Analysis of Water Hammer Using Method of Characteristics for Different Pipes Material

Mostafa Mahdy

Department of Mechanical Design, Faculty of Engineering, Mataria, Helwan University, P.O. Box 11718, Helmeiat - Elzaton, Cairo, Egypt

***Abstract*: This paper displays plan to the system for the method of characteristics for the cases included. In the last sections, the recommendation of the study and future scope of work taking this work as a base is displayed. A comparable system is set down for different cases including more intricate circumstances like transient inception by pumps, and many time valves closing. Likewise, the model is compared with other studies for validation. In the first section; the requirement for concentrate the water hammer state is set up. Agreeing with the formula provided by the *Juokowski* equation, the pressure variances formed by unexpected changes in flow speed in a standard water distribution system are of huge amount and possess potential to damage of the system components. By analyzing the water hammer caused by such quick changes in flow conditions, the damage can be diminished. The study on water hammer state instigated when researchers like *Newton* and *Lagrange* studied about the propagation of waves in water and it came into light after *Juokowski* gave expression for pressure surge consistent to a rapid change in flow speed. Afterward, in mid twentieth century *Allievi* [1] exhibited the differential equations for water hammer state after which several methods have changed for solving water hammer state problems. Then, following a half of century in 1950s *Gray* [2] introduced the strategy of the method of characteristics for computational study of the Water-hammer problem afterward following 10 years in 1962 *V.L. Streeter* and *C. Lai* [3] introduced a paper which broadened *Gray's* work and made the *method of characteristics* more popular. This section also talks about the evolution of methods for solving the water hammer problem, in a in a concise way.**

In second part, brief summaries for all the writing considered for setting up the last model for method of characteristics are put down, stating the purpose of study and the notable points. In the third part, the water hammer state theory is established beginning with the assumptions involved in classical water hammer theory, display in all the numerical models defined by researchers for water hammer state investigation. Then, the fundamental hyperbolic partial differential equations of continuity and momentum, which oversee the flow conditions during water hammer state for different pipes material, are expressed.

I. INTRODUCTION

Water-hammer is a term utilized for alluding to a state of flow, in which the pressure and flow speed fluctuate quickly, basically because of huge differences in pressures included. Water-hammer happen when the flow is constrained starting with one stable condition to another cases, they happen when a valve is closed suddenly in a pipe. [4]

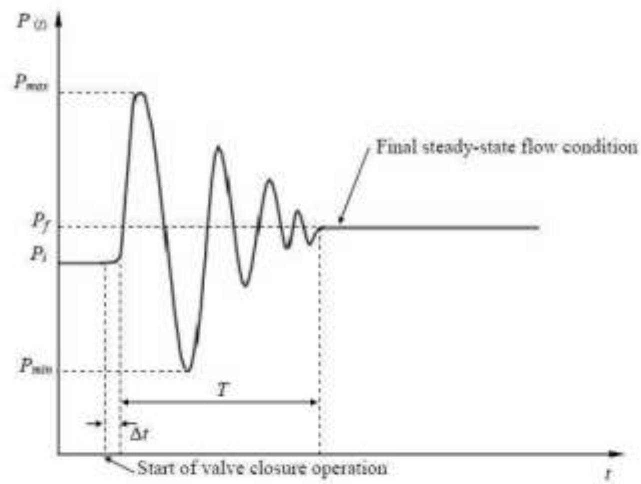

Figure 1: Water-hammer waves

As appeared in the figure (1) a damped sinusoidal performance can be seen amid the transient routine, showing towards nearness of pressure waves.

Water-hammer waves are difficult to stay away from and their event is usually caused by rapid action of valves and pumps. Most helpless regions to the water hammer are regions with high pressures, with either low or high static pressures. And regions distant from the tank. [5]

As indicated by *Juokowski* [6] when there is prompt change in fluid's flow speed a comparing change in pressure intensity is additionally seen which are linked as:

$$\Delta H = \frac{a}{g}\Delta V \tag{1}$$

The extraordinary changes in pressure and fluid speed are a risk to the hydraulic system included - valves, the pipe walls or other parts. Subsequently, it is important to investigate the water hammer possibilities in a system structures and model them for accomplishing a more secure design of the system structures.

