Application of Adomian Decomposition Method for Solving Highly Nonlinear Initial Boundary Value Problems

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Abstract:- **In this paper, Adomian Decomposition Method is applied to solve various forms of Highly Nonlinear Initial Boundary Value Problem .Even this method is a non-numeric method , it can be adapted for solving nonlinear partial differential equations.The nonlinear parameters can be obtained by using Adomian polynomials.It follows that non-linearities in the equation can be handled easily and accurate solution may be obtained for any physical problem.We illustrate this technique with the help of example and represent solutions graphically by Mathematica software.**

***Keywords*: Nonlinear Initial Boundary Value Problem, Adomian Decomposition Method, Adomian Polynomials, Mathematica.**

I. INTRODUCTION

Nonlinear Partial Differential Equations have remarkable developments in different areas like gravitation ,chemical reaction ,fluid dynamics,dispersion,nonlinear optics,plasma physics,acoustics etc. Nonlinear wave propagation problems have provided solutions of different physical structures than solutions of linear wave equations . Nonlinear Partial Differential equations have been widely studied throughout recent years [1, 4, 5, 6, 7]. The importance of obtaining the exact solutions of nonlinear equations in mathematics is still a significant problem that needs new research work.

The Partial Differential Equation is termed as Initial Boundary Value Problem if both initial conditions and boundary conditions are prescribed.In this paper we will study the one dimensional heat flow.We will discuss the heat conduction in a rod and temperature distribution of a rod , which is governed by an Initial Boundary Value Problem.

Recently considerable attention has been given to Adomian Decomposition Method for solving Nonlinear Initial Boundary Value Problem. The ADM was introduced by Adomian [2, 3] in the early 1980 to solve nonlinear ordinary and partial differential equations. This method also discuss the appearance of Noise Terms in inhomogeneous equations. It is used for obtaining solution in a closed form with only two successive iterations. It avoids artifical boundary conditions , linearization and yields an efficient numerical solution with high accuracy.

The organization of this paper as follows . In Section 2 basic idea of Nonlinear Initial Boundary Value Problem is presented, In section 3 , we describe the ADM to solve Nonlinear Initial Boundary Value Problem . In section 4 , we present examples to show the efficiency of using ADM to solve Nonlinear Initial Boundary Value Problem. Finally , relevant conclusions are drawn in section 5.

II. NONLINEAR INITIAL BOUNDARY VALUE PROBLEM

In this section, we study about Nonlinear Initial Boundary Value Problem.

Definition 2.1 *The Nonlinear Initial Boundary Value Problem in its standard form is given by*

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 F(u(x,t))}{\partial x^2}$$

$$\text{initialcondition:} u(x,0) = f(x)$$

$$\text{boundaryconditions} u(0,t) = 0,\ u(1,t) = 0, t \ge 0$$

Where $F(u(x,t)) = u^m, m \in N$ is nonlinear term. In the next section, we develop the Adomain decomposition method for Nonlinear Initial Boundary Value Problem.

III. THE ADOMIAN DECOMPOSITION METHOD (ADM)

To illustrate the basic idea of this method , we consider a general Nonlinear Initial Boundary Value Problem as follow

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 F(u(x,t))}{\partial x^2} \quad (3.1)$$

$$\text{initialcondition:} u(x,0) = f(x) \quad (3.2)$$

$$\text{boundaryconditions} u(0,t) = 0,\ u(1,t) = 0, t \ge 0 \quad (3.3)$$

In an operator form equation (3.1) can be rewritten as

$$L_t u(x,t) = L_{xx} F(u(x,t)) \quad (3.4)$$

$$\text{initial condition}: u(x,0) = f(x) \quad (3.5)$$

$$\text{boundary conditions}\ u(0,t) = 0,\ u(1,t) = 0, t \geq 0 \quad (3.6)$$

Where L_t is a first order differential operator and the inverse operator L_t^{-1} is an integral operator defined by

$$L_t^{-1} = \int_0^t (.)dt \quad (3.7)$$

Applying L_t^{-1} to both sides of equation (3.4),we have

$$u(x,t) = f(x) + L_t^{-1}[L_{xx} F(u(x,t))] \quad (3.8)$$

Now, we decompose the unknown function $u(x,t)$ into sum of an infinite number of components given by the decomposition series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad (3.9)$$

The nonlinear terms $Nu(x,t)$ are decomposed in the following form:

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n \quad (3.10)$$

where the Adomian polynomial can be determined as follows:

$$A_n = \frac{1}{n!}[\frac{d^n N}{d\lambda^n}(\sum_{k=0}^{n} \lambda^k u_k)]_{\lambda=0} \quad (3.11)$$

where A_n is called Adomian polynomial and that can be easily calculated by Mathematica software.

