>
<td>268</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>0.5</td>
<td>1.20</td>
<td>1027</td>
<td>535</td>
</tr>
</tbody>
</table>

Comparison between predicted and experimental value of 0.38% carbon steel.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.38% carbon steel</th>
<th>MATLAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>553.1</td>
<td>552</td>
</tr>
<tr>
<td>B</td>
<td>600.8</td>
<td>604</td>
</tr>
<tr>
<td>C</td>
<td>0.0134</td>
<td>0.0131</td>
</tr>
<tr>
<td>n</td>
<td>0.234</td>
<td>0.231</td>
</tr>
<tr>
<td>m</td>
<td>1</td>
<td>0.95</td>
</tr>
<tr>
<td>Co</td>
<td>5.4</td>
<td>6</td>
</tr>
<tr>
<td>phi</td>
<td>21.82°</td>
<td>20.99°</td>
</tr>
</tbody>
</table>

The predicted results are compared with the experimental results for 0.38% Carbon steel and AL6061-T6. The comparison shows that the predictions are in close agreement.

V. CONCLUSIONS

The developed model can be used to determine the constants of JC flow stress model in a reverse approach using orthogonal cutting test data instead of experimentally intensive SHPB test. Orthogonal cutting test data for two materials namely 0.38% carbon steel and AL6061-T6 from the available literature are used to validate the present work and results are found in a good agreement with the experimental results.

ACKNOWLEDGEMENT

I would like to thank Dr. D. I. Lalwani (NIT, Surat) for his valuable guidance and great support to carry out this work.

Notation

- \(A\): Yield strength in JC flow stress model
- \(B\): Strength coefficient in JC flow stress model
- \(C\): Strain rate constant in JC flow stress model
- \(\text{C}_\text{O}\): It is defined as ratio of shear plane length\((l)\) to thickness of primary shear zone
- \(\text{C}_\text{P}\): Specific heat of work-piece material
- \(F\): Friction force
- \(F_c\): Cutting force in velocity direction
- \(F_s\): Shear force along the shear plane \(AB\)
- \(h\): Tool chip interface length
- \(k_{\text{AB}}\): Shear flow stress at the shear plane \(AB\)
- \(k_{\text{chip}}\): Shear flow stress along the tool chip interface
- \(K\): Thermal conductivity of workpiece material
- \(l\): Length of shear plane \(AB\)
**REFERENCES**


