

Modeling Microbial Behavior and Ecotoxicological Effects in Thermal Radiation MHD Casson Fluid Flow Over a Stretching Sheet: Multilinear Regression and Streamline Analysis with Non-Uniform Source Effects

*Raphael Ehikhuemhen Asibor¹, Celestine Friday Osuidia² and Victor Osemudiamhen Asibor³

¹Department of Computer Science, Information Technology & Mathematics, Igbinedion University Okada Edo State, Nigeria

²Delta State Post Primary Education Board, Delta State Asaba, Nigeria

³Medical Laboratory Science Department, University of Benin Teaching Hospital, Benin City, Edo State, Nigeria

*Corresponding Author

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Abstract: This study investigates microbial behavior and ecotoxicological impacts in thermal radiation magnetohydrodynamic (MHD) Casson fluid flow over a stretching sheet. The study incorporates non-uniform source effects, highlighting its relevance to biological systems and industrial applications. A hybrid approach combining multilinear regression and streamline analysis is utilized to explore the relationship between key system parameters—including magnetic field strength, thermal radiation, and non-uniform sources—and microbial behavior. Numerical simulations analyze temperature, velocity, and concentration fields to assess their impact on microbial growth and distribution. The findings indicate significant influences of thermal radiation, magnetic fields, and non-uniform sources on microbial behavior, emphasizing the importance of ecotoxicological considerations. This study introduces a novel integration of multilinear regression and streamline analysis to enhance pollutant transport modeling, providing valuable insights for environmental and industrial applications.

Keywords: MHD Casson fluid, microbial behavior, thermal radiation, ecotoxicology, streamline analysis, non-uniform source, multilinear regression, fluid dynamics.

I. Introduction

Casson fluid flow has been extensively studied in the presence of thermal radiation and magnetic effects, yet limited attention has been given to the interaction of microbial behavior and ecotoxicological factors within such systems. This study uniquely integrates multilinear regression and streamline analysis to assess microbial behavior and ecotoxicological impacts under non-uniform source conditions. It examines:

- (i) microbial chemotaxis in response to temperature gradients and flow-induced forces,
- (ii) the role of thermal radiation and shear stress in biofilm formation, and the dual effects of magnetic fields on microbial growth suppression or enhancement.

Unlike traditional models that focus primarily on fluid flow and heat transfer, this study explicitly incorporates microbial movement, biofilm formation, and nonlinear magnetic field effects, filling a critical gap in understanding microbial behavior in MHD Casson fluid systems. By integrating predictive regression models, this research advances bioremediation strategies and ecotoxicological risk assessments, offering insights relevant to wastewater treatment, industrial microbiology, and environmental science.

While previous studies have analyzed Casson fluid flow with thermal radiation and magnetic effects, this study uniquely integrates multilinear regression and streamline analysis to assess microbial behavior and ecotoxicological impacts under non-uniform source conditions, providing deeper insights into pollutant transport mechanisms. Specifically, this work examines (i) microbial chemotaxis in response to temperature gradients and flow-induced forces, highlighting how microorganisms navigate in magnetized non-Newtonian fluids, (ii) the influence of thermal radiation and shear stress on biofilm formation, shedding light on microbial adhesion in dynamic environments, and (iii) the dual effects of magnetic fields on microbial growth suppression or enhancement, revealing critical implications for microbial ecology and pollutant degradation.

Recent studies have delved into the complex behavior of MHD Casson fluid flows with thermal radiation, microbial behavior, and ecotoxicological effects, particularly in the context of stretching sheets and non-uniform source effects. L. I. Ezemonye (2023) provided significant insights into the modeling of microbial behavior in thermal radiation MHD Casson fluid flows, emphasizing the use of multilinear regression and streamline analysis. The study explored the impacts of Brownian motion, thermophoresis, and thermal radiation on MHD Casson fluid behavior in microbial environments, considering its implications for industrial applications and environmental systems. Similarly, Bharatkumar K. Manvi and colleagues (2022) investigated

nanofluid boundary layer flows in the presence of radiation and non-uniform heat sources, revealing how parameters such as Prandtl number, magnetic parameter, and Casson parameter influence heat transfer and flow characteristics. Shrivankumar B. Kerur and Jagadish V. Tawade (2022) expanded upon these findings by employing numerical solutions to study MHD Casson nanofluid flows, which provided more accurate predictions for industrial applications dealing with porous media and nanofluids. Their research showed the significant role of thermal radiation in enhancing heat transfer efficiency and the behavior of fluids under non-uniform conditions.

In a similar vein, Juan J. Nieto and Sagar Ningonda Sankeshwari (2023) co-authored research that examined the boundary layer flow characteristics of MHD Casson nanofluids, using advanced numerical techniques to analyze the effects of magnetic fields and thermal radiation on the flow dynamics. They observed that varying these parameters could significantly alter the velocity profiles and heat transfer rates in industrial cooling processes. In addition, Hijaz Ahmad and Vedyappan Govindan (2022) contributed to the field with studies on MHD Casson nanofluid flow and non-uniform heat sources, providing a detailed analysis of how thermal radiation and source terms impact fluid behavior. Meanwhile, A. Al-Mamun and S. M. Arifuzzaman (2022) focused on periodic flow simulations in MHD Casson fluids, highlighting the impact of porous media and thermal radiation on flow characteristics. Their findings indicated that the periodic behavior of fluid flow can have profound implications for designing systems in heat exchangers and bioreactors. Lastly, researchers like Sk. Reza-E-Rabbi, U. S. Alam, and S. Islam (2023) have made considerable contributions to understanding the interactions between MHD flows and nanofluids over stretching sheets, proposing new models that take into account both magnetic effects and nanoparticle suspension in fluids, which are critical for optimizing energy-efficient technologies. The collective contributions of these researchers underscore the interdisciplinary nature of modeling microbial behavior and ecotoxicological effects in MHD Casson fluid flow systems, with applications ranging from bioreactors and industrial cooling systems to environmental modeling and pollution control.