The investigation on the Water-hammer has been completed for more than hundred years, and the models and the strategies to accomplish water hammer state solution for flow characteristics have advanced enormously. Be that as it may, the basics principles continue as before. The following area respects the elementary assumptions, overseeing equations and the pressure wave concept which form the basis of each model or strategy.

1.1Various Methods Utilized for Water-Hammer Investigation

In the course of recent years, analysts have proposed strategies for solving water hammer state problems with different assumptions and conditions. As expressed in the past segment it was after the coming of twentieth century when genuinely precise mathematical models came into picture. In 1902 *Allievi* [1] proposed the **arithmetical technique** in which he solved the differential equations after neglect the effects of friction, despite the outcomes of this strategy were inaccurate but this strategy used as the base for future studies. In view of *Juokowski's* theory, *N.R. Gibson* [7] extended Allievi's technique by consider the nonlinear friction losses and named it *Gibson Method*, however the outcome were as yet inexact.

In 1928, *Löwy* [8] proposed the use of **graphical strategy**, which was built up by *Gespard Monge* [9] in 1798, for solve the water-hammer problems. He was the first to comprise the friction part into the differential equations. Later *L. Bergeron* [10] stretched out this technique to decide the flow conditions at intermediate points in the pipe, not just at the valve or tank. The technique was additionally enhanced by *J. Parmakian* [11] in 1950s. This technique solves the problem assuming the quasi-steady friction model but for consider the dynamic effects a "*correction term*" was added in solution. Despite the method produced fast outcomes but the method was inaccurate, it just gave exact values for the primary wave-period for later stages the friction *correction term* was not satisfactory.

During the 1950s, with the appearance of PCs, the research became directed towards gating methods for computerized investigation of water hammers problems. The first was *Gray* [2] who proposed a calculation to execute strategy for **method of characteristics** to give computational investigation of Water-hammer. His design was later amended by the works of *V.L. Streeter* and *C.Lai* [3].In this technique, the fundamental governing equations which are *hyperbolic* and *partial first order* in nature are changed over into *ordinary first order*, along uncommon lines named the *characteristic* lines. This technique becomes quite accurate when the unsteady friction considered with it. It is, thus the most widely accepted model till date. Be that as it may, the limitations of *stability* and *convergence* [4] of the finite-difference technique pose a limit to the *range of applicability* of this technique. In the present situation, the frequency domain analysis of the water hammer has approached. In 1989, *E.B. Wylie* along with L. *Suo* [12] published a paper exhibiting **impulse response method,** which incorporates frequency dependent friction and wave velocity. This method involves use of inverse Fourier transform (IFT) to solve for the flow conditions. This strategy is considerably quicker than method of characteristics, however there is loss of accuracy since it linearizes the friction which is not suitable for each situation.

II. LITERATURE REVIEW

Earlier investigations that **method of characteristics** is the most broadly utilized strategy for numerically solving most water hammer state problems. Throughout time, Researchers have developed the use of this technique to get exact and flexible computational methodology. The technique in a general compacts the two simple *Partial Differential Equations* (PDEs) into *Ordinary Differential Equations* (ODEs) along the characteristic lines called C^+ and C^-. Then a proper numerical difference method is connected to get the answer for the ODEs. But, the numerical methodology limits the stability of the solution, as appeared by *Perkins et al.* [13] that the *time- increase* (Δt) has a lower limit of $a\ \Delta x$. This condition is named **Courant's Stability** condition. Further, to consolidate different parts such as turbines and pumps, used a suitable boundary conditions is necessary. Though the second half of 20^{th} century, widespread research was completed to enhance the strategy.

