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Sway Mechanism in Frame Structures; Using Concept of Quantum Displacement

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Abstract: Sway displacement is the characteristic virtual and lateral displacement of frame structures, as a result of their response to load applications and internal actions within the structural system, and the attainment of dynamic and static equilibrium expected for the system. (eg, $\Sigma F = 0$, and $\Sigma M = 0$). Similarly, non-sway frames are structures considered to have small or negligible inter-storey displacements in structural mechanics analysis, which allows for the computation of the effect of applied loads, such as, deformation, deflection, displacement and internal forces within the structure, that can also ensure the stable equilibrium condition in real life structural loadings. The paper is theoretical evaluation of cause of sway and lateral movement in structures, since such movement can affect the stability of the system. The quantum state will provide a probable assessment for the outcome of measurement on the structural system performance, such as the tolerable critical displacement and deformation limits. Quantum realm and scale, is the condition where the action or angular momentum is quantized, since the expected lateral movement and sway in structural frames are negligible and relatively very small in scalar magnitude. Displacement is a vector, whose length is the shortest distance from the initial to a final position of point P, and the displacement vector is usually expressed as the difference between the final and initial position vectors (V_{ijk} , U_{ijk}). Constraint to lateral movement can be provided using the shear wall, within the structural system to act as restraint and to reduce the sway movements to minimal, negligible and acceptable magnitude. In conclusion, action of forces on structures, must not cause significant motion (ie static equilibrium) nor excessive deformation, therefore critical evaluation of all loading conditions that could affect the performance must be ensured adequate, such as to minimized and curtailed lateral movement as a result of excessive deformation and reduction in structural ability and strength over time period.

Keywords: Sway Mechanism, Displacement, Structures, Rigid body, Static equilibrium, Deformation

I. Introduction

Rigid bodies are solid objects, which deformation is relatively and very small, and can be neglected, since the distance between any two given points (or, particles) on the rigid body, is assumed to remain constant over time, regardless of the effect of external forces (Eugster et al, 2020). A structure is considered a rigid body because it can only accommodate negligible and permissible deformation, and also must remain (at rest) in the position of load application (Lindstrom and Lindstrom 2021). This condition, according to Newton's motion laws, is possible when the sum of all forces acting on the system is equal to zero (ie, $\Sigma F = 0$, $\Sigma M = 0$), a condition known as static equilibrium state (Erikson and Nordmark, 2019)). Frame structures are structures having the combination of beams and columns, connected monolithically at their ends, to resist applied loads and forces (Memon, 2024). Frames are classified as, (i) A rigid frame, defined as the structures in which beams and columns are connected monolithically and act together to resist the applied structural loads. Frames (ie, in structural design), can provide more stability and also could resist the applied forces, shear forces, bending moments and torsion more effectively. (ii) This is when bracing are provided between the beams and columns (Frames), to increase their resistance against lateral forces and sideway movement, due to the applied structural loads. Braced frame system can provide more efficient resistance against lateral (or, seismic) and wind forces, and are usually more effective than the rigid frame system. The structural configuration of the frame systems, is defined by the geometrical and loading symmetry, and when either of this is lacking or inadequate, the system becomes unsymmetrical, which can subject the Frame into instability, which is not a stable equilibrium, and can also, lead to lateral or side-way movement and sway. Sway is an unstable condition and must be prevented, by restraining the degree of movement to attain stability (Agrawal and Badgire, 2017), since structures are only functional at stable equilibrium conditions during loading (ie minimal and negligible displacement). A non-sway frame, is a structure which for stability considerations, and with respect to applied forces and internal actions, can be assumed to have negligible lateral and inter-story displacement (Maurice et al, 1012). Displacement is a vector, whose length is the shortest distance from the initial to final position of a point P. It quantifies both the distance and direction of motion along a straight line, and defined as the difference between the final and initial position vectors, ie, $\Delta s = S_f - S_i$ (Lindstrom and Lindstrom, 2021). Principle of least displacement (ie, $\Delta s \approx 0$), or more precisely the principle of minimal displacement action, indicates that, displacement of a rigid body must be relatively minimal and negligible, for it to be assumed stationery and/or at rest position. It is variational principle, that when applied to the action of a physical system can be used to obtain the equations of motion (ie, $\Sigma F = ma = 0$, for rest conditions), of the system, using the virtual work method (Erikson and Nordmark, 2019). In relativity principle, a different action must be maximized or minimized, as functional to attain stable equilibrium. Functionals are often expressed as definite integrals involving functions and their derivatives. Thus, to minimized lateral displacement, the action caused by the applied forces on the structure, can be optimized, in order to determine, the extrema functions, that will make the functional attain a maximum or minimum value (eg, maximum force or minimum displacement expected and required for stability). Constraint

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systems, are parameter that can provide resistance to lateral movement of framed system (Mansouri and Maheri, 2019). In Hamiltonian mechanics, a Primary constraint is a relation between the coordinates and momenta (ie, m Δv), that holds, without employing the equation of motion. Similarly, Secondary constraint, is one that is not primary, and holds when the equation of motion are satisfied. Also, the Holonomic constraint depends only, on the coordinates X_i , and time t, and does not depend on the velocities or any higher derivatives with respect to time.

Ie, $f(X_1, X_2, X_3, \dots X_n, t) = 0$

Constraint that cannot be expressed in the form indicated, is non-holonomic constraint, whose state depends on the path taken in order to achieve the position or present stat (Erikson and Nordmark, 2019). In design of Structures, "Shear wall", is a structural system composed of braced panels (also known as shear panels), to counter the effects of "lateral forces and action" acting on the structure. Shear walls resist in-plane loads, that are applied along its height. A shear wall is stiffer in its principal (or cross sectional) axis than its longitudinal axis. It is considered as a primary structure, which provides relatively stiff resistance to vertical and horizontal forces acting in its plane. The design parameter for effective transmission of forces, within the shear-wall system includes, the geometric configuration (wall thickness, moment of inertia), structural loads, material properties (elastic and shear modulus), restraint capacity and construction methods.

II. Literature Review

2.1: Rigid body dynamics studies the movement of systems of interconnected bodies under the action of external forces, using the law of kinematics (ie, Newton's laws), and by application of the "second law of motion" or the "derivative form" in the Lagrangian mechanics, the solution of these equations can provide the description of position, motion, and acceleration of individual components of the system as a function of time. Thus, constraint systems are parameter that provides resistance to movement of rigid body system Newton's laws for a rigid system of N particles, ie, P_i, I = 1, 2,..N, is considered satisfied, because structures are expected to be in static equilibrium during load application, and no motion (or, displacement) is anticipated in that direction. Hence, the resultant force and torque at a reference point R is defined as,

$F = \Sigma\, m.a = 0$, and, $T = \Sigma\, (r-R)\, m.a = 0$,

where, r_i, denotes the planar trajectory of each particle, m_i, = mass of particles and a_i = acceleration.. Limit state principle ensures that structures