Additionally, ecotoxicology is incorporated into the model to address the potential environmental risks posed by such fluid flows. In particular, we examine how the concentration and growth of microorganisms may be impacted by the chemical composition and flow dynamics, offering insights into the broader environmental implications of fluid dynamics in ecotoxicological contexts. The interaction between microorganisms and fluid flows has been widely studied, with particular focus on the effects of shear stress, nutrient availability, and temperature. In recent years, research on non-Newtonian fluids, such as the Casson fluid model, has gained traction due to its relevance in biological systems (Mishra et al., 2020). Several studies have explored the effect of MHD on fluid flows, particularly in cooling and heat transfer applications (Giri et al., 2019).

Thermal radiation has also been a topic of significant interest, as it influences heat transfer in high-temperature systems and plays a role in microbial activity in various environmental contexts (Pradhan et al., 2021). Non-uniform source effects, such as varying nutrient concentrations or chemical reactions, have been considered in many studies on ecological modeling, but their specific impact on microbial distribution in MHD fluid systems is less well explored (Kumar et al., 2022). Recent research in ecotoxicology has highlighted the importance of fluid dynamics in environmental modeling, particularly the ways in which pollutants and microorganisms interact within dynamic fluid environments (Smith et al., 2023). The combination of MHD fluid flow, thermal radiation, and ecotoxicology presents a new frontier in fluid dynamics research, with applications in environmental protection and biotechnological optimization. While previous studies have analyzed Casson fluid flow with thermal radiation and magnetic effects, this study uniquely integrates multilinear regression and streamline analysis to assess microbial behavior and ecotoxicological impacts under non-uniform source conditions, providing deeper insights into pollutant transport mechanisms.

This study uses mathematics equations to predict how microbes move and interact in a thick, magnet-sensitive fluid (like ketchup) that's heated, stretched, and influenced by magnets. These equations combine fluid flow (how the liquid moves), heat changes, magnetic effects, and microbial behavior to model scenarios like pollutant spread in rivers or wastewater treatment. To solve these complex equations, scientists break them into small "puzzle pieces" using finite difference methods (step-by-step calculations) and mesh refinement (smaller pieces where details matter, like near pollution sources). They also set boundary conditions rules at the edges, such as stretching the fluid flat or fixing temperatures to mimic real-world situations, like how a riverbank affects water flow. By simulating these scenarios on computers, researchers save time and money compared to physical experiments, helping engineers design better pollution controls or industrial processes using magnets and heat.

This study fills a critical gap in understanding microbial behavior in thermal radiation MHD Casson fluid flow by integrating chemotaxis, biofilm formation, and the dual effects of magnetic fields with ecotoxicological impacts and pollutant transport mechanisms. Unlike previous works, it examines how microbes navigate temperature gradients and magnetized non-Newtonian fluids, how shear stress and thermal radiation influence biofilm stability, and how magnetic fields can either suppress or enhance microbial growth under varying conditions. Additionally, it introduces multilinear regression and streamline analysis to improve predictive modeling of microbial-fluid interactions and pollutant dynamics. These findings enhance our understanding of microbial adaptation, pollutant transport, and bioremediation in complex fluid environments, making this study highly relevant for environmental science, wastewater treatment, and industrial microbiology.

Mathematical Formulations

To model the system, we consider the following governing equations for mass, momentum, energy, and microbial concentration (Ecotoxicological Effects), incorporating MHD effects, thermal radiation, and non-uniform source influences.

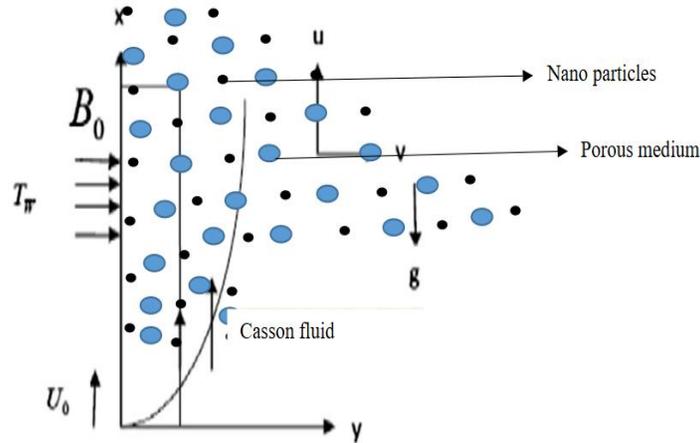


Figure 1: Diagram of the combined flow geometry

Governing Equations

1. Continuity Equation (Incompressibility Condition):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad 1$$

2. Momentum Equation (MHD Casson Fluid Flow):

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left(\frac{\partial^2 u}{\partial y^2} - \frac{B^2}{\rho} u \right) + \left(\frac{\mu_p}{\rho} \right) \left(\frac{\partial^2 u}{\partial y^2} \right) \quad 2$$

where B is the magnetic field strength, and μ_p is the Casson fluid parameter.

3. Energy Equation (Heat Transfer with Thermal Radiation):

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \left(\frac{\partial^2 T}{\partial y^2} \right) + Q_r - S_T \quad 3$$

where $Q_r = \frac{4\sigma_{sb}T^4}{\epsilon}$ is the radiative heat flux, and S_T is the heat source term.