Next literature enumerates key enhancements and progressions to the method of characteristics:

E.L. Holmboe and *W.T. Rouleau* [14] (1967) introduced an experimental work study on the commonness of effect of viscous shear on Water hammer in a pipe. The test setup incorporated a copper-tube with fast shutting valve connected at one end and a constant head tank at other. Pressure transducers were joined at the valve location and center of the pipe for recording the information. The pipe was inserted in a concrete to invalidate the pipe vibration, which is important to capture the water hammer conditions. In further part the information is graphed and compared with frictionless investigation. The information provided by this test has been base for checking numerical models by numerous scientists.

E.B. Wylie [15] (1983) set out the essential plan for the use of method of characteristics through a microcomputer, by apply the forward difference numerical procedure for getting computational arrangement. The paper likewise puts advancing methodologies for efficient calculation and storage of the flow conditions, in a microcomputer. A *staggered grid* method, which includes variable component length and time-increment to lessen calculation time and move focus to critical positions likewise displayed. In concluding sections, the numerical outcomes for an experiment case are diagramed and also furthermore contrasted with experimental data along with results of basic frictionless investigation.

B.W. Karney et al. [16] (1992) presented the *linearization constant* "which gives a superior forward difference estimate for the steady friction term in the design of characteristic calculations. The paper talks about the variety of pressure and flow rate for a simple valve/orifice closure situation, with a constant head tank. Then, the strategy is executed on a more complex grid and a calculation is put forward for reproducible outcomes.

*G. Pezzinga*and *P. Secundra* [17] (1995) introduced that the water hammer motions can be reduced by using extra polymeric pipes to the original pipes. The experiments confirm that the pressure surges has been reduced and decline rate accelerates by adding of such pipes, for instance *High-Density Polyethylene* (HDPE) which is used in the test setup. The paper exhibits an exceptionally detailed depiction of exploratory setup and furthermore some numerical models are portrayed which are able to predict flow characteristics. The data for both with and without the additional pipe is graphed. This paper has filled in as confirmation standard for different papers such as *M.S. Ghidaoui et al.* [18].

W.F. Silva-Araya et al. [19] (1997) assessed the radial difference of flow speed for circular pipes. The paper introduced a numerical model for water hammer state and created plots of pressure head at valve versus time and speed versus radial distance at valve, for basic water hammer cases such as quick valve closure with a constant head tank and contrasted the results with experimental information. This paper presented the energy dissipation term in the method of characteristics formulation in the form of a *Dissipation Integral* and also established process for radial variation of flow speed.

G. Pezzinga [20] (1999) set up a Quasi-2D model display for water hammer state. Firstly, an arrangement of 2D PDEs for radial(r) and axial(x) difference of flow characteristics. Then the numerical pattern and computational strategy for solve the model, utilizing a procedure like method of characteristics, is introduced. Then an experimental setup utilized for verification of the computed outcomes is depicted. Then graphs comparing the pressure head deviation calculated by the Quasi-2D model and previous 1D models alongside experimental information are introduced. In finishing up comments, it is expressed that the 2D model exhibited is many time consuming and takes about 3.5 hours of analysis time, which is multiple times of the past 1D models by 30 times, anyway the accuracy has been increased.

A. Bergant et al. [21] (2001) introduced an audit of different models utilized for the friction modeling during water hammer state. The paper starts with stating that the most business packages of water hammer flow modeling employ an essential friction model by assume a constant *Darcy-Weisbach* friction factor at each time step, which is a mistaken presumption. Then a series of models is listed with different essential presumption. The paper portrays two of the strategies, beginning with *Zielke* [22] (1968) who was the first to propose a time-varying (*frequency-dependent*) friction factor, however just for laminar flows. Then the model figured by *Brunone* (1991) however effectively detailed by *Vitkovsky*, co-author of the paper, in 1998 utilizing the *Vardy-Brown* [23] demonstrate for shear-stress decay. Afterward, an experimental mechanical assembly is appeared, which is used for contrasting the above-said models with experimental information.

