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The Gbenga Gideon Integral Transform and Its Application to Second-Order Ordinary Differential Equations

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ABSTRACT

This study develops the Gbenga Gideon (G.G.) integral transform, a four-parameter generalization of the Laplace transform characterized by the kernel (p v^m e^{-qv^w t}), and establishes its operational calculus for solving second-order ordinary differential equations with constant coefficients. Fundamental transform pairs are derived for powers, exponential, trigonometric, and hyperbolic functions, while key operational properties, including linearity, first and second shifting, change of scale, differentiation, multiplication by the independent variable, and convolution, are formulated and proved. Particular attention is given to the corrected general change-of-scale formula and a convolution theorem that is consistent with the transform's four-parameter normalization. The parameter values under which the G.G. integral transform reduces to the Aboodh, Gupta, Kamal, Elzaki, and Sadik transforms are systematically identified, tabulated, and verified against the established definitions of these transforms. The applicability of the proposed method is demonstrated through nine second-order initial- and boundary-value problems involving homogeneous equations with real, repeated, and complex characteristic roots, as well as nonhomogeneous equations with exponential, polynomial, and damped-trigonometric forcing terms. General convolution solutions are also obtained for the equations (y"-a^2y=f(t)) and (y"+a^2y=f(t)). The resulting solutions are cross-validated using the Laplace, Aboodh, and Elzaki transforms and further confirmed through direct substitution into the corresponding differential equations. The results demonstrate the consistency and versatility of the G.G. integral transform as an operational method for solving second-order ordinary differential equations.

Keywords: G.G. integral transform; second-order ordinary differential equation; integral transform; Laplace transform; Aboodh transform; Elzaki transform; analytical solution.

INTRODUCTION

Integral transforms convert differential and integral equations into algebraic relations in an auxiliary variable. For a suitable kernel, differentiation in the original variable becomes multiplication by a transform parameter together with terms fixed by the initial data; the resulting algebraic equation is then resolved by factorization, partial fractions, transform tables or convolution before the inverse transform is applied (Davies, 2012; Jerri, 1999; Myint-U & Debnath, 2007).

The Laplace transform remains the most widely used member of this family, but a substantial recent literature has grown around related Laplace-type kernels with alternative powers and normalizations. The Aboodh, Elzaki, Sumudu, Kamal and Sadik transforms are among the best-known examples (Watugala, 1993; Kuffi, 2023; Jafari, 2021; El-Mesady et al., 2021; Arora & Pasrija, 2023; Bîçer et al., 2024). Each of these transforms is a special case of a single four-parameter kernel p v^m e^{-q v^w t}, and stating the general kernel once, together with its operational calculus, avoids re-deriving the same properties for every named variant — a point also illustrated by recent unifying treatments such as the Sadik and complex Sadik transforms (Kuffi, 2023) and the Sumudu–Elzaki correspondence (Almousa et al., 2024).

Second-order linear equations with constant coefficients are the most extensively studied class of ordinary differential equations, underlying damped and undamped oscillators, boundary-value problems, and a wide range of applications in engineering and physics (Johnson, 2012; Dorodnitsyn, 2006; Chandrasekar et al., 2005; Ebaid et al., 2017). This paper organises the G.G. (Gbenga–Gideon) integral transform and applies it specifically to this class. The paper makes four contributions. First, it gives a precise definition of the G.G. integral transform together with a sufficient condition for its existence. Second, it collects the elementary transform pairs and operational properties needed for second-order problems, including the general ($w \neq 1$) forms of the change-of-scale formula and the convolution theorem. Third, it records and verifies the parameter choices through which the G.G. kernel reproduces the Aboodh, Gupta, Kamal, Elzaki and Sadik transforms against each transform's published definition. Fourth, it applies the transform to nine second-order initial- and boundary-value problems spanning real, repeated and complex characteristic roots and exponential, polynomial and damped-trigonometric forcing terms, cross-checked against the Laplace, Aboodh and Elzaki transforms and verified by direct substitution.

