INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING,
MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VIII, August 2026

A Study of Minimum Dominating Maximum Eccentricity Energy in Globe and Bistar Graphs

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DOI: https://doi.org/10.51583/IJLTEMAS.2026.150800099

Received: 01 September 2026; Accepted: 08 September 2026; Published: 18 September 2026

ABSTRACT

In this paper, we compute the minimum dominating maximum eccentricity energy of globe graphs, bistar graphs and several related graph families. The corresponding matrices along with their eigenvalues are obtained leading to explicit expressions for these energies. These energies are compared across the graph families to assess the effect of graph structure.

Keywords: Bi-star Graph, Globe Graph, Maximum Eccentricity, Domination Energy.

INTRODUCTION

Graph energy, introduced by Gutman [6], is a significant concept in the field of graph theory, which bridges graph structures with linear algebra and chemical applications. Traditionally defined as the sum of the absolute eigenvalues of the adjacency matrix, it has been extended into various graph related matrices, giving rise to numerous energies based spectral parameters. Graph energy and its associated properties have been extensively studied in the existing literature [1, 2, 7, 10, 11, 13].

For any two vertices $u, v \in V(G)$, the distance $d(u, v)$ is defined as the length of a shortest path connecting u and v in G. If there is no path connecting the two vertices, their distance is taken to be ∞. In particular, for G being a finite connected graph, the distance between every pair of vertices is finite. The eccentricity of $v \in V(G)$, denoted by $e(v)$, is the maximum distance between v and any other vertex of G, that is, $e(v) = max\ d(u, v) \in V(G)$. Naji and Soner [7] introduced the maximum eccentricity matrix $M_e(G)$ for a connected graph G. This matrix is defined as follows:

$$e_{ij} = \begin{cases} max\{e(v_i), e(v_j)\}, & if\ \ v_i v_j \in E(G) \\ 0, & otherwise \end{cases}$$

The maximum eccentricity eigenvalues of G are the eigenvalues of $M_e\,(G)$. We consider finite, simple, undirected graphs and recall the necessary basic graph-theoretic definitions below:

Definition 1: Let $G =$ be a simple graph. A set $D \subseteq V$ is a dominating set if every vertex outside D is adjacent to a vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of such a set.

Definition 2: The maximum eccentricity energy of G, denoted by $EM_e(G)$, is defined as $EM_e(G) = \sum_{i=1}^{p} |\lambda_i|$ where $\lambda_1, \lambda_2, \dots, \lambda_p$ are the eigenvalues of $M_e\,(G)$.

Definition 3: A globe graph $Gl(p)$ is obtained by connecting two vertices with p internally disjoint paths of length two. Equivalently, if the two vertices are u and v, then for each $i = 1, 2, \dots, p$, a new vertex w_i is introduced, and the edges uw_i and $w_i v$ are added. Thus, $Gl(p) \cong K_{2,p}$. The globe graph has $|V(Gl(p))| = (p + 2)$ vertices and $|E(Gl(p))| = 2p$ edges respectively.

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Definition 4: The bistar graph $B_{m,n}$ is a tree formed by joining the central vertices of two stars $K_{1,m}$ and $K_{1,n}$. It has $(m+n+2)$ vertices and $(m+n+1)$ edges.

Definition 5: For a graph G, its square G^2 has the same vertex set as G, where two distinct vertices are adjacent if their distance is at most two; that is, $uv \in E(G^2)$ if and only if $d_G(u,v) \leq 2$.

Definition 6: The crown graph Cr_p is obtained from $K_{p,p}$ by deleting a perfect matching. With bipartition sets

$V_1 = \{u_1, u_2, \dots, u_p\}$ and $V_2 = \{v_1, v_2, \dots, v_p\}$, u_i is adjacent to v_j whenever $i \neq j$. Thus, $|V(Cr_p)| = 2p$, and $|E(Cr_p)| = p(p-1)$.

Definition 7: The splitting graph of G, denoted by $S(G)$, is constructed by adding a new vertex v' corresponding to each vertex $v \in V(G)$. Each newly introduced vertex v' is joined to all the neighbors of its corresponding vertex v in G. Consequently, for every $v \in V(G)$, the neighborhood of v' in $S(G)$ satisfies $N_S(v') = N_G(v)$.