A. Adamkowski [24] (2003) exhibited the usage of method of characteristics for the case of pipes with differing cross-sectional area. In initial few sections, a progression of literature is introduced which are utilized being developed of model exhibited in the paper. The model is depends on the 1D theory of water hammer as created by *E.B. Wylie* [15] and assumes that the cross section area of the pipe is varying with axial distance. Then, governing equation, appropriate in this case, are exhibited after which, the detailing of forward difference characteristic line is illustrated. The formulation includes calculation of parametric constants (consider the variation of area) for different types of cross-section variety. In closing sections, the numerical result of the model is contrasted with those of the past models such as E.B.Wylie's to build up that numerical solutions turns out to be more precise with previous discussed model, albeit computational time is expanded marginally. The comparison is one by plotting the pressure peaks at different time steps for both the strategies.

M.S. Ghidaoui et al. [25] (2005) exhibited a survey article on the Water-hammer analysis. The paper starts with some historical background on the water hammer, and then 1D mathematical model is introduced for water hammer state analysis beginning with the fundamental governing equations. Then, divider shear stress models are set down expressing improvements in an essential model from *quasi-steady* model to *unsteady* shear stress model. Then, in last sections different numerical graphs, including method of characteristics graph, are examined, expressing limitations and advantages for every of them. Likewise, different methods of assessing these graphs such as *Mass Balance Approach* and *Energy Approach*. In following sections, a comparable investigation is done for 2D model of water hammer state flow, beginning with mathematical model, then discussed the numerical techniques and graphs to assess them. Next, a list of different software packages, which summon comparative mathematical models and provide numerical analysis of water hammer state by accept one of the methods and graphs for assessment, is exhibited. The list incorporates packages such as *TRANSAM*, *LIQT* and *WHAMO*. A considerable lot of these packages can produce water hammer state flow characteristic information for different causes of water hammer state such as valve closure, pump failure etc. In finishing up sections, dialog on utilization of such investigation like *leak detection* in pipelines is finished. Finally, scope for future research is set somewhere near particular areas like visualizing and displaying the physical systems behind development *helical* *type vortex* in water hammer state flows. *A. Adamkowski et al.* [26] (2006) exhibited an experimental investigation of different unsteady friction models like *Zielke's* [22] model, *Vardy-Brown* [23] model and *Brunoneet al.* [27] model. In started sections, a basic method of characteristics procedure is given utilization of different unsteady friction models. Then, the experimental test rig is described, a sloped pipeline with two constant head tanks located at both of its ends (High pressure tank at upstream end and low pressure tank at downstream end). A valve is put at downstream end. Pressure transducers are situated at four equidistant locations for information recovery. Then, in last sections the experimental outcomes are compared with the numerical outcomes produced by applying unsteady friction models. In closing comments, the paper views the *Vardy-Brown* model as the most accurate model for water hammer flows.

A.K. Soares et al. [28] (2012) broke down the water hammer state flows in a circular pipe caused by a pump failure/stoppage. The paper at first expresses an experimental setup utilized to deliver information for comparison with numerical outcomes. The setup includes a long pipeline appended to a constant head tank. A centrifugal pump is used to feed the pipeline with water extracted from the tank. The water is re-circulated to the tank for keeping up the constant pressure head. Then, in last sections the strategy for method of characteristics based computational procedure, which utilizes an unsteady friction model, is put forward. The pump characteristics are likewise obliged in the numerical methodology by using pump information for different specific velocities (Ns). Then, in coming sections the numerical outcomes are contrasted with experimental information and the comparison between pressure heads is diagramed. In last section, a case study done on water hammer state formed in a local pumping station is exhibited, indicating predicted and real pressure head variety.

Other than the above expressed writing, following books published in the second half of twentieth century have filled in as incredible store of different studies done on water hammer state flows called - *"Applied Hydraulic Transients"* [4] by *M.H. Chaudhary* in 1979 and *"Fluid Transients"* [29] by *E.B. Wylie* and *V.L. Streeter* in 1978.

III. THE WATER HAMMER STATE THEORY

3.1 Classical Assumptions

The current fundamental theory for the water hammer flows in pipelines is drawn on following classical assumptions: [29] [4].

