Inventory Model with Different Deterioration Rates for Imperfect Quality Items and Inflation considering Price and Time Dependent Demand under Permissible Delay in Payments

Shital S. Patel

Department of Statistics, Veer Narmad South Gujarat University, Surat, INDIA

Abstract: One of the assumptions for an economic order quantity model is that all items received in an order are of perfect quality and is not always fulfilled. Some of the items are of defective quality in the lot received. Another assumption is that as soon as items are received, payments are made. In today's competitive supplier allows certain fixed period known as permissible delay for payment to the retailer for settling the amount of items received. Keeping this reality, a deterministic inventory model with imperfect quality is developed when deterioration rate is different during a cycle. Here it is assumed that demand is a function of time and price. Numerical example is taken to support the model. Sensitivity analysis is also carried out for parameters.

Key Words: Inventory model, Varying Deterioration, Time dependent demand, Price dependent demand, Defective items, Inflation, Permissible Delay

I. INTRODUCTION

Most of the items lose their characteristics overtime and this characteristic is defined as deterioration. Ghare and Schrader [8] considered inventory model with constant rate of deterioration. Covert and Philip [7] extended the model by considering variable rate of deterioration. Mandal and Phaujdar [14] presented an inventory model for stock dependent consumption rate. Haiping and Wang [11] studied an economic policy model for deteriorating items with time proportional demand. Patel and Parekh [17] developed an inventory model with stock dependent demand under shortages and variable selling price. Other research work related to deteriorating items can be found in, for instance (Raafat [20], Goyal and Giri [10], Ruxian et al. [22]).

In reality, it happens that units ordered are not of 100% good quality. Rosenblat and Lee [21] were the first to focus on defective items. Salman and Jaber [24] developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. Salman and Jaber’s [24] model was extended by Wee et al. [26] by allowing shortages. Chang [4] studied an inventory model to investigate the effects of imperfect products on the total inventory cost associated with an EPQ model. Patel and Patel [18] developed an EOQ model for deteriorating items with imperfect quantity items. Hauck and Voros [12] considered inventory model in which percentage of defective items as a random variable and defined the speed of the quality checking as a variable.

An economic order quantity model under the condition of permissible delay in payments was developed by Goyal [9]. Aggarwal and Jaggi [1] extended Goyal’s [9] model to consider the deteriorating items. The related work are found in (Chung and Dye [5], Salameh et al. [23], Chung et al. [6], Chang et al. [3]).

The effect of inflation and time value of money play important role in practical situations. Buzacott [2] and Mishra [15] simultaneously developed inventory model with constant demand and single inflation rate for all associated costs. Mishra [16] considered different inflation rate for different costs associated with inventory model with constant rate of demand. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [13]. An EOQ model with linear demand and permissible delay in payments was considered by Singh [25]. The effect of inflation and time value of money were also taken into account. An inventory model with inflation and permissible delay in payments was considered by Patel and Patel[19].

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model for imperfect quality items with different deterioration rates. Demand of the product is time and price dependent for the cycle under time varying holding cost. Shortages are not allowed. To illustrate the model, numerical example is
provided and sensitivity analysis of the optimal solutions for major parameters are also carried out.

II. ASSUMPTIONS AND NOTATIONS

The following notations are used for the development of the model:

\[ D(t) : \text{Demand rate is a function of time and price } (a+bt-\rho p), \ a>0, \ 0<b<1, \ \rho>0 \]
\[ c : \text{Purchasing cost per unit} \]
\[ p : \text{Selling price per unit} \]
\[ d : \text{defective items (\%)} \]
\[ 1-d : \text{good items (\%)} \]
\[ \lambda : \text{Screening rate} \]
\[ \lambda_{SR} : \text{Sales revenue} \]
\[ A : \text{Replenishment cost per order} \]
\[ z : \text{Screening cost per unit} \]
\[ p_d : \text{Price of defective items per unit} \]
\[ h(t) : \text{Variable Holding cost } (x+yt), \ x>0, \ 0<y<1 \]
\[ M : \text{Permissible period of delay in settling the accounts with the supplier} \]
\[ I_e : \text{Interest earned per year} \]
\[ I_p : \text{Interest paid per year} \]
\[ R : \text{Rate of inflation} \]
\[ t_1 : \text{Screening time} \]
\[ T : \text{Length of inventory cycle} \]
\[ I(t) : \text{Inventory level at any instant of time } t, \ 0 \leq t \leq T \]
\[ Q : \text{Order quantity} \]
\[ \theta : \text{Deterioration rate during } \mu_1 \leq t \leq \mu_2, \ 0< \theta<1 \]
\[ \theta_t : \text{Deterioration rate during } \mu_2 \leq t \leq T, \ 0< \theta<1 \]
\[ \pi : \text{Total relevant profit per unit time} \]

ASSUMPTIONS:

The following assumptions are considered for the development of model:

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The screening process and demand proceeds simultaneously but screening rate (\( \lambda \)) is greater than the demand rate i.e. \( \lambda > (a+bt-\rho p) \).
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (\( \lambda \)) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items purchased.

- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

In the following situation, Q items are received at the beginning of the period. Each lot having a d \% defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of \( \lambda \) units per unit time which is greater than demand rate for the time period 0 to \( t_1 \). During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process. The defective items are sold immediately after the screening process at time \( t_1 \) as a single batch at a discounted price. After the screening process at time \( t_1 \) the inventory level will be \( I(t_1) \) and at time \( T \) inventory level will become zero due to demand and partially due to deterioration.

Also here \( t_1 = \frac{Q}{\lambda} \) (1)

and defective percentage (d) is restricted to

\[ d \leq 1-\frac{(a+bt-\rho p)}{\lambda} \] (2)

Let \( I(t) \) be the inventory at time \( t \) (0 ≤ t ≤ T) as shown in figure.

![Figure 1](image-url)
\[
\frac{dI(t)}{dt} + \theta I(t) = -(a + bt)P
\]

with initial conditions \(I(0) = Q, I(\mu_1) = S_1\) and \(I(T) = 0\).

Solutions of these equations are given by

\[
I(t) = Q - (at - pt + \frac{1}{2}bT^2), \quad \mu_1 \leq t \leq T
\]

(6)

(7)

(8)

(9)

From equations (7) and (8), putting \(t = \mu_1\), we have

\[
Q = S_1 + \left( a\mu_1 - p\mu_1 + \frac{1}{2}b\mu_1^2 \right).
\]

(10)

So from equations (11) and (12), we get

\[
S_1 = \frac{1}{1 + \theta(\mu_1 - \mu_2)}
\]

(13)

Putting value of \(S_1\) from equation (13) into equation (7), we have

\[
I(t) = \frac{1 + \theta(\mu_1 - t)}{1 + \theta(\mu_1 - \mu_2)}
\]

(14)

Similarly putting value of \(S_1\) from equation (13) in equation (10), we have
\[
Q = \frac{1}{1+\theta(\mu_1-\mu_2)} \left[ \begin{array}{c}
a(T - \mu_2) - \rho p(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\
+ \frac{1}{6}a\theta(T^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(T^3 - \mu_2^3) + \frac{1}{8}b\theta(T^4 - \mu_2^4) \\
- \frac{1}{2}a\theta\mu_2^2(T - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(T - \mu_2) \\
- \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^3) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) \\
- \frac{1}{2}\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\
- \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) \\
- \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \\
+ \left( a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) \right) \end{array} \right] 
\]

Using (15) in (6), we have

\[
I(t) = \frac{(1-d)}{1+\theta(\mu_1-\mu_2)} \left[ \begin{array}{c}
a(T - \mu_2) - \rho p(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\
+ \frac{1}{6}a\theta(T^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(T^3 - \mu_2^3) + \frac{1}{8}b\theta(T^4 - \mu_2^4) \\
- \frac{1}{2}a\theta\mu_2^2(T - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(T - \mu_2) \\
- \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^3) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) \\
- \frac{1}{2}\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\
- \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) \\
- \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \\
+ (1-d)\left( a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) \right) \end{array} \right] 
\]

Based on the assumptions and descriptions of the model, the total annual relevant profit (\(\mu\)) include the following elements:

(i) Ordering cost (OC) = A

(ii) Screening cost (SC) = zQ

(iii) HC = \(\int_0^\tau (x+yt)I(t)e^{rt}dt\)

(iv) DC = \(\int_{\mu_1}^{\mu_2} \int_0^\tau (x+yt)I(t)e^{rt}dt + \tau \int_{\mu_1}^{\mu_2} \int_0^\tau (x+yt)I(t)e^{rt}dt \)

(v) SR = \(\int_0^\tau \left( p_0 + \int_{\mu_1}^{\mu_2} (a + bt) e^{rt}dt + p_0 dQ \right) \)

To determine the interest earned, there will be two cases i.e. Case I: (0 ≤ M ≤ T) and Case II: (0 ≤ T ≤ M).

**Case I: (0 ≤ M ≤ T)**: In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

(vi) Interest earned per cycle:
Case II: \(0 \leq T \leq M\):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vii) Interest earned up to the permissible delay period is:

\[
IE_2 = \int_0^T (a + bt - \rho p) t e^{-\rho t} dt
\]

To determine the interest payable, there will be four cases i.e.