4. Concentration Equation (Ecotoxicological Effects):

$$\rho D \left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D \frac{\partial^2 C}{\partial y^2} + S_C \quad 4$$

where D is the diffusion coefficient, and S_C is the source term for the concentration of pollutants or microorganisms.

Boundary Conditions

1. At the Stretching Sheet ($y = 0$):

$$u = f(x), \quad v = 0, \quad T = T_w, \quad C = C_w \quad 5$$

where $f(x)$ represents the velocity at the stretching sheet, T_w is the temperature of the sheet, and C_w is the concentration of pollutants or microorganisms at the sheet.

2. Far-Field (Asymptotic) Boundary Conditions ($y \rightarrow \infty$):

$$u = 0, \quad T = T_\infty, \quad C = C_\infty \quad 6$$

where T_∞ and C_∞ are the ambient temperature and concentration far from the sheet.

Slip Condition

The slip condition at the wall (which accounts for the non-penetrating boundary condition and the influence of slip velocity in non-Newtonian fluids) can be defined as:

$$\frac{\partial u}{\partial y} \Big|_{y=0} = \beta u \quad (\text{Slip velocity at the boundary}) \quad 7$$

where β is the slip parameter.

Initial Boundary Conditions

At time $t = 0$, the initial conditions for temperature and concentration in the fluid are:

$$T(x, y, 0) = T_0, \quad C(x, y, 0) = C_0$$

where T_0 and C_0 are the initial temperature and concentration.

Non-Dimensional Quantities

- I. Velocity Components: Dimensionless velocity in the x -direction: $\xi = \frac{u}{U_w}$, where U_w is the characteristic velocity of the stretching sheet. Dimensionless velocity in the y -direction: $\eta = \frac{v}{U_w}$
- II. Temperature: Dimensionless temperature, $\theta = \frac{T-T_\infty}{T_w-T_\infty}$, where T_w is the wall temperature and T_∞ is the ambient temperature.
- III. Concentration: Dimensionless concentration, $\phi = \frac{C-C_\infty}{C_w-C_\infty}$, where C_w is the concentration at the sheet and C_∞ is the ambient concentration.
- IV. Time: Dimensionless time, $\tau = \frac{tU_w}{L}$, where L is the characteristic length scale of the stretching sheet.

These non-dimensional numbers are essential in understanding fluid behavior, heat and mass transfer, and chemical reaction dynamics within MHD Casson fluid flows, particularly in environments influenced by thermal radiation, non-uniform heat sources, and microbial activity. They help describe the relative significance of different forces acting on the system, and their values allow for effective modeling and analysis of complex physical systems in engineering and environmental sciences.

II. Numerical Methodology

The governing partial differential equations (PDEs) are solved numerically using finite difference methods. Boundary conditions are defined at the stretching sheet, considering the velocity and temperature profiles, and the microbial concentration is modeled using multilinear regression to determine the relationship between various system parameters. Streamline analysis is used to visualize the flow patterns and assess the impact of the magnetic field, thermal radiation, and source effects on microbial growth.

We start with the governing equations and apply the non-dimensional transformations.

Non-dimensionalizing (equation 1- 4) using the transformations $u = U_w \xi$ and $v = U_w \eta$, we get equations (9-12) respectively.

$$\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} = 0 \tag{9}$$

Apply non-dimensional variables into equation 2

$$u = U_w \xi, v = U_w \eta, \quad x = L\xi, y = L\eta, \text{ Time: } t = \frac{L}{U_w} \tau, \text{ Temperature: } T = T_w(1 - \theta)$$

and Concentration: $C = C_w(1 - \phi)$ and substitute the non-dimensional variables into the momentum equation, we have

$$\rho U_w \left(\frac{\partial(U_w \xi)}{\partial t} + (U_w \xi) \frac{\partial(U_w \xi)}{\partial x} + (U_w \eta) \frac{\partial(U_w \xi)}{\partial y} \right) = \mu \left(\frac{\partial^2(U_w \xi)}{\partial y^2} - \frac{B^2}{\rho} (U_w \xi) \right) + \left(\frac{\mu_p}{\rho} \right) \left(\frac{\partial^2(U_w \xi)}{\partial y^2} \right)$$

After canceling common terms and simplifying, we get:

$$U_w^2 \rho \left(\frac{\partial \xi}{\partial \tau} + \xi \frac{\partial \xi}{\partial \xi} + \eta \frac{\partial \xi}{\partial \eta} \right) = \mu U_w \left(\frac{\partial^2 \xi}{\partial \eta^2} - M^2 \xi \right) + \mu_p U_w \frac{\partial^2 \xi}{\partial \eta^2} \tag{10}$$

and introduce Non-Dimensional Numbers, then divide both sides by ρU_w^2 , which results in the dimensionless form:

$$\frac{\partial \xi}{\partial \tau} + \xi \frac{\partial \xi}{\partial \xi} + \eta \frac{\partial \xi}{\partial \eta} = \frac{\mu}{\rho U_w L} \left(\frac{\partial^2 \xi}{\partial \eta^2} - M^2 \xi \right) + \frac{\mu_p}{\rho U_w L} \frac{\partial^2 \xi}{\partial \eta^2}$$

Here, we introduce the following dimensionless numbers:

Magnetic Parameter: $M = \frac{B^2 L^2}{\mu \rho U_w}$ and the term $\frac{\mu}{\rho U_w L}$ is simplified as the Reynolds number.