The fluid-flow is single dimensional in other words, characteristic quantities are averaged over the cross section of pipe

The fluid-flow is low compressible that is, it elastically deformation under high pressures with negligible relative changes in density

The Dynamic liquid-pipe interactions are neglected and a quasi-steady interaction between pipe and liquid is assumed i.e., **quasi-steady friction model***

The liquid flow speed is very small as compared to the pressure water hammer wave speed

*The quasi-steady friction model's accuracy is limited and consequently, the present theories incorporate the dynamic interactions between liquid and pipe material known as **unsteady friction model**.

IV. THEORY AND MODEL IMPLEMENTATION

4.1 Water Hammer Equations and Method of Characteristics

The water hammer phenomenon is portrayed by the pressure head and the flow in the pipe system are computed utilizing the conservation of mass (also known as the continuity equation), Equation (2), and conservation of linear momentum, Equation (3) [30].

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} - \frac{2c^2\upsilon}{gE}\frac{\partial \sigma_x}{\partial t} = 0 \tag{2}$$

$$\frac{\partial Q}{\partial t} + Ag\frac{\partial H}{\partial x} + \frac{f}{2DA}Q\left|Q\right| = 0 \tag{3}$$

Where g is the gravitational acceleration, t is time, H is the piezometric head, A is the cross-sectional area of the pipe, Q is the volumetric flow rate, x is position in the axial direction. In the equations, it is assumed that the cross sectional area and the wave speed, a, are constant, and that a >> v which means that the convective terms can be ignored.

The term $\frac{2\upsilon}{gE}\frac{\partial \sigma_x}{\partial t}$, where σ_x represents the axial stress in the pipe, is neglected if not considering axial stress or strain, an assumption that generally is made. The following analysis are done under this assumption, however will later be altered [31].

It is additionally assumed that the flow is one dimensional and that the compressibility and flexibility of the pipe is accounted for in the pressure wave speed as seen in Equation (4).

$$a^2 = \frac{K/\rho}{1+\left[\left(K/E\right)\left(D/e\right)\right]c_1} \tag{4}$$

Where r is the density of the fluid, K is the bulk modulus of the fluid, E is Young's modulus of the pipe, D is the inner pipe diameter, e is the pipe wall thickness, and c1 is a coefficient describing the relation to Poisson's ratio, ν.

The method used to solve the set of Partial Differential Equations (PDEs) is the MOC.

The MOC transforms the set of PDEs into the four Ordinary Differential Equations (ODEs) seen in Equations (5) and (6).

$$C^+ : \begin{cases} \frac{g}{a}\frac{dH}{dt} + \frac{1}{A}\frac{dQ}{dt} + \frac{1}{A}\frac{fQ|Q|}{2D} = 0 \\ \frac{dx}{dt} = +a \end{cases} \quad (5)$$

$$C^- : \begin{cases} -\frac{g}{a}\frac{dH}{dt} + \frac{1}{A}\frac{dQ}{dt} + \frac{1}{A}\frac{fQ|Q|}{2D} = 0 \\ \frac{dx}{dt} = -a \end{cases} \quad (6)$$