Minimum Dominating Maximum Eccentricity Energy of A Graph

Let G be a graph of order p and size q, with $V(G) = \{v_1, v_2, \dots, v_p\}$ and edge set $E(G)$. A set $D \subseteq V$ is dominating if every vertex in $(V-D)$ is adjacent to some vertex in D. The minimum size of such a set is called the maximum eccentricity domination number, denoted by $\gamma_{D_e}^{max}(G)$. The associated minimum dominating maximum eccentricity matrix $A_{D_e}(G) = a_{ij}$ is a $p \times p$ matrix defined with respect to a minimum maximum eccentricity dominating set, where

$$a_{ij} = \begin{cases} max\{e(v_i), e(v_j)\}, & if \ \ v_i v_j \in E(G) \\ 1, & if \ v_i \sim v_j \ and \ \ v_i \in D \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_{D_e}(G)$ is $f_p(G,\beta) = det\left(\beta I - A_{D_e}(G)\right)$. The eigenvalues of $A_{D_e}(G)$ are called the minimum dominating maximum eccentricity eigenvalues of G. Since $A_{D_e}(G)$ is a real symmetric matrix, all its eigenvalues are real. Let these eigenvalues be ordered as $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_p$. The minimum dominating maximum eccentricity energy is defined as

$$E_{D_e}(G) = \sum_{i=1}^{p} |\lambda_i|$$

Examples

The concept of minimum dominating maximum eccentricity energy is demonstrated in the following examples.

Bull Graph: Let G be a simple bull graph with $p = 5$ vertices and $q = 5$ edges, obtained by attaching two pendant edges to two vertices of a triangle, as shown in Figure 1.1.

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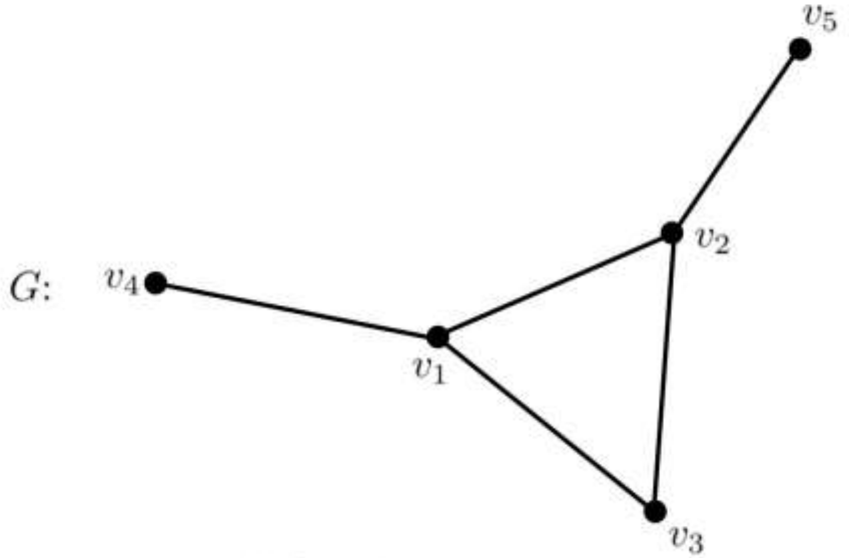

Figure 1.1. Bull Graph with 5 vertices and 5 edges

The vertices $\{v_1, v_2, v_3\}$ have maximum eccentricity 2, whereas the vertices in $\{v_4, v_5\}$ have maximum eccentricity 3. Let $D = \{v_1, v_2\}$ be a dominating maximum eccentricity set. The corresponding minimum dominating maximum eccentricity matrix is given by

$$A_{D_e}(G) = \begin{pmatrix} 1 & 2 & 2 & 3 & 0 \\ 2 & 1 & 2 & 0 & 3 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \end{pmatrix}_{5\times 5}$$

The characteristic equation of $A_{D_e}(G)$ is $\beta(\beta^2 - 3\beta - 17)(\beta^2 + \beta - 9) = 0$. The minimum dominating maximum eccentricity eigenvalues are $\beta_1 = 0$, $\beta_2 = 5.88748$, $\beta_3 = -2.88748$, $\beta_4 = 2.54138$, and $\beta_5 = -3.54138$. Therefore, the minimum dominating maximum eccentricity energy of the bull graph is $E_{D_e}(G) \approx 14.85772$.

Triangular oxide network $\boldsymbol{TOX(p)}$: The minimum dominating maximum eccentricity energy of the triangular oxide network $TOX(p)$ when $p = 2$ is approximately 18.04435.