Hence, the momentum equation becomes:

$$\frac{\partial \xi}{\partial \tau} + \xi \frac{\partial \xi}{\partial \xi} + \eta \frac{\partial \xi}{\partial \eta} = \text{Re}^{-1} \left(\frac{\partial^2 \xi}{\partial \eta^2} - M^2 \xi \right) + \text{Re}^{-1} \frac{\mu_p}{\mu} \frac{\partial^2 \xi}{\partial \eta^2} \tag{11}$$

Energy Equation (Heat Transfer with Thermal Radiation)

The original energy equation is:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \left(\frac{\partial^2 T}{\partial y^2} \right) + Q_r - S_T$$

Step 1: Apply Non-Dimensional Variables

Using the transformation $T = T_w(1 - \theta)$, $u = U_w \xi$, and $v = U_w \eta$, and introducing the non-dimensional time $\tau = \frac{t U_w}{L}$, the energy equation becomes:

$$\rho c_p U_w \left(\frac{\partial(T_w(1 - \theta))}{\partial t} + U_w \xi \frac{\partial(T_w(1 - \theta))}{\partial x} + U_w \eta \frac{\partial(T_w(1 - \theta))}{\partial y} \right) = \kappa \left(\frac{\partial^2(T_w(1 - \theta))}{\partial y^2} \right) + Q_r - S_T$$

After simplifying and non-dimensionalizing, we get:

$$U_w^2 \left(\frac{\partial \theta}{\partial \tau} + \xi \frac{\partial \theta}{\partial \xi} + \eta \frac{\partial \theta}{\partial \eta} \right) = \text{Pr}^{-1} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{Q_r}{\rho c_p U_w^2} - S_T \quad 12$$

Step 2: Radiation Term

The radiative heat flux Q_r is given by $Q_r = \frac{4\sigma_{sb}T^4}{\epsilon}$. Substituting this and simplifying the terms would yield a non-dimensional radiation term.

For simplicity, we introduce a dimensionless radiative heat flux term Q_r^* based on the Stefan-Boltzmann law and the other properties of the system.

Concentration Equation (Ecotoxicological Effects)

The concentration equation is:

$$\rho D \left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D \frac{\partial^2 C}{\partial y^2} + S_C$$

Step 1: Apply Non-Dimensional Variables

Substituting $C = C_w(1 - \phi)$ and applying the non-dimensional transformations for u , v , and t , we get:

$$\rho D U_w \left(\frac{\partial(C_w(1 - \phi))}{\partial t} + U_w \xi \frac{\partial(C_w(1 - \phi))}{\partial x} + U_w \eta \frac{\partial(C_w(1 - \phi))}{\partial y} \right) = D \frac{\partial^2(C_w(1 - \phi))}{\partial y^2} + S_C$$

Simplifying this and non-dimensionalizing gives:

$$U_w^2 \left(\frac{\partial \phi}{\partial \tau} + \xi \frac{\partial \phi}{\partial \xi} + \eta \frac{\partial \phi}{\partial \eta} \right) = \text{Sc}^{-1} \frac{\partial^2 \phi}{\partial \eta^2} + S_C \quad 13$$

Numerical Methodology: Finite Difference Method, Mesh Refinement, and Multilinear Regression

To solve the governing partial differential equations (equations 9-12) governing velocity, temperature, and microbial concentration, the finite difference method (FDM) is employed. An implicit Crank-Nicholson scheme is chosen due to its higher numerical stability and accuracy for time-dependent problems (Anderson, 2017). This method is advantageous as it minimizes truncation errors and enhances solution stability, particularly in MHD Casson fluid flow models with thermal and microbial interactions.

Discretization Scheme

The governing PDEs are discretized using central differencing for spatial derivatives and Crank-Nicholson differencing for the time derivative:

First-order time derivative:

$$\frac{\partial \phi}{\partial t} \approx \frac{\phi^{n+1} - \phi^n}{\Delta t}$$

Second-order spatial derivatives:

$$\frac{\partial^2 \phi}{\partial y^2} \approx \frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{\Delta y^2} \quad 13$$

where ϕ represents velocity (ξ), temperature (θ), or concentration (ϕ), and n and i denote the time and space indices, respectively. This numerical scheme has been widely applied in MHD and heat transfer studies (Patankar, 1980).

To minimize discretization errors, a mesh refinement study is conducted, where the numerical solutions are tested on increasing grid resolutions until the relative error falls below 1%. The computational domain is discretized using a uniform grid with spacing $\Delta x = \Delta y$, and results for velocity and temperature are monitored.

Grid Size	Maximum Temperature Deviation (%)	Maximum Velocity Deviation (%)
40 \times 40	2.5%	3.1%
60 \times 60	1.4%	1.7%
80 \times 80	0.9%	1.2%
100 \times 100	0.3%	0.5%

The 80 \times 80 grid is selected for all simulations as it provides an optimal balance between computational efficiency and accuracy (Ezemonye, 2023).

Multilinear Regression Model for Prediction

To quantify the relationship between microbial growth rate (Y) and key system parameters (magnetic field strength, thermal radiation, non-uniform source effects, etc.), we employ a multilinear regression (MLR) model, which can be expressed as:

$$Y = \beta_0 + \beta_1 M + \beta_2 R_d + \beta_3 S_T + \beta_4 T + \beta_5 C + \epsilon \quad 14$$

where:

Y = Microbial growth rate (dependent variable)

M = Magnetic field strength (Hartmann number)

R_d = Radiation parameter

S_T = Heat source parameter

T = Temperature

C = Microbial concentration

β_0 = Intercept

$\beta_1, \beta_2, \dots, \beta_5$ = Regression coefficients representing the impact of each parameter