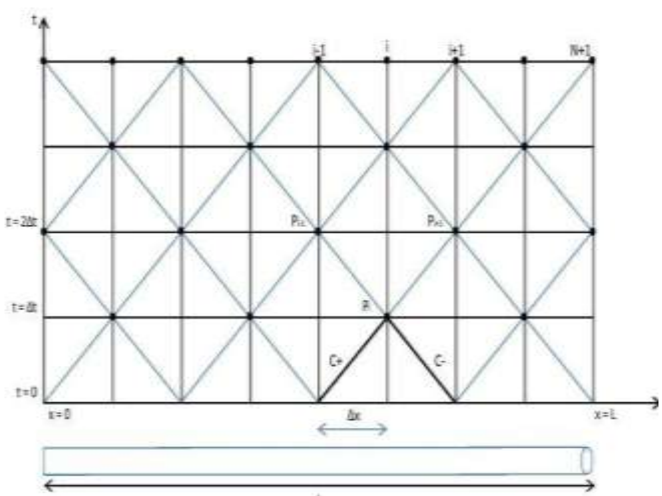

Figure 2: Characteristic lines in the x-t plane

The equations in Equation (5) can be solved along the characteristic lines with the slope determined by Equation (6), as is illustrated in Figure (2). By utilization of the characteristic lines, point P can be discovered utilizing point A and B. This is finished by the positive characteristic line, C^+, corresponding to a positive a and the negative characteristic line, C^- corresponding to a negative a. For this situation the characteristic lines are linear, since it is assumed that a is constant.

Approximating Equation (4) with finite differences and integrating, a pipe of length L is separated into N number of components, giving N + 1 number of nodes. For each time step Δt, the pressure and flow is computed in each node. The time step is determined by the pipe length and the wave speed according to: $\Delta t = \Delta x / a$, along the positive characteristic line and along the negative characteristic line yield Equations (7) and (8), where C_P and C_m are described with Equations (9) and (10), respectively and $B = \frac{a}{gA}$.

$$C^+ : H_i = C_P - B_P Q_i \quad (7)$$

$$C^- : H_i = C_M + B_M Q_i \quad (8)$$

$$B_P = B + \frac{f\,\Delta x}{2gDA^2} \quad (9)$$

$$\mathrm{B}_M = B + \frac{f\,\Delta x}{2gDA^2} \quad (10)$$

$$C_P = H_{i-1} + BQ_{i-1} \quad (11)$$

$$C_m = H_{i+1} - BQ_{i+1} \quad (12)$$

The main compatibility equation is valid along the characteristic line $\Delta x = c.\Delta t$, getting C_P and B_P at the distance $i-1$ from the point of interest, i.e. at the previous time step $t - \Delta t$. The other one is valid along $\Delta x = -c$, with C_M and B_M at the distance. The referenced quantities can be calculated through Equation 9-12.

Utilizing the expressions above in combination with Equation 7 and 8, the pressure head and flow can be computed for each interior node as indicated by Equation 13 and

Equation 14, respectively [30].

$$H_i = \frac{C_P B_M + C_M B_P}{B_P + B_M} \tag{13}$$

Q_P can then be calculated with Equation (13).

$$Q_i = \frac{C_P - C_M}{B_P - B_M} \tag{14}$$

A rectangular mesh is picked over the less computationally intensive diamond grid to enhance the plotting of the pressure pattern. Note that the rectangular mesh, which consists of two independent diamond grids, tends to generate small unphysical oscillations, which are visible on the graph.

Note: These equations can be solved at the same time, just for space points which are inside the boundaries. For the boundary point's individual characteristic equation along with appropriate boundary condition must be used for calculating flow characteristics. Normally, steady-state is assumed as the initial condition.

4.2 Boundary Conditions

To get the pressure head and flow at the boundary nodes, there are different boundary condition that can be applied depending on which element the pipe is appended to. Which equation to use depends on how the pipe is connected to the element i.e. if the boundary condition is upstream or downstream end of the pipe. Having a downstream boundary condition the C^+ equation is utilized, and for upstream boundary condition the C^- equation is utilized [32].