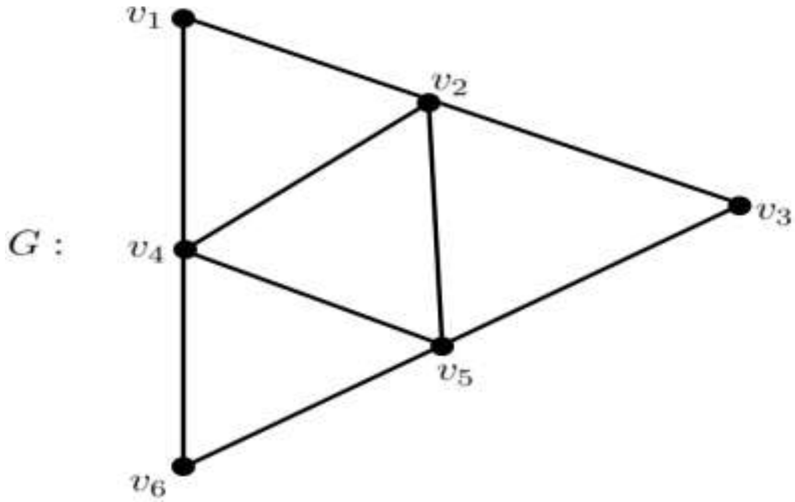

Figure 1.2. Triangular Oxide Netwok TOX(2)

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A Triangular oxide network $TOX(p)$ is a graph model of a silicate structure where the silicon ions are removed, leaving only the triangularly arranged oxygen atoms. For a given length p, it consists of $\frac{p^2+3p+2}{2}$ vertices and $\frac{3(p^2+p)}{2}$ edges (Oxygen atoms). $TOX(p)$ network is a mathematical structure used to study the properties of certain oxides by analyzing its topology. It is an important area of chemical graph theory that employs mathematical techniques, particularly topological indices, to investigate and characterize the structural properties of specific oxide compounds.

The set $\{v_1, v_2, \ldots, v_6\}$ forms a maximum eccentricity set, with each vertex having eccentricity 2. Let $D = \{v_3, v_4\}$ denote the minimum dominating maximum eccentricity set. The corresponding dominating maximum eccentricity matrix is given by:

$$A_{D_e}(G) = \begin{pmatrix} 0 & 2 & 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 1 & 0 & 2 & 0 \\ 2 & 2 & 0 & 1 & 2 & 2 \\ 0 & 2 & 2 & 2 & 0 & 2 \\ 0 & 0 & 0 & 2 & 2 & 0 \end{pmatrix}_{6\times 6}$$

The corresponding characteristic polynomial of $A_{D_e}(G)$ is $(\beta^2 + 2\beta - 4)(\beta^4 - 4\beta^3 - 23\beta^2 + 14\beta + 76) = 0$. The eccentricity eigenvalues are $\beta_1 = 1.23607$, $\beta_2 = -3.23607$, $\beta_3 = -2.59305$, $\beta_4 = -2.19306$, $\beta_5 = 1.95699$, and $\beta_6 = 6.82911$. Therefore, the minimum dominating maximum eccentricity energy of the triangular oxide network is $E_{D_e}(G) \approx 18.04435$.

Petersen graph: Let G be a Petersen graph, which is a simple graph with 10 vertices and 15 edges, as represented in Figure 1.3. The vertex set forms a maximum eccentricity set, with each vertex having eccentricity 2.