Substituting the decomposition series (3.9) and (3.10) into both sides of equation (3.8) gives

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + L_t^{-1}[L_{xx} \sum_{n=0}^{\infty} A_n] \quad (3.12)$$

The components $u_n(x,t)$, $n \geq 0$ of the solution $u(x,t)$ can be recursively determined by using the relation as follows:

$$u_0(x,t) = f(x),$$

$$u_{k+1}(x,t) = L_t^{-1}[L_{xx} A_k]$$

That leads to

$$u_0(x,t) = f(x)$$
$$u_1(x,t) = L_t^{-1}[L_{xx} A_0]$$
$$u_2(x,t) = L_t^{-1}[L_{xx} A_1]$$
$$u_3(x,t) = L_t^{-1}[L_{xx} A_2]$$
$$\vdots$$

This completes the determination of the components of $u(x,t)$ .Based on this determination ,the solution in a series form is obtained.In many cases a closed form solution can be obtained. In the next section, we illustrate some examples.

IV. APPLICATIONS

ADM for Nonlinear Initial Boundary Value Problem:

In order to elucidate the solution procedure of the ADM,we consider Nonlinear Initial Boundary Value Problem.

Test Problem (i): Consider the following Nonlinear Initial Boundary Value Problem

$$L_t u(x,t) = L_{xx} u^3$$

$$\text{initial condition}: u(x,0) = sinx$$

$$\text{boundary conditions}\ u(0,t) = 0,\ u(1,t) = 0, t \geq 0$$

By using ADM, we have following recursive relation

$$u_0(x,t) = sinx,$$

$$u_{k+1}(x,t) = L_t^{-1}[L_{xx} A_k]$$

That leads to

$$u_0(x,t) = sinx$$

Calculating the values of u_1, as follow-

$$u_1(x,t) = L_t^{-1}[L_{xx} A_0]$$
$$A_0 = u_0^3,$$
$$A_0 = (sinx)^3,$$
$$u_1(x,t) = L_t^{-1}[L_{xx}(sinx)^3],$$
$$u_1(x,t) = [\frac{-3}{4} sinx + \frac{9}{4} sin3x]t,$$

Calculating the values of u_2, as follow-

$$u_2(x,t) = L_t^{-1}[L_{xx} A_1]$$
$$A_1 = 3u_0^2 u_1,$$
$$A_1 = [\frac{-9}{4} sin^3 x + \frac{27}{4} sin^2 x sin3x]t,$$

$$u_2(x,t) = L_t^{-1}[L_{xx}(\frac{-9}{4} sin^3 x + \frac{27}{4} sin^2 x sin3x)t],$$

$$u_2(x,t) = [\frac{-567}{16} sin3x + \frac{675}{16} sin5x]\frac{t^2}{2},$$

$$\vdots$$

Therefore, the series solution for the IBVP is given by

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots.$$

Substituting values of components in above equation, we get the solution of Nonlinear Initial Boundary Value Problem.

$$u(x,t) = sinx + [\frac{-3}{4} sinx + \frac{9}{4} sin3x]t + [\frac{-567}{16} sin3x + \frac{675}{16} sin5x]\frac{t^2}{2} + \cdots$$

The graphical representation of solutions by Mathematica software:

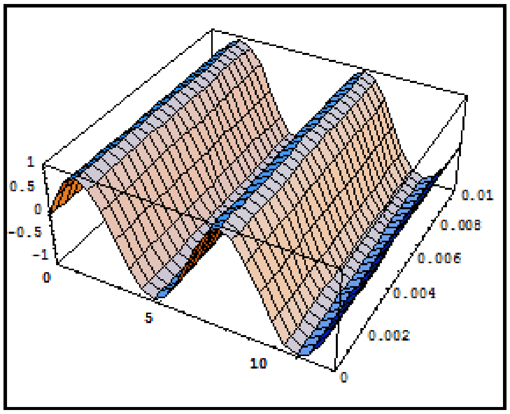

Fig.4.1: The solution of Nonlinear Initial Boundary Value Problem

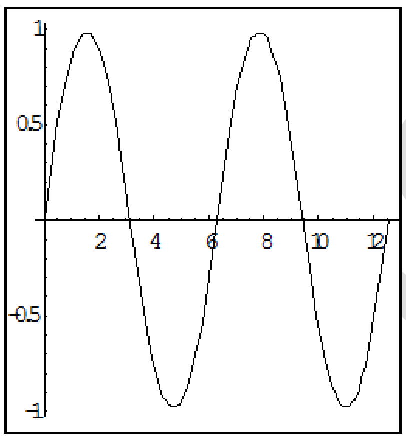

Fig.4.1: The solution of Nonlinear Initial Boundary Value Problem

V. CONCLUSIONS

The main objective of this work is to obtain a solution for Nonlinear Initial Boundary Value Problem. We observe that ADM is a powerful method to solve Nonlinear Initial Boundary Value Problem. To, show the applicability and efficiency of the proposed method ,the method is applied to obtain the solutions of several examples. The obtained results demonstrate the reliability of the algorithm. It is worth mentioning that the proposed technique is capable of reducing the volume of the computational work as compared to the classical methods. Finally, we come to the conclusion that the ADM is very powerful and efficient in finding solutions for wide class of Nonlinear Initial Boundary Value Problem.

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