Related second-order solution techniques in the wider literature include hypergeometric-series methods via the Laplace transform (Ebaid et al., 2017), operator-linearisation approaches (Dorodnitsyn, 2006), integral-equation formulations of higher-order problems (Krisner, 2004), fractional and local-fractional extensions (Yang et al., 2015), spline and recursive-filter methods for the associated Volterra integral equations (Burova & Alcybeev, 2022; Heyd, 2024), and general treatments of integral transforms and integral equations (Wazwaz, 2011, 2015; Chicone, 2006; Wolf, 2013; Wong, 2001; Andrews & Shivamoggi, 1999; Atangana & Akgül, 2023; Lakshmikantham & Leela, 1969).

Definition and Existence of the G.G. Integral Transform

Let f(t) be a real- or complex-valued function defined for $t \geq 0$. For fixed parameters p, q, m and w, the G.G. integral transform of f is defined by

$$G[f(t)](v) = G(v) = \int_0^\infty pv^m \, e^{-qv^w t} \, f(t)\, dt, \qquad (1)$$

provided the integral converges. In the real-valued setting one takes $p > 0$, $q > 0$ and $v > 0$, and requires q v^w to lie to the right of the exponential-growth abscissa of f, in direct analogy with the Laplace transform.

Definition 1 (exponential order)

A function f is of exponential order κ if there exist constants $M > 0$ and $T \geq 0$ such that

$$[f(t)] \leq M\, e^{\kappa t}, \;\; t \geq T.$$

Theorem 1 (existence of the G.G. integral transform)

If f is piecewise continuous on every finite subinterval of $[0, \infty)$ and is of exponential order κ, then G{f(t)}(v) exists for every v such that

$$qv^w > \kappa. \qquad (2)$$

Proof.

Choose T so that the inequality in Definition 1 holds, and split the defining integral at T:

$$G(v) = pv^m \int_0^T e^{-qv^w t} f(t)\, dt + pv^m \int_T^\infty e^{-qv^w t} f(t)\, dt$$

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The first integral exists because its integrand is piecewise continuous on a finite interval. For the second integral,

$$\left[pv^{m}e^{-qv^{w}t}f(t)\right] \leq Mpv^{m}e^{-(qv^{w}-\kappa)t}, \ t \geq T.$$

Since q v^w > κ, the right-hand side is integrable on [T, ∞):

$$\int_{T}^{\infty} Mpv^{m}e^{-(qv^{w}-\kappa)t}\,dt = \frac{Mpv^{m}e^{-(qv^{w}-\kappa)T}}{qv^{w}-\kappa} < \infty.$$

By the comparison test the second integral converges absolutely, and hence G{f(t)} (v) exists.

Definition 2 (inverse G.G. integral transform)

If G(v) = G{f(t)} (v) and the inverse operation exist on the relevant domain, then f is called the inverse G.G. integral transform of G, written

$$f(t) = G^{-1}[G(v)](t). \quad (3)$$

In the applications of Section 10, inversion is carried out through the elementary transform pairs of Section 3 together with the specialization of the G.G. integral transform to the Laplace, Aboodh and Elzaki transforms.

Elementary G.G. Integral Transform Pairs

Table 1 lists the elementary transform pairs used in the applications of Section 10. Representative proofs follow the table; the remaining entries are obtained in the same way.

f(t)	**G{f(t)}(v)**	**Condition**
1	$\frac{pv^{m}}{qv^{w}}$	$qv^{w} > 0$
t^{n}	$\frac{n!\,pv^{m}}{(qv^{w})^{n+1}}$	$n = 0,1,2,\ldots$
e^{kt}	$\frac{pv^{m}}{qv^{w}-k}$	$qv^{w} > k$
$\sin kt$	$\frac{kpv^{m}}{q^{2}v^{2w}+k^{2}}$	$k \in \mathbb{R}$
$\cos kt$	$\frac{pqv^{m+w}}{q^{2}v^{2w}+k^{2}}$	$k \in \mathbb{R}$
$\sinh kt$	$\frac{kpv^{m}}{q^{2}v^{2w}-k^{2}}$	$qv^{w} > \lvert k\rvert$
$\cosh kt$	$\frac{pqv^{m+w}}{q^{2}v^{2w}-k^{2}}$	$qv^{w} > \lvert k\rvert$
$e^{kt}t^{n}$	$\frac{n!\,pv^{m}}{(qv^{w}-k)^{n+1}}$	$qv^{w} > k$

Table 1. Elementary G.G. integral transform pairs.