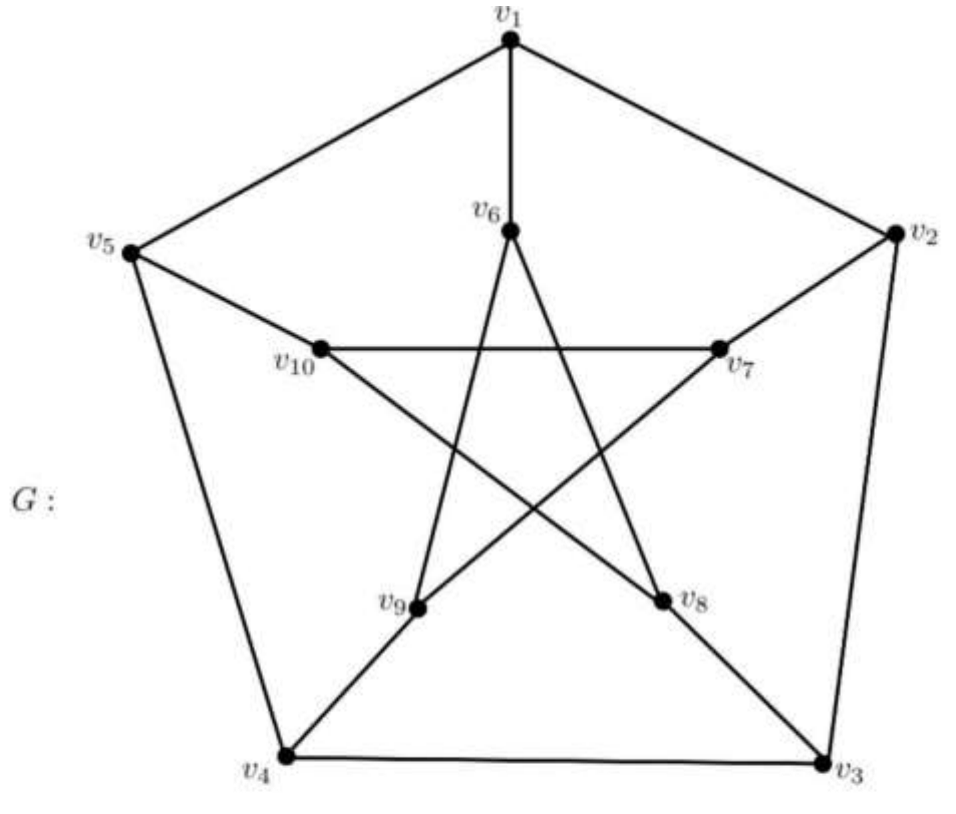

Figure 1.3. Petersen Graph with 10 vertices and 15 edges

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Consider $D = \{v_1, v_3, v_7\}$ as the minimum dominating maximum eccentricity set. The corresponding dominating maximum eccentricity matrix is

$$A_{D_e}(G) = \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \end{pmatrix}_{10\times 10}$$

The corresponding characteristic polynomial of $A_{D_e}(G)$ is $(\beta + 4)(\beta - 2)^2(\beta^2 + \beta - 10)^2(\beta^3 - 5\beta^2 - 16\beta + 48) = 0$. The minimum dominating maximum eccentricity eigenvalues are $\beta_1 = -4, \beta_2 = 2$, $\beta_3 = 2$, $\beta_4 = 2.70156$, $\beta_5 = 2.70156$, $\beta_6 = -3.70156, \beta_7 = -3.70156, \beta_8 = -3.49776, \beta_9 = 6.32972$ and $\beta_{10} = 2.16804$. Therefore, the minimum dominating maximum eccentricity energy $E_{D_e}(G) \approx 32.80176$.

Complete Graph (K_5): Let G be a simple complete graph with 5 vertices and 10 edges, as shown in Figure 1.4. The set $\{v_1, v_2, \ldots, v_5\}$ forms a maximum eccentricity set, and every vertex has eccentricity 1.

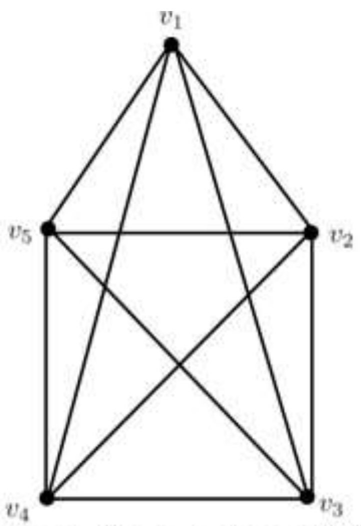

Figure 1.4. Complete Graph (K_5)

Let $D = \{v_1\}$ be the minimum dominating maximum eccentricity set, and let the corresponding dominating maximum eccentricity matrix be

$$A_{D_e}(K_5) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}_{5\times 5}$$

The characteristic polynomial of $A_{D_e}(K_5)$ is $(\beta + 1)^3(\beta^2 - 4\beta - 1) = 0$. The minimum dominating maximum eccentricity eigenvalues are $\beta_1 = -1, \beta_2 = -1, \beta_3 = -1, \beta_4 = 4.23607$ and $\beta_5 = -0.23607$. Hence, $E_{D_e}(K_5) \approx 7.47214$.