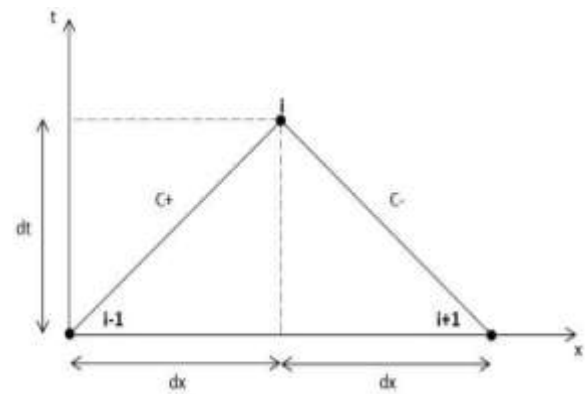

Figure 3: x-t Diagram

Figure (4) shows the change of the Normalized Piezometric head at the valve for different pipes marital for five materials were studied PVC, concrete, asbestos cement (AC), ductile iron and steel. The other variables were taken as constant fluid density (ρ) =1000 kg/m3 (fluid is water), liquid bulk modulus (K_{bm}) = 2.15 Gpa, pipe length is 2500 m, pipe diameter is 50 cm, a_0 is 1300 m/s and the pipe wall thickness for the all pipe materials were taken 2 cm. The used materials data is shown in table (1) and were adopted from Jones and Bosserman [35, 36], Richard and Svindland [37] and Sharp and Sharp [38]

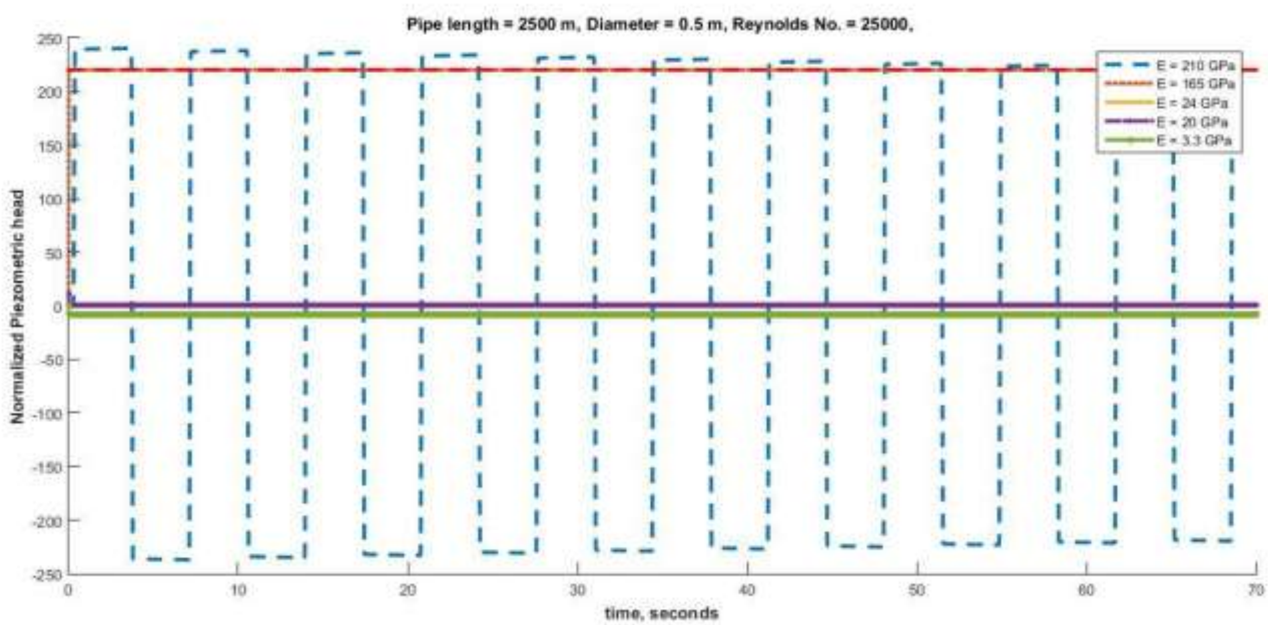

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Figure 4: Normalized Piezometric head at the valve - dependency on Elasticity modulus

V. CONCLUSIONS

It is clear from the results got with the pipes modules of elasticity. However, it is important to add that the elasticity modulus was modified as suggested by Simpson [5] on the basis of the measured sonic velocity: value E = 0.75x1011 N/m2 was used in our calculation instead of 1.2x1011 N/m2 (copper). Exact reason for that discrepancy is not given in [39].

Pressure waves in single-phase systems are weaker if pipe elasticity is taken into account.