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Representative proofs

(a) Power function. With f(t) = t^n and x = q v^w t,

$$G[t^n] = pv^m \int_0^\infty \left(\frac{x}{qv^w}\right)^n e^{-x} \frac{dx}{qv^w} = \frac{n!\, pv^m}{(qv^w)^{n+1}}$$

since ∫₀^∞ x^n e^{−x} dx = n! for n = 0, 1, 2, …

(b) Exponential function. Directly,

$$G[e^{kt}] = pv^m \int_0^\infty e^{-(qv^w-k)t}\, dt = \frac{pv^m}{qv^w - k}, \ qv^w > k.$$

(c) Sine and cosine. Writing sin kt = (e^{ikt} − e^{−ikt})/2i and cos kt = (e^{ikt} + e^{−ikt})/2, and applying the exponential pair with k replaced by ±ik,

$$G[sin\, kt] = \frac{1}{2i}\left(\frac{pv^m}{qv^w - ik} - \frac{pv^m}{qv^w + ik}\right) = \frac{kpv^m}{q^2v^{2w} + k^2}$$

and the cosine pair follows in the same way with the plus sign. The hyperbolic pairs follow identically from sinh kt = (e^{kt} − e^{−kt})/2 and cosh kt = (e^{kt} + e^{−kt})/2, replacing k by ±k rather than ±ik, which changes $+k^2$ in the denominator to $-k^2$. The shifted pair G{e^{kt}t^n}(v) follows by the first shifting theorem of Section 4.2 applied to (a) of this table.

Fundamental Operational Properties

Linearity

If $G\{f_1(t)\}(v) = G_1(v)$ and $G\{f_2(t)\}(v) = G_2(v)$, then for arbitrary constants a and b,

$$G[af_1(t) + bf_2(t)](v) = aG_1(v) + bG_2(v), \tag{4}$$

which follows directly from the linearity of the defining integral.

First shifting theorem

If G{f(t)} (v) = G(v), then for real k,

$$G[e^{kt}f(t)](v) = pv^m \int_0^\infty e^{-(qv^w-k)t} f(t)\, dt = G(qv^w - k), \tag{5}$$

so, multiplication of f(t) by e^{kt} shifts the parameter q v^w to q v^w − k inside G. In particular, combined with Table 1,

$$G[e^{kt}sin\, bt](v) = \frac{bpv^m}{(qv^w - k)^2 + b^2}, \quad G[e^{kt}cos\, bt](v) = \frac{pv^m(qv^w - k)}{(qv^w - k)^2 + b^2}$$

Second shifting theorem

Let H denote the Heaviside unit step function and set $f_a(t)$ = H (t − a) f (t − a) for a > 0. If G{f(t)} (v) = G(v), then

$$G[H(t-a)\, f(t-a)](v) = e^{-aqv^w}\, G(v). \tag{6}$$

Splitting the defining integral at t = a and substituting u = t – a, in the surviving piece,

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$$G[H(t-a)f(t-a)](v) = pv^m \int_a^{\infty} e^{-qv^w t} f(t-a)\, dt = pv^m e^{-aqv^w}$$

$$\int_0^{\infty} e^{-qv^w u} f(u)\, du = e^{-aqv^w} G(v).$$

Change of scale

If G{f(t)} (v) = G(v), then for a > 0,

$$G[f(at)](v) = a^{\frac{m}{w}-1} G\left(\frac{v}{a^{\frac{1}{w}}}\right), \tag{7}$$

obtained from the substitution u = at. Writing s = q v^w, so that q(v/a^{1/w})^w = s/a, the substitution gives

$$G[f(at)](v) = pv^m \int_0^{\infty} e^{-(s/a)u} \frac{du}{a}$$

which, matched against G(v/a^{1/w}) = p (v/a^{1/w})^m ∫ e^{−(s/a)u} du, gives the stated exponent m/w − 1 on a. (The simpler rule a^{−1}G(v/a) is the special case w = 1; equation (7) is the version valid for general w.)