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Minimum Dominating Maximum Eccentricity Energy: Results For Standard Graphs

Theorem 1: For a bistar graph $B_{p,p}$ with $p \geq 2$

$$E_{D_e}(B_{p,p}) = \sqrt{45p^2 - 96p + 100} + \sqrt{45p^2 - 100p + 96}$$

Proof: Let $V(B_{p,p}) = \{v, v_1, v_2, \dots, v_p, u, u_1, u_2, \dots, u_p\}$ be the vertex set of $B_{p,p}$. The graph $B_{p,p}$ is formed by joining two central vertices u and v, where each central vertex is adjacent to p pendant vertices. Hence, $B_{p,p}$ has $(2p+2)$ vertices and $(2p+1)$ edges, as displayed in Figure 1.5.

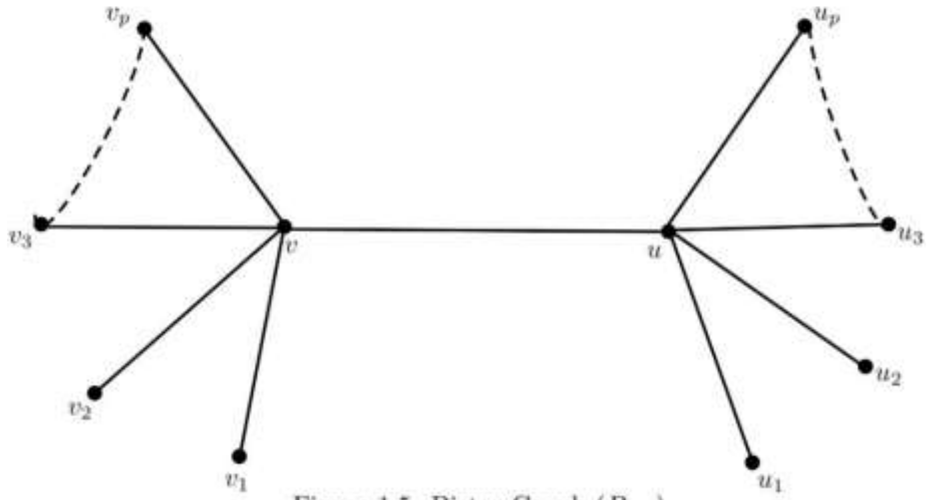

Figure 1.5. Bistar Graph ($B_{p,p}$)

Consider $D = \{v, u\}$ as a minimum dominating maximum eccentricity set. The corresponding adjacency matrix is

$$A_{D_e}(B_{p,p}) = \begin{pmatrix} 1 & (p+1) & \cdots & (p+1) & (p+1) & 0 & \cdots & 0 \\ (p+1) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (p+1) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ (p+1) & 0 & \cdots & 0 & 1 & (p+1) & \cdots & (p+1) \\ 0 & 0 & \cdots & 0 & (p+1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & (p+1) & 0 & \cdots & 0 \end{pmatrix}_{(2p+2)\times(2p+1)}$$

The corresponding spectral polynomial of $A_{D_e}(B_{p,p})$ is

$$f_p(B_{p,p}, \beta) = \beta^{2p-2}(\beta^2 - (p+2)\beta - (11p^2 - 25p + 24))(\beta^2 - p\beta - (11p^2 - 25p + 24)) = 0$$

The minimum dominating maximum eccentricity spectrum of the bistar graph $B_{p,p}$ is

$$Spec_{D_e}(B_{p,p}) =$$

$$\begin{pmatrix} \frac{(p+2)-\sqrt{45p^2-96p+100}}{2} & \frac{-p-\sqrt{45p^2-96p+100}}{2} & 0 & \frac{(p+2)+\sqrt{45p^2-96p+100}}{2} & \frac{-p+\sqrt{45p^2-96p+100}}{2} \\ 1 & 1 & (2p-2) & 1 & 1 \end{pmatrix}$$

Thus, the minimum dominating maximum eccentricity energy corresponding to $B_{p,p}$ is

$$E_{D_e}(B_{p,p}) = \sqrt{45p^2 - 96p + 100} + \sqrt{45p^2 - 100p + 96}$$

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Theorem 2: For a square of a bistar graph $B_{p,p}^2$ with $p \geq 2$

$$E_{D_e}\left(B_{p,p}^2\right) = 2p + \sqrt{644p^2 - 1640p + 1540}$$

Proof: The square of the bistar graph $B_{p,p}$, denoted by $B_{p,p}^2$, is obtained by joining two vertices whenever their distance in $B_{p,p}$ is at most two. Thus, $B_{p,p}^2$ retains all the adjacencies of $B_{p,p}$ and further adds edges between pairs of vertices that are separated by a distance of two. Let $V\left(B_{p,p}^2\right) = \{v, v_1, v_2, \dots, v_p, u, u_1, u_2, \dots, u_p\}$ be the vertex set of $B_{p,p}^2$. Thus, $B_{p,p}^2$ has $(2p+2)$ vertices and (p^2+3p+1) edges, as shown in Figure 1.6.