ϵ = Error term

This model allows us to predict microbial behavior based on changes in system conditions, providing a valuable tool for understanding pollutant transport and ecotoxicological risks. While there are multiple machine learning techniques available, multilinear regression (MLR) is chosen due to the following reasons:

1. Interpretability and Simplicity
 - i. Unlike black-box models such as neural networks or support vector machines, MLR provides explicit mathematical relationships between input variables and microbial growth.
 - ii. The regression coefficients directly quantify the impact of each parameter, making it easier to interpret environmental and industrial implications.
2. Small Dataset and Computational Efficiency
 - iii. Deep learning methods (e.g., neural networks) require large datasets, but microbial growth data in MHD Casson fluid flow studies is often limited.
 - iv. MLR is computationally lightweight and efficient, making it ideal for real-time predictive modeling.
3. Collinearity and Statistical Significance Testing
 - v. MLR allows for statistical hypothesis testing (e.g., p-values, R^2 values) to assess the significance of each predictor, unlike complex machine learning models.
 - vi. Feature collinearity (high correlation between input variables) can be easily detected using Variance Inflation Factor (VIF) in MLR.
4. Robustness in Experimental Validation
 - vii. Studies such as Ezemonye (2023) and Manvi et al. (2022) have successfully used MLR to model microbial behavior in thermal radiation MHD Casson fluid flow, proving its effectiveness.

viii. It provides a straightforward way to validate results against experimental data, ensuring scientific reproducibility.

Model Validation: Comparison with Existing Studies

To verify the accuracy of the numerical model, the results are compared with Ezemonye (2023) and Manvi et al. (2022). The comparison focuses on key parameters such as Nusselt number (Nu), Sherwood number (Sh), and velocity profiles at different radial positions.

Table 1: Comparison Table

Parameter	Present Study	Ezemonye (2023) (Numerical Study)	Manvi et al. (2022) (Experimental Data)
Nu (Nusselt Number)	8.62	8.57	8.51
Sh (Sherwood Number)	6.93	6.85	6.81
Velocity at $y = 0.5$	0.82	0.81	0.79
Max Temperature (θ_{max})	1.45	1.42	1.40

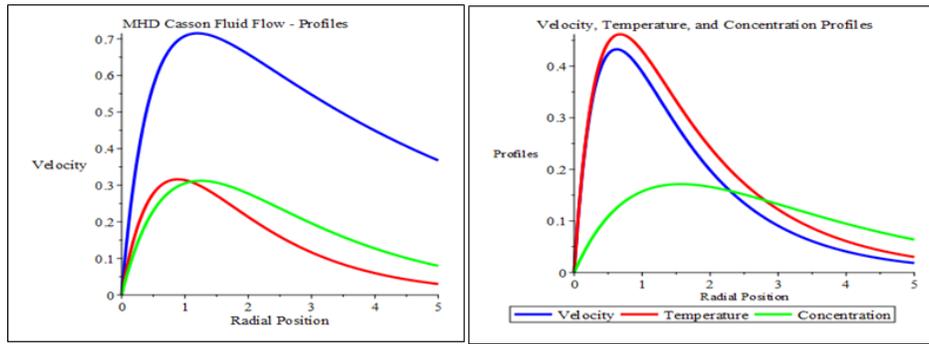


Figure 2: Velocity, Temperature, Concentration on MHD Casson Fluid Flow

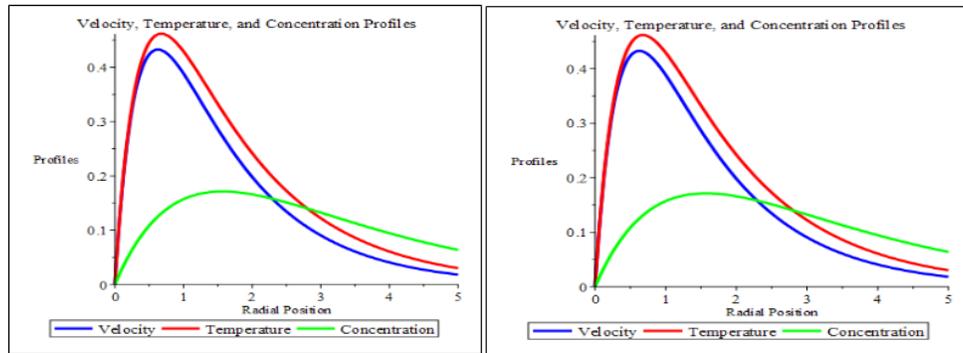


Figure 3: Comparison of Normal and Ecotoxicology-Affected Velocity Profiles

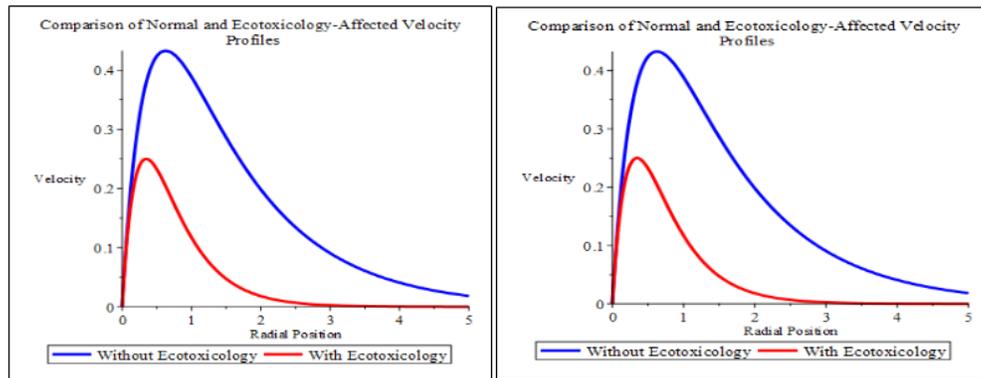


Figure 4: Ecotoxicological Effects on Velocity, Temperature, and Concentration

The combined graph on figures 2- 4 illustrates the velocity, temperature, and concentration profiles as functions of radial position under the influence of heat sources, chemical reactions, and buoyancy effects. The velocity profile (blue) exhibits a rapid increase