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MATLAB Program

```
 clc;
clear all;
close all
global Ro Vct;
%% Input Parameters
L=2500;%length of pipe
d=0.5;%diameter of pipe
e=25e-3;%thickness of the walls
rho=1000;%Density
nu=2e-6;%Kinematic Viscoscity
muM=0;%Poisson coefficient
re=25000;% Reynolds no
E=[210e9 165e9 24e9 20e9 3.3e9];%Modules of elasticity
Kbm = 2.15e9; %liquid bulk modulus
for kk=1:length(E)
a(kk) = ((Kbm/rho)^0.5)*(1+(1-muM^2)*(Kbm*d/E(kk)*e))^-0.5;%wave speed
C=1;
Vct=0.1;% Valve Closing time
Cv=1-C;
%Ro=E/d;
Ro=0;% Roughness Coefficient
Vo=re*nu/d;
mju=2e-3;
f=colebrook(re,Ro); %Colebrook Friction factor
Po=(f*L*rho*Vo^2)/(2*d);
%% MOC Grid Creation
n=100;h=L/(n-1);V(1:n)=Vo;P(1:n)=Po;
dt(kk)=h/a(kk);tmax=70;itmax(kk)=tmax/dt(kk);
for i=2:n
P(i)=P(i-1)-f*rho*Vo^2*h/(2*d);
end
%% Calculation of Friction terms
A=sqrt(1/4*pi);ki=re^(-0.0567);k=log10(15.29*ki);B=((re)^k)/12.86;% smooth pipe
delt=4*nu*dt/d^2;
sum=0;
for k=1:10
sum=sum+(A*m(k))*(exp(-(ni(k)+B)*delt));
end
lambda(kk)=(16*nu*sum(kk)*h)/(a(kk)*d^2);
%% Calculation of Velocities and Presussre at evry point as Per MOC-Grid
for it=1:itmax(kk)
t(kk)=it*dt(kk);
for i=2:n-1
Pa=P(i-1);Pb=P(i+1);Va=V(i-1);Vb=V(i+1);
Fa=Friction(Va,d,nu);
Fb=Friction(Vb,d,nu);
Ea(kk)=Fa*rho*h/(2*a(kk)*d)*Va*abs(Va);
Eb(kk)=Fb*rho*h/(2*a(kk)*d)*Vb*abs(Vb);
Pc(kk,i)=a(kk)/2*((Pa+Pb)/a(kk)+rho*(Va-Vb)+(Eb(kk)-Ea(kk)));
Vc(kk,i)=0.5*(((Pa-Pb)/a(kk))+rho*(Va+Vb)-Eb(kk)-Ea(kk))/(rho+lambda(kk));
end
Pc(kk,1)=Po;
Vb=V(2);Pb=P(2);
Fb=Friction(Vb,d,nu);
```

```
Eb(kk)=Fb*h/(2*a(kk)*d)*Vb*abs(Vb);
Vc(kk,1)=(rho*Vb+(Pc(1)-Pb)/(a(kk))-rho*Eb(kk))/(rho+lambda(kk));
Vc(kk,n)=Vo*VctFunctn(t(kk));
Va=V(n-1);Pa=P(n-1);
Fa=Friction(Va,d,nu);
Pc(kk,n)=(Pa-(rho+lambda(kk))*a(kk)*Vc(kk,n) + rho*a(kk)*Va - a(kk)*Ea(kk));
vmatrix(it,1:n)=Vc(1:n);
pmatrix(it,1:n)=Pc(1:n);
P=Pc;V=Vc;
end
%% Plotting of Graphs
hold on
time=linspace(0,tmax,itmax(kk));
Pressure=((pmatrix(:,n))/Po);
NNN=length(time)
plot(time(1:NNN),Pressure(1:NNN),'linewidth',2.5)
legend('\bfClassic WH','\bfMOC Model','Location','Best')
xlabel('\bftime, seconds')
ylabel('\bfNormalized Piezometric head')
title(['\bfPipe length = ' num2str(L) ' m, ' '\bfDiameter = ' num2str(d)])
format shortg
end
```