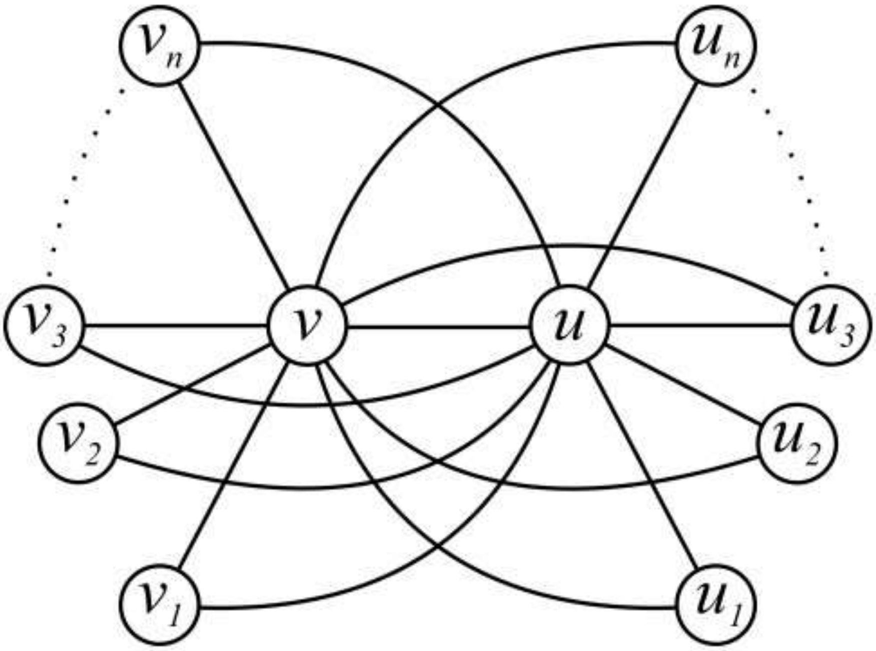

Figure 1.6. Square of Bistar Graph

Let $D = \{v, u\}$ be the minimum dominating maximum eccentricity set. The corresponding adjacency matrix is given by

$$A_{D_e}\left(B_{p,p}^2\right) = \begin{pmatrix} 1 & (2p+1) & \cdots & (2p+1) & (2p+1) & (2p+1) & \cdots & (2p+1) \\ (2p+1) & 0 & \cdots & 0 & (2p+1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (2p+1) & 0 & \cdots & 0 & (2p+1) & 0 & \cdots & 0 \\ (2p+1) & (2p+1) & \cdots & (2p+1) & 1 & (2p+1) & \cdots & (2p+1) \\ 0 & 0 & \cdots & 0 & (2p+1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (2p+1) & 0 & \cdots & 0 & (2p+1) & 0 & \cdots & 0 \end{pmatrix}_{(2p+2)\times(p^2+3p+1)}$$

The spectral polynomial of $A_{D_e}\left(B_{p,p}^2\right)$ is

$$f_p\left(B_{p,p}^2, \beta\right) = \beta^{2p-1}(\beta + 2p)(\beta^2 - (2p+2)\beta - (160p^2 - 412p + 384)) = 0$$

The minimum dominating maximum eccentricity spectrum of $B_{p,p}^2$ is

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$$Spec_{D_e}\left(B^2_{p,p}\right)$$
$$= \begin{pmatrix} \frac{(2p+2)-\sqrt{644p^2-1640p+1540}}{2} & 0 & -2p & \frac{(2p+2)+\sqrt{644p^2-1640p+1540}}{2} \\ 1 & (2p-1) & 1 & 1 \end{pmatrix}$$

Consequently, the minimum dominating maximum eccentricity energy of $B^2_{p,p}$ is given by

$$E_{D_e}\left(B^2_{p,p}\right) = 2p + \sqrt{644p^2 - 1640p + 1540}$$

Theorem 3: For a Globe graph $Gl(p)$ with $p \geq 2$

$$E_{D_e}[Gl(p)] = 1 + \sqrt{8p^3 + 1}$$

Proof: Let $V[Gl(p)] = \{u, v, v_1, v_2, \dots, v_p\}$ be vertex set of $Gl(p)$. The graph $Gl(p)$ is formed by joining two non-adjacent vertices u and v through p internally disjoint paths, each of length two. Consequently, $Gl(p)$ has $(p+2)$ vertices and $2p$ edges, as shown in Figure 1.7.