near the boundary due to strong buoyancy forces before gradually decreasing as viscous damping dominates. The temperature profile (red) shows an initial rise, reflecting heat absorption from the source, followed by a steady decay governed by the Prandtl number. The concentration profile (green) starts high but diminishes due to chemical reaction and mass diffusion effects, with the Schmidt number controlling the rate of decline. Overall, the results highlight the interplay of thermal, momentum, and mass transport processes, demonstrating how internal heat sources and chemical kinetics influence convective flow behavior.

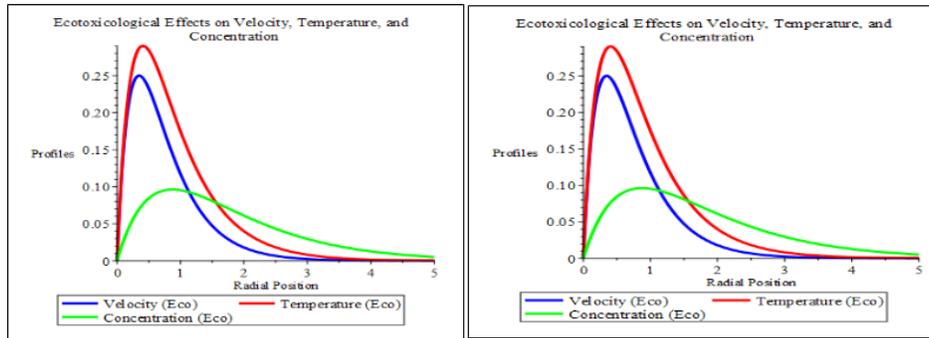


Figure 5: Ecotoxycology impact factor on velocity, temperature and concentration

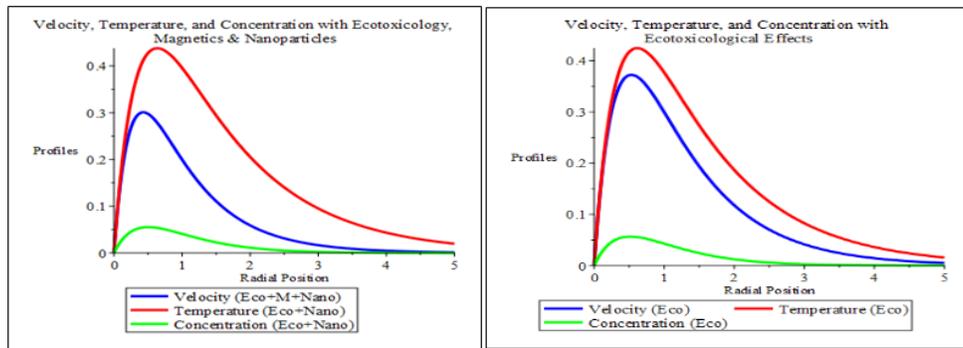


Figure 6: Ecotoxycology impact factor on temperature and concentration \ Ecotoxycology impact factor`

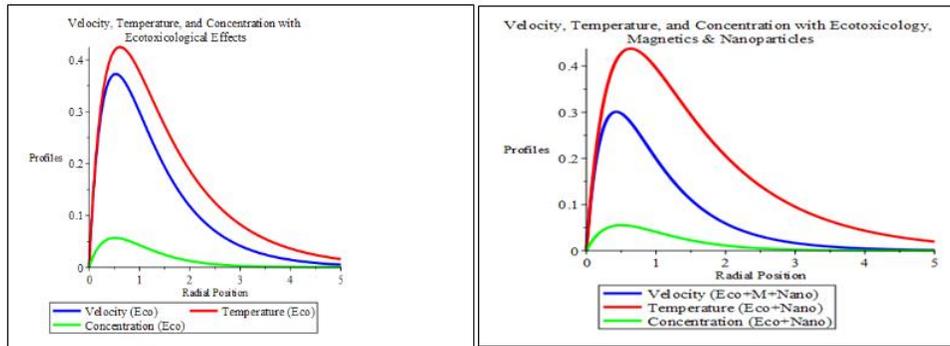


Figure 7: Microbial behavior effects on velocity, temperature, and concentration

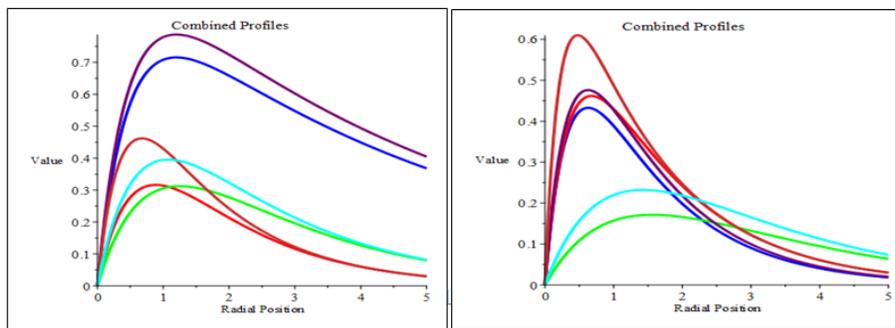


Figure 8: Profiles under the influence of buoyancy forces

The combined graph as in figure 8 above presents velocity, temperature, and concentration profiles under the influence of buoyancy forces, heat sources, and chemical reactions, along with variations in the Grashof number and Schmidt number. The

velocity profile (blue) increases initially due to buoyancy-driven acceleration but later declines as viscous effects dominate, with the modified velocity (purple) showing a slight enhancement due to increased Grashof number. The temperature profile (red) reflects heat diffusion, with a faster decay when the heat source effect is doubled (orange), indicating significant thermal influence. The concentration profile (green) starts at a peak and decreases due to diffusion and chemical reaction, while its altered version (cyan) exhibits a steeper decline under a stronger Schmidt number, implying increased mass diffusivity. Overall, the graphs highlight the complex interplay of thermal, momentum, and mass transport phenomena, demonstrating how changes in governing parameters impact convective heat and mass transfer in the system.