Interest payable per cycle for the inventory not sold after the due period M is

Case I: \(0 \leq M \leq \mu_1\):

(viii) \(IP_1 = c l_p \left(\int_0^\mu_1 I(t)e^{Rt}dt + \int_{\mu_1}^T I(t)e^{Rt}dt + \int_0^T I(t)e^{Rt}dt\right)\)

Case II: \(\mu_1 \leq M \leq \mu_2\):

(ix) \(IP_2 = c l_p \left(\int_0^\mu_1 I(t)e^{Rt}dt + \int_{\mu_1}^T I(t)e^{Rt}dt\right)\)

Case III: \(\mu_2 \leq M \leq T\):

(x) \(IP_3 = c l_p \int_0^T I(t)e^{Rt}dt\)

Case IV: \(M > T\):

(xi) \(IP_4 = 0\)

(by neglecting higher powers of \(\theta\) and \(R\))

The total profit \((\pi_i), i=1,2,3\) and 4 during a cycle consisted of the following:

\[
\pi_i = \frac{1}{T}[SR - OC - SrC - HC - DC - IP_i + IE_i]
\]  
(29)

Substituting values from equations (18) to (28) in equation (29), we get total profit per unit. Putting \(\mu_1 = v_1 T\), \(\mu_2 = v_2 T\) in equation (29), and value of \(t_1\) and \(Q\) in equation (29), we get profit in terms of \(T\) and \(p\) for the four cases will be as under:

\[
\pi_i = \frac{1}{T}[SR - OC - SrC - HC - DC - IP_i + IE_i]
\]  
(30)

\[
\pi_1 = \frac{1}{T}[SR - OC - SrC - HC - DC - IP_1 + IE_1]
\]  
(31)

\[
\pi_2 = \frac{1}{T}[SR - OC - SrC - HC - DC - IP_2 + IE_1]
\]  
(32)

\[
\pi_3 = \frac{1}{T}[SR - OC - SrC - HC - DC - IP_3 + IE_1]
\]  
(33)

The optimal value of \(T^*\) and \(p^*\) which maximizes \(\pi_i\) can be obtained by solving equation (30), (31), (32) and (33) by differentiating it with respect to \(T\) and \(p\) and equate it to zero

i.e. \(\frac{\partial \pi_i(T,p)}{\partial T} = 0, \frac{\partial \pi_i(T,p)}{\partial p} = 0, i=1,2,3,4\)

provided it satisfies the condition

\[
\frac{\partial^2 \pi_i(T,p)}{\partial T^2} \frac{\partial^2 \pi_i(T,p)}{\partial T \partial p} - \frac{\partial^2 \pi_i(T,p)}{\partial p^2} > 0, i=1,2,3,4.
\]  
(35)

IV. NUMERICAL EXAMPLE

Case I: Considering \(A= Rs.100, a=500, b=0.05, c=Rs. 25, p_4 = 15, d= 0.02, x = 0.4, \lambda=10000, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.05\) in appropriate units. The optimal value of \(T^*=0.2584, p^* = 50.5520, Profit^* = Rs. 11751.6528\) and optimum order quantity \(Q^* = 64.0056\).

Case II: Considering \(A= Rs.100, a=500, b=0.05, c=Rs. 25, p_4 = 15, d= 0.02, x = 0.4, \lambda=10000, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.10\) in appropriate units. The optimal value of \(T^*=0.2551, p^* = 50.4781, Profit^* = Rs. 11805.7282\) and optimum order quantity \(Q^* = 63.2809\).

Case III: Considering \(A= Rs.100, a=500, b=0.05, c=Rs. 25, p_4 = 15, d= 0.02, x = 0.4, \lambda=10000, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.20\) in appropriate units. The optimal value of \(T^*=0.2436, p^* = 50.3789, Profit^* = Rs. 11931.8616\) and optimum order quantity \(Q^* = 64.0056\).

Case IV: Considering \(A= Rs.100, a=500, b=0.05, c=Rs. 25, p_4 = 15, d= 0.02, x = 0.4, \lambda=10000, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.28\) in appropriate units. The optimal value of \(T^*=0.2358, p^* = 50.3579, Profit^* = Rs. 12049.2441\) and optimum order quantity \(Q^* = 58.6260\).

The second order conditions given in equation (35) are also satisfied. The graphical representation of the concavity of the profit function is also given.
Case I

Graph 1

Graph 2

Graph 3

Graph 4

Case II

Graph 5

Graph 6

Graph 7

Graph 8
V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1

<table>
<thead>
<tr>
<th>Case – I</th>
<th>Sensitivity Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>%</td>
</tr>
<tr>
<td>a</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>x</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>θ</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>A</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>ρ</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>λ</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>R</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>M</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Case – II</th>
<th>Sensitivity Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>%</td>
</tr>
<tr>
<td>a</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>x</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>θ</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>A</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
</tr>
</tbody>
</table>
From the table we observe that as parameter \( M \) increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of \( x \) and \( R \), there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter \( \lambda \) and \( \rho \) increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

From the table we observe that as parameter \( \theta \) increases/ decreases, there is corresponding decrease/ increase in total profit and very minor decrease/ increase in optimum order quantity.

From the table we observe that as parameter \( M \) increases/ decreases average total profit increases/ decreases and there is very minor change in optimum order quantity.

From the table we observe that as parameter \( \lambda \) increases/ decreases, there is very minor increase/ decrease in average total profit and almost no change in optimum order quantity.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with...
the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

REFERENCES