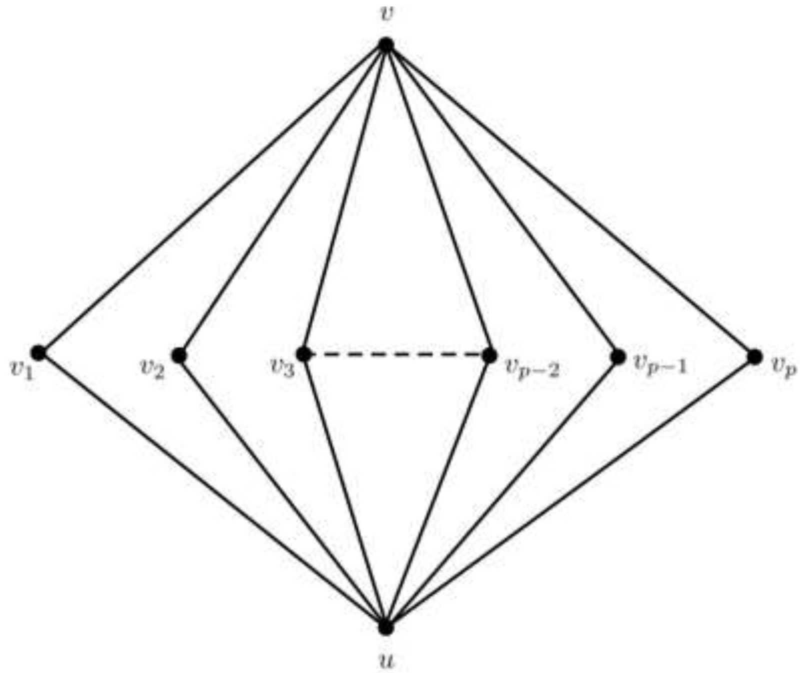

Figure 1.7. Globe Graph $Gl(p)$

Let $D = \{u, v\}$ be the minimum dominating maximum eccentricity set. The corresponding adjacency matrix is

$$A_{D_e}[Gl(p)] = \begin{pmatrix} 1 & 0 & p & p & \cdots & p & p \\ 0 & 1 & p & p & \cdots & p & p \\ p & p & 0 & 0 & \cdots & 0 & 0 \\ p & p & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p & p & 0 & 0 & \cdots & 0 & 0 \\ p & p & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{(p+2)\times 2p}$$

The corresponding spectral polynomial of $A_{D_e}[Gl(p)]$ is

$$f_p(Gl(p), \beta) = (-1)^p \beta^{p-1}(\beta - 1)(\beta^2 - \beta - 2p^3) = 0$$

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The minimum dominating maximum eccentricity spectrum of $Gl(p)$ is

$$Spec_{D_e}[Gl(p)] = \begin{pmatrix} \frac{1-\sqrt{8p^3+1}}{2} & 0 & 1 & \frac{1+\sqrt{8p^3+1}}{2} \\ 1 & (p-1) & 1 & 1 \end{pmatrix}$$

Thus, the minimum dominating maximum eccentricity energy of $Gl(p)$ is

$$E_{D_e}[Gl(p)] = 1 + \sqrt{8p^3+1}$$

Theorem 4: For a crown graph Cr_p with $p \geq 3$

$$E_{D_e}(Cr_p) = 2(p^2-3p+2) + \sqrt{p^4-6p^3+47p^2-182p+237} + \sqrt{p^4-6p^3+51p^2-194p+245}$$

Proof: Let $V(Cr_p) = \{v_1, v_2, \dots, v_p, u_1, u_2, \dots, u_p\}$ denote the vertex set of Cr_p. The crown graph Cr_p consists of $2p$ vertices and $p(p-1)$ edges. Let $D = \{v_1, u_1\}$ be a minimum dominating maximum eccentricity set. The corresponding adjacency matrix is

$$A_{D_e}(Cr_p) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & (p-1) & \cdots & (p-1) \\ 0 & 0 & \cdots & 0 & (p-1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & (p-1) & (p-1) & \cdots & 0 \\ 0 & (p-1) & \cdots & (p-1) & 1 & 0 & \cdots & 0 \\ (p-1) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (p-1) & (p-1) & \cdots & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{(2p)\times p(p-1)}$$