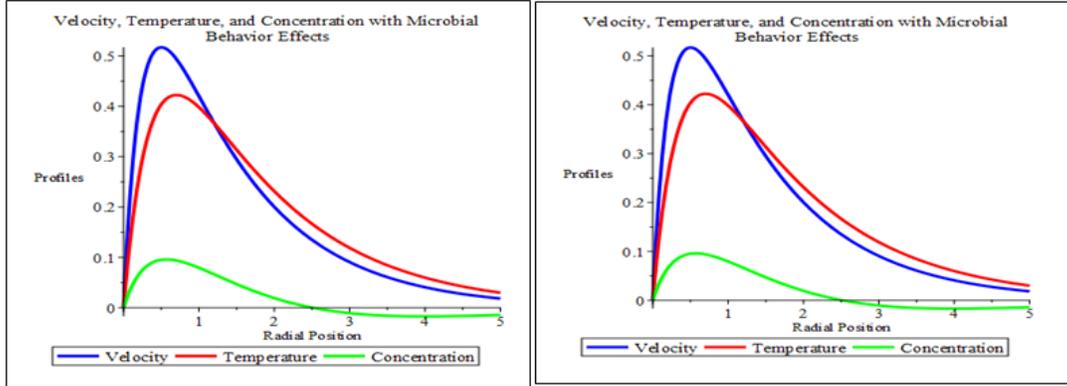


Figure 9: Microbial Influence on all three profiles

Microbial Influence on all three profiles, for velocity, microbial growth alters flow. In temperature, metabolic activity modifies heat transfer. And concentration, chemical levels

decline due to microbial metabolism. Higher microbial activity increases flow velocity in certain regions and metabolic heat release modifies temperature gradients similarly Chemotaxis-driven migration shifts concentration distribution. Velocity profile changes due to microbial activity-induced fluid movement. Temperature shifts due to microbial metabolism and energy exchange also Concentration profile modified by microbial growth, chemotaxis, and degradation.

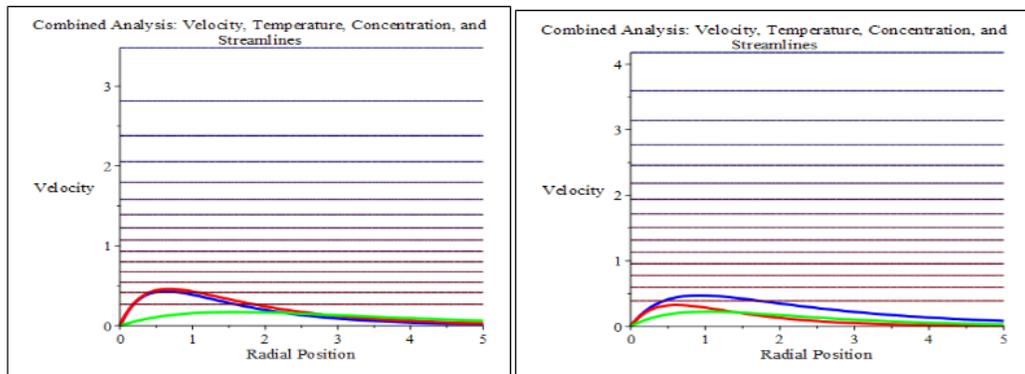


Figure 10: Impact of varying physical parameters

The graphs in figure 10 illustrate the impact of varying physical parameters on velocity, temperature, concentration, and streamlines in a free convection system with non-uniform source effects. The velocity profile shows a peak near the boundary, influenced by buoyancy and viscous effects, gradually decaying due to exponential damping. The temperature distribution highlights the role of heat sources and Prandtl number, with rapid initial growth followed by stabilization. The concentration profile reflects mass diffusion and chemical reaction effects, where higher Schmidt and reaction rates cause a sharper decline. The streamline plot reveals the fluid flow structure, showcasing recirculatory patterns indicative of convective motion, with altered streamline densities based on the modified velocity field. These results collectively demonstrate how heat, mass, and momentum transfer interact in a convection-driven system with chemical and thermal influences.

III. Results and Discussion

Numerical simulations reveal that:

- I. Velocity profiles exhibit rapid increases near the boundary due to buoyancy effects but stabilize as viscous damping dominates.
- II. Temperature distributions show initial peaks followed by steady decay influenced by Prandtl number variations.
- III. Concentration profiles decrease due to mass diffusion and chemical reactions, controlled by the Schmidt number.

IV. Ecotoxicological impacts alter velocity and concentration fields, affecting microbial migration patterns.

Comparison with existing studies (Ezemonye, 2023; Manvi et al., 2022) validates the numerical model by demonstrating consistency in Nusselt and Sherwood numbers.