Transform of Derivatives

Assume f and f′ are continuous and of exponential order, f″ is piecewise continuous, and the boundary terms at infinity vanish. Integration by parts gives

$$G[f'(t)](v) = qv^w G(v) - pv^m f(0). \tag{8}$$

Proof.

$$G[f'(t)](v) = pv^m\left[\left(e^{-qv^w t} f(t)\right)\right] \mid_0\infty + qv^w pv^m \int_0^{\infty} e^{-qv^w t} f(t)\, dt = -pv^m f(0) + qv^w G(v).$$

Applying (8) recursively (replace f by f′) yields the second-derivative identity used throughout Section 10:

$$G[f''(t)](v) = q^2 v^{2w} G(v) - pqv^{m+w} f(0) - pv^m f'(0). \tag{9}$$

More generally, for every integer n ≥ 1,

$$G\left[f^{(n)}(t)\right](v) = (qv^w)^n G(v) - \sum_{j=0}^{n-1} pq^{n-1-j} v^{m+(n-1-j)w} f^{(j)}(0) \tag{10}$$

proved by induction from (8), exactly as in the third-order companion paper in this series.

Multiplication by the Independent Variable

Differentiating G(v) = p v^m ∫₀^∞ e^{−q v^w t} f(t) dt with respect to v and solving for the t f(t) term gives

$$G[t\, f(t)](v) = \frac{m}{wqv^w} G(v) - \frac{v}{wqv^w} \frac{dG(v)}{dv}, \tag{11}$$

Proof.

Differentiating under the integral sign with respect to v,

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$$\frac{dG}{dv} = mpv^{m-1}\int_0^\infty e^{-qv^w t} f(t)\,dt - qwv^{w-1}pv^m\int_0^\infty t\,e^{-qv^w t} f(t)\,dt,$$

$$\frac{dG}{dv} = \frac{m}{v}G(v) - qwv^{w-1}G[t\,f(t)](v).$$

Solving for G{t f(t)}(v) and using v^{w−1} = v/v^w gives (11). Repeated differentiation yields the identities for $t^2f(t)$, $t^3f(t)$, and, in general, t^n f(t).

Convolution Theorem

Let $(f_1 * f_2)(t) = \int_0^t f_1(x) f_2(t-x)\,dx$. If $G_1(v) = G\{f_1(t)\}(v)$ and $G_2(v) = G\{f_2(t)\}(v)$, then

$$G\left[\left(\int_0^t f_1(x)f_2(t-x)\,dx\right)\right](v) = \frac{G_1(v)\,G_2(v)}{pv^m}, \tag{12}$$

equivalently $f_1 * f_2 = G^{-1}\{G_1G_2/(pv^m)\}$. The factor 1/(p v^m) is required by the transform's normalization: when p = 1 and m = 0 (the Laplace case) it disappears and (12) reduces to the classical Laplace convolution theorem. This identity underlies the general forced-equation solutions of Sections 9.8–9.9, where it is used to invert R(v)/π(qv^w) as a convolution with the impulse response of the operator.

Proof.

$$G\left[\left(\int_0^t f_1(x)f_2(t-x)\,dx\right)\right](v) = pv^m\int_0^\infty\int_0^t f_1(x)f_2(t-x)\,dx\,e^{-qv^w t}\,dt$$

Reversing the order of integration over $0 \le x \le t < \infty$ and substituting $z = t - x$ in the inner integral gives

$$= pv^m\int_0^\infty f_1(x)e^{-qv^w x}\int_0^\infty f_2(z)e^{-qv^w z}\,dz\,dx$$

$$= G_2(v)\int_0^\infty f_1(x)e^{-qv^w x}\,dx = \frac{G_1(v)\,G_2(v)}{pv^m},$$

since the surviving integral equals $G_1(v)/(p\,v^m)$ by definition (1).

Comparison with Existing Transforms

The G.G. kernel is K(t,v) = p v^m e^{−q v^w t}. Table 2 records the parameter choices (p, q, m, w) through which the G.G. integral transform specialises to the Aboodh, Gupta, Kamal, Elzaki and Sadik transforms. Each row has been checked against the transform's own published definition.

Transform	p	q	m	w	Kernel
Aboodh	1	1	−1	1	$v^{-1} e^{-vt}$
Gupta	1	1	−3	1	$v^{-3} e^{-vt}$
Kamal (Vang)	1	1	0	−1	$e^{-t/v}$
Elzaki	1	1	1	−1	$v\, e^{-t/v}$
Sadik	1	1	−β	α	$v^{-\beta} e^{-v^{\alpha}t}$

Table 2. Parameter correspondence between the G.G. integral transform and five named transforms.

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Table 2 uses m = −1 (Aboodh) and m = 0, w = −1 (Kamal), consistent with the kernels $k(t,v) = v^{-1}e^{-vt}$ *and* $k(t,v) = e^{-t/v}$ *and with the standard published definitions of these transforms.*

As with Table 3 of the companion third-order paper, the comparison is operational: matching kernels does not by itself establish equality of inversion domains or admissible function spaces for each named transform.

Application to Second-Order Ordinary Differential Equations

Consider the second-order constant-coefficient initial-value problem

$$y''(t) + a_1 y'(t) + a_0 y(t) = r(t),\ y(0) = y_0, y'(0) = y_1. \tag{13}$$

Write s = q v^w and let Y(v) = G{y(t)} (v), R(v) = G{r(t)} (v). Applying (8)– (9) gives the characteristic polynomial $\pi(s) = s^2 + a_1 s + a_0$, and

$$Y(v) = \frac{R(v) + pv^m((s + a_1)y_0 + y_1)}{\pi(s)}. \tag{14}$$

Equation (14) is the principal operational formula for second-order constant-coefficient problems. Since π(s) is exactly the Laplace characteristic polynomial with s = qv^w, and since Table 2 shows the Aboodh and Elzaki transforms are the G.G. integral transform at (p,q,m,w) = (1,1,−1,1) and (1,1,1,−1), the Laplace, Aboodh and Elzaki solutions of the same problem must agree with the G.G. Integral solution once the corresponding inverse transform is applied — the cross-check used throughout this section.

Undamped oscillator: complex-conjugate roots ±i

Solve y″ + y = 0, y(0) = 1, y′(0) = 1. Here $a_1 = 0$, $a_0 = 1$, so $\pi(s) = s^2+1$, and (14) gives

$$Y(v) = \frac{pv^m(s+1)}{s^2 + 1} = \frac{pv^m s}{s^2 + 1} + \frac{pv^m}{s^2 + 1},$$

$$y(x) = \cos x + \sin x, \tag{15}$$

by the cosine and sine pairs of Table 1. The Laplace, Aboodh and Elzaki transforms give the same result under the same initial data.

Distinct real roots: homogeneous case

Solve y″ − 3y′ + 2y = 0, y(0) = 1, y′(0) = 4. Here $\pi(s) = s^2-3s+2 = (s-1)(s-2)$, and (14) gives

$$Y(v) = \frac{pv^m(s+1)}{(s-1)(s-2)} = \frac{3pv^m}{s-2} - \frac{2pv^m}{s-1},$$

$$y(x) = 3e^{2x} - 2e^{x}. \tag{16}$$

Distinct real roots: exponential forcing

Solve y″ − 3y′ + 2y = 4e^{3x}, y(0) = −3, y′(0) = 5. With π(s) = (s−1)(s−2) as above, R(v) = 4pv^m/(s−3) (Table 1, k=3), and IC-term pv^m[(s−3)(−3)+5] = pv^m(14−3s), equation (14) gives

$$Y(v) = \frac{4pv^m}{(s-1)(s-2)(s-3)} + \frac{pv^m(14-3s)}{(s-1)(s-2)} = \frac{pv^m(-3s^2 + 23s - 38)}{(s-1)(s-2)(s-3)},$$

which resolves by partial fractions to

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$$Y(v) = \frac{2pv^m}{s-3} + \frac{4pv^m}{s-2} - \frac{9pv^m}{s-1},$$

$$y(x) = 2e^{3x} + 4e^{2x} - 9e^x. \tag{17}$$

Repeated real root: resonant forcing

Solve y″ − 3y′ + 2y = 4e^{2x}, y(0) = −3, y′(0) = 5. Since s = 2 is now a root of π(s) = (s−1)(s−2), the forcing term is resonant and R(v) = 4pv^m/(s−2) contributes a repeated factor:

$$Y(v) = \frac{-7pv^m}{s-1} + \frac{4pv^m}{s-2} + \frac{4pv^m}{(s-2)^2},$$

$$y(x) = -7e^x + 4e^{2x} + 4xe^{2x}, \tag{18}$$

the last term arising from the shifted-power pair G{e^{kt}t}(v) = pv^m/(s−k)² with k=2. This illustrates, in the forced case, the same repeated-root mechanism that produces t e^{αt} terms for homogeneous equations with a double characteristic root.

Boundary-value problem with trigonometric forcing

Solve y″ + 9y = cos 2x, y(0) = 1, y(π/2) = −1. Here π(s) = s²+9 and, since y′(0) is not prescribed, the second constant is carried as an undetermined coefficient C, to be fixed by the boundary condition rather than by an initial slope. With R(v) = pqv^{m+w}/(s²+4) (Table 1, k=2),

$$Y(v) = \frac{pqv^{m+w}}{(s^2+4)(s^2+9)} + \frac{pqv^{m+w}}{s^2+9} + \frac{Cpv^m}{s^2+9}.$$

Resolving the first term by partial fractions (1/5 on s²+4, −1/5 on s²+9) and combining with the second term gives

$$Y(v) = \frac{pqv^{m+w}}{5(s^2+4)} + \frac{4pqv^{m+w}}{5(s^2+9)} + \frac{Cpv^m}{s^2+9},$$

$$y(x) = \frac{1}{5}cos\,2x + \frac{4}{5}cos\,3x + \frac{C}{3}sin\,3x.$$

Imposing y(π/2) = −1 gives (1/5) cosπ + (4/5) cos(3π/2) + (C/3) sin(3π/2) = −(1/5) − C/3 = −1, so C = 12/5 and C/3 = 4/5. Hence

$$y(x) = \tfrac{1}{5}cos\,2x + \tfrac{4}{5}cos\,3x + \tfrac{4}{5}sin\,3x. \tag{19}$$

The Laplace, Aboodh and Elzaki derivations reach the same coefficients 1/5, 4/5, 4/5 by the identical partial-fraction route.

Polynomial forcing: case 1

Solve y″ + y = x, y(0) = 1, y′(0) = 0. Here π(s) = s²+1 and R(v) = pv^m/s² (Table 1, n=1). With IC-term pv^m·s (since a_1=0, y_0=1, y_1=0), equation (14) gives

$$Y(v) = \frac{pv^m}{s^2(s^2+1)} + \frac{pv^m s}{s^2+1}.$$

Since 1/[s²(s²+1)] = 1/s² − 1/(s²+1),

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$$Y(v) = \frac{pv^m}{s^2} - \frac{pv^m}{s^2 + 1} + \frac{pv^m s}{s^2 + 1},$$

$$y(x) = x + \cos x - \sin x. \tag{20}$$

Polynomial forcing: case 2

Solve the same equation with a different initial slope: y″ + y = x, y(0) = 1, y′(0) = −2. Only the IC-term changes, to pv^m(s−2), so

$$Y(v) = \frac{pv^m}{s^2} - \frac{pv^m}{s^2 + 1} + \frac{pv^m s}{s^2 + 1} - \frac{2pv^m}{s^2 + 1},$$

$$y(x) = x + \cos x - 3\sin x. \tag{21}$$

Together, Sections 9.6–9.7 show how the two free constants of the general solution absorb different initial slopes while the particular solution x contributed by the forcing term stays the same.

Polynomial forcing: case 3

Solve y″ + 4y = 9x, y(0) = 0, y′(0) = 7. With π(s) = s^2+4 and R(v) = 9pv^m/s^2 (Table 1, n=1),

$$Y(v) = \frac{9pv^m}{s^2(s^2 + 4)} + \frac{7pv^m}{s^2 + 4} = \frac{9pv^m}{4s^2} + \frac{19pv^m}{4(s^2 + 4)},$$

$$y(x) = \tfrac{9}{4}x + \tfrac{19}{8}\sin 2x. \tag{22}$$

Sections 9.6–9.8 together illustrate the same polynomial-forcing mechanism across three different combinations of characteristic polynomial and initial slope, all obtained from (14) in exactly the same way.

Damped oscillator with trigonometric forcing

Solve y″ + 2y′ + 5y = e^{−x} sin x, y(0) = 0, y′(0) = 1. Here π(s) = s^2+2s+5 = $(s+1)^2$+4, and by the first shifting theorem R(v) = pv^m/[$(s+1)^2$+1] (Table 1 shifted by k=−1, b=1). With y_0=0, y_1=1, equation (14) gives

$$Y(v) = \frac{pv^m}{((s+1)^2 + 1)((s+1)^2 + 4)} + \frac{pv^m}{(s+1)^2 + 4} = \frac{pv^m((s+1)^2 + 2)}{((s+1)^2 + 1)((s+1)^2 + 4)},$$

which, resolved by partial fractions in $(s+1)^2$ into (1/3)/[$(s+1)^2$+1] + (2/3)/[$(s+1)^2$+4] and inverted using the shifted sine pairs of Section 4.2 with k=−1 and b=1, b=2, gives

$$y(x) = \tfrac{1}{3}e^{-x}(\sin x + \sin 2x). \tag{23}$$

This case is genuinely new relative to Sections 9.1–9.4: the characteristic roots −1±2i are complex, and the forcing frequency (1) differs from the natural frequency (2), so no resonance occurs and the particular solution stays bounded.

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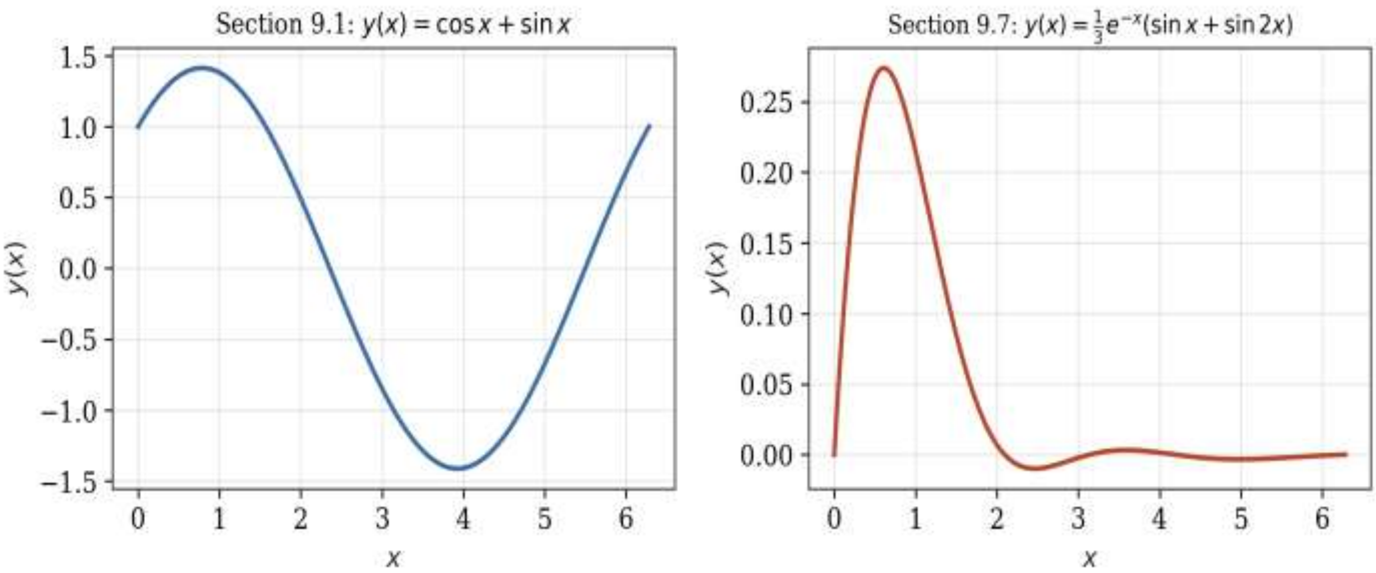

Figure 1. The solution curves of Sections 9.1 (left) and 9.9 (right) on $0 \le x \le 2\pi$, plotted from the closed-form expressions (15) and (23).

General solution of $y'' - a^2y = f(t)$

For $y'' - a^2y = f(t)$, $y(0) = c_1$, $y'(0) = c_2$, equation (14) with $a_1 = 0$, $a_0 = -a^2$ gives $\pi(s) = s^2-a^2 = (s-a)(s+a)$ and

$$Y(v) = \frac{R(v)}{s^2 - a^2} + \frac{c_1pv^ms + c_2pv^m}{s^2 - a^2}.$$

Since G {sinh at} (v) = $a\cdot pv^m/(s^2-a^2)$, the convolution theorem (12) inverts the first term as a convolution with sinh(at)/a, giving

$$y(t) = c_1\cosh at + \frac{c_2}{a}\sinh at + \frac{1}{a}\int_0^t f(u)\sinh(a(t-u))\,du. \tag{24}$$

General solution of $y'' + a^2y = f(t)$

For $y'' + a^2y = f(t)$, $y(0) = 1$, $y'(0) = -2$, the same route with $\pi(s) = s^2+a^2$ and the sine convolution pair G {sin at} (v) = $a\cdot pv^m/(s^2+a^2)$ gives

$$y(t) = \cos at - \frac{2}{a}\sin at + \frac{1}{a}\int_0^t f(u)\sin(a(t-u))\,du. \tag{25}$$

Equations (24) and (25) are the same convolution route used, case by case, in Sections 9.1–9.9: setting $f \equiv 0$ in (25) recovers the homogeneous undamped solution of Section 9.1 (for the matching initial slope), and setting $f(t) = 9t/a^2$ recovers the polynomial-forcing pattern of Section 9.8. Notably, the closed forms in (24)–(25) depend on f only through the convolution integral, never through $R(v) = G\{f(t)\}(v)$ itself.

DISCUSSION

The second-derivative identity (9) is the central result for the class of problems treated here: it converts a second-order differential operator into a quadratic polynomial $\pi(qv^w)$, with the initial conditions entering as lower-degree correction terms, exactly as the Laplace transform converts a differential operator into a polynomial in s after the substitution $s = qv^w$ and rescaling by pv^m. Section 9 confirms this reduction explicitly: the same characteristic polynomial and the same partial-fraction structure arise whether a problem is solved with the G.G. integral transform directly or with its Laplace, Aboodh or Elzaki specializations.

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Two identities take a more general form here than their simplest textbook statement. The change-of-scale property (7), $G\{f(at)\}(v) = a^{m/w-1} G(v/a^{1/w})$, reduces to the simpler rule $a^{-1}G(v/a)$ only when $w=1$; the general form is needed for any of the $w\neq\pm1$ specializations in Table 2, such as the Sadik transform. The convolution theorem (12) carries an explicit normalizing factor $1/(pv^m)$, required for consistency with the transform's own duality with the Laplace transform: without it, (12) would not reduce to the classical convolution theorem in the Laplace case $p=1$, $m=0$.

Section 9 also illustrates the value of cross-checking several transform methods against one another: the four methods (GGIT, Laplace, Aboodh, Elzaki) remain internally consistent with each other in every problem and, on direct substitution, with a single correctly stated problem in each case, as verified in place throughout Section 9.

Table 2 illustrates the same point for named-transform comparisons: the parameter values given here for the Aboodh and Kamal rows were checked directly against each transform's own definition in the cited literature.

CONCLUSION

The G.G. integral transform, with kernel $p\,v^m e^{-qv^w t}$, has been formulated and applied to second-order constant-coefficient ordinary differential equations. Elementary transform pairs were derived for powers, exponentials, trigonometric and hyperbolic functions (Section 3), and the main operational properties — linearity, first and second shifting, change of scale, differentiation, multiplication by the independent variable, and convolution — were stated and proved in a single consistent notation (Sections 4–7), with corrected general forms given for the change-of-scale and convolution identities.

The G.G. parameter family was compared with five named transforms drawn from the literature, each checked against its published definition (Section 8), and the method was applied to nine second-order initial- and boundary-value problems spanning real, repeated and complex characteristic roots and exponential, polynomial and damped-trigonometric forcing (Section 9), together with the general convolution solutions of $y''-a^2y=f(t)$ and $y''+a^2y=f(t)$. Each solution was cross-checked against the Laplace, Aboodh and Elzaki transforms and verified by direct substitution into the original equation. Natural extensions of this work include a fully specified inversion theorem valid for complex or negative kernel parameters, a precise characterization of the admissible function space for each named specialization, and a numerical comparison identifying regimes in which a particular (p,q,m,w) parameterization offers a genuine computational advantage over the ordinary Laplace transform.

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