The corresponding spectral polynomial of $A_{D_e}(Cr_p)$ is

$$f_p(Cr_p, \beta) = (\beta-(p-1))^{p-2}\,(\beta+(p-1))^{p-2}\,\left(\beta^2-(p^2-3p+3)\beta-(8p^2-41p+57)\right)\left(\beta^2 + (p^2-3p+1)\beta-(10p^2-47p+61)\right) = 0$$

The minimum dominating maximum eccentricity spectra of the crown graph Cr_p is

$$Spec_{D_e}(Cr_p) = \begin{pmatrix} \frac{(p^2-3p+3)-X}{2} & \frac{-(p^2-3p+1)-Y}{2} & -(p-1) & (p-1) & \frac{(p^2-3p+3)+X}{2} & \frac{-(p^2-3p+1)+Y}{2} \\ 1 & 1 & (p-2) & (p-2) & 1 & 1 \end{pmatrix}$$

where $X = \sqrt{p^4-6p^3+47p^2-182p+237}$ and $Y = \sqrt{p^4-6p^3+51p^2-194p+245}$

Therefore, the minimum dominating maximum eccentricity energy of Cr_p is

$$E_{D_e}(Cr_p) = 2(p^2-3p+2) + \sqrt{p^4-6p^3+47p^2-182p+237} + \sqrt{p^4-6p^3+51p^2-194p+245}$$

Theorem 5: For the splitting graph of the star $S'(K_{1,p})$ with $p \geq 2$, the maximum eccentricity energy is given by

$$E_{D_e}[S'(K_{1,p})] = \sqrt{1+2(9+\sqrt{65}p^3)} + \sqrt{1+2(9-\sqrt{65}p^3)}$$

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Proof: Let $V[S'(K_{1,p})] = \{v_1, v_2, \dots, v_p, u_1, u_2, \dots, u_p\}$ denote the vertex set of $S'(K_{1,p})$. The splitting graph $S'(K_{1,p})$ is constructed from the star graph $K_{1,p}$. Hence, $S'(K_{1,p})$ contains $(2p+1)$ vertices and $3p$ edges. Let $D = \{v_1, u_1\}$ be a minimum dominating maximum eccentricity set. The corresponding adjacency matrix is

$$A_{D_e}[S'(K_{1,p})] = \begin{pmatrix} 1 & 2p & \cdots & 2p & 0 & 2p & \cdots & 2p \\ 2p & 0 & \cdots & 0 & p & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2p & 0 & \cdots & 0 & p & 0 & \cdots & 0 \\ 0 & p & \cdots & p & 1 & 0 & \cdots & 0 \\ 2p & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2p & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{(2p+1)\times 3p}$$

The spectral polynomial of $A_{D_e}[S'(K_{1,p})]$ is

$$f_p(S'(K_{1,p}), \beta) = \beta^{2p-2}\left(\beta - \frac{1 \pm \sqrt{1 + 2(9 \pm \sqrt{65}p^3)}}{2}\right) = 0$$

The minimum dominating maximum eccentricity spectra of $S'(K_{1,p})$ is

$$Spec_{D_e}[S'(K_{1,p})] =$$

$$\begin{pmatrix} \frac{1+\sqrt{1+2(9+\sqrt{65}p^3)}}{2} & \frac{1+\sqrt{1+2(9-\sqrt{65}p^3)}}{2} & 0 & \frac{1-\sqrt{1+2(9+\sqrt{65}p^3)}}{2} & \frac{1-\sqrt{1+2(9-\sqrt{65}p^3)}}{2} \\ 1 & 1 & (2p-2) & 1 & 1 \end{pmatrix}$$

The minimum dominating maximum eccentricity energy of $S'(K_{1,p})$ is

$$E_{D_e}[S'(K_{1,p})] = \sqrt{1 + 2(9 + \sqrt{65}p^3)} + \sqrt{1 + 2(9 - \sqrt{65}p^3)}$$

CONCLUSION

Nowadays, the study of domination and graph energy has become an important area of research in graph theory, attracting considerable attention from researchers. In recent years, several new domination parameters have been introduced to explore the structural and spectral properties of graphs. In this article, we introduce and investigate the minimum dominating maximum eccentricity parameter, determine its exact values for standard families of graphs, and establish suitable bounds in terms of graph order, degree, and other structural parameters.

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ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VIII, August 2026

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