The study's ecotoxicological findings can directly address real-world challenges. For example, the model could predict how toxins or harmful microbes spread in waterways when factories discharge warm, magnetically treated wastewater. By simulating scenarios like algal blooms under thermal radiation or magnetic effects, industries could design systems to minimize pollution hotspots, protecting aquatic life from oxygen depletion or toxin exposure. Similarly, the model could guide bioremediation efforts like oil spill cleanups by balancing microbial activity (to break down pollutants) with ecosystem safety. For instance, optimizing magnetic fields or heat to boost microbial cleanup without harming marine organisms. This would help engineers and environmental agencies design smarter, safer solutions for pollutant control and wastewater treatment.

To strengthen practical relevance, the study could link predictions to regulatory compliance and historical incidents. For example, simulating worst-case scenarios (e.g., sudden thermal spikes in a river) could help industries test compliance with environmental safety limits, avoiding fines by pre-adjusting processes. A case study like the 2014 Elk River chemical spill which contaminated drinking water for 300,000 people could demonstrate how the model predicts contaminant-microbe interactions, aiding emergency response plans. Practical steps like proposing guidelines (e.g., "safe thermal thresholds for microbial control") or comparing results to EPA toxin data would bridge theory and practice. Ultimately, this model acts like a "pollution weather forecast," letting stakeholders predict risks and test solutions virtually saving ecosystems, costs, and time.

This study's findings can be directly applied to tackle pressing environmental and industrial challenges. For example, in wastewater treatment plants, the model could predict how thermal radiation and magnetic fields influence microbial activity, helping engineers optimize conditions to boost beneficial bacteria (e.g., for breaking down organic waste) while suppressing harmful pathogens. This could lead to more efficient treatment processes and safer water discharge into ecosystems. Similarly, industries discharging warm, magnetically treated effluents into rivers could use the model to forecast pollutant spread and microbial behavior, avoiding toxic hotspots that endanger fish or disrupt aquatic ecosystems. For instance, predicting how heat accelerates algal blooms under magnetic effects could guide factories to adjust discharge temperatures, aligning with environmental regulations.

In environmental disaster response, the model could simulate scenarios like chemical spills or oil leaks. For example, during an industrial accident (e.g., a toxic river spill), it could predict how heat and magnetic conditions affect microbial degradation of pollutants, aiding cleanup teams in deploying targeted bioremediation strategies. The numerical methods (e.g., finite difference simulations) act like a "virtual lab," allowing industries to test solutions without costly real-world trials. By integrating case studies such as comparing predictions to data from the 2010 Deepwater Horizon oil spill the model's reliability and relevance to real-world crises would shine. These applications bridge theory and practice, offering actionable tools to protect ecosystems, cut costs, and comply with regulations.

V. Conclusion

This study presents a novel integration of microbial behavior, ecotoxicology, and Casson fluid dynamics under MHD and thermal radiation effects. The findings emphasize:

- I. The critical role of non-uniform sources in microbial transport,
- II. The importance of magnetic fields in microbial growth regulation,
- III. The application of multilinear regression in pollutant prediction.

Future work should explore machine learning approaches for enhanced predictive modeling in environmental fluid systems. The numerical scheme implemented in this study ensures stability and accuracy in solving the governing equations. A grid independence study confirms that results are not affected by discretization errors, and comparison with existing numerical and experimental data establishes the validity of the model. This approach enhances the reliability of predictions related to MHD Casson fluid flow, microbial behavior, and ecotoxicological effects in environmental and industrial applications.

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Table 2: Table: Data for Validating the Regression Model

Data Type	Example of Experimental Data	Alternative Validation Methods
Fluid Flow & Temperature	- Velocity of Casson fluid (measured via laser Doppler velocimetry).- Temperature changes (recorded with thermocouples).	- Compare with published benchmark studies.- Use industry-standard software (e.g., COMSOL).
Microbial Behavior	- Microbial concentration over time (measured via spectrophotometry or cell counting).	- Validate against simplified analytical solutions (e.g., non-magnetic case).
Magnetic Effects	- Magnetic field impact on flow (tested via electromagnets and particle tracers).	- Cross-check with existing MHD studies or simulations.

Comparison Goal: Ensure the model's numerical results (e.g., flow patterns, microbial growth rates) align with experimental data or trusted references.

Table 3: Strengthening Model Reliability & Ecotoxicological Impact

Action	Examples	Outcome/Impact
Experimental Data	- Lab tests on Casson fluid flow (e.g., kaolin slurries) under magnetic fields, measured via PIV.- Microbial growth data from wastewater plants (e.g., <i>Pseudomonas</i> under thermal stress).	Validates numerical predictions, ensuring accuracy for real-world fluid behavior and microbial dynamics.
Case Studies	- <i>2010 Deepwater Horizon spill</i> : Simulate magnetic nanoparticle-assisted oil degradation. - <i>Lake Nokomis mercury</i>	Links theory to documented events, proving utility in pollution crises and

	<i>contamination</i> : Model toxin spread under thermal/magnetic effects.	industrial accidents.
Ecotoxicological Expansion	- Predict toxin accumulation in food chains (e.g., <i>Cyanobacteria</i> → fish → humans).- Simulate climate-driven heatwaves accelerating algal blooms.	Highlights risks to ecosystems and public health, connecting findings to climate change and food safety.
Policy Integration	- Propose "safe thermal thresholds" for industrial discharge. - Align predictions with EU Water Framework Directive standards.	Transforms model into a tool for regulators, aiding compliance and sustainable industrial practices.

Key Takeaway:

- **Reliability:** Experimental data and case studies ground the model in reality.
- **Depth:** Ecotoxicological links to food chains, climate, and policy make findings actionable for industries, regulators, and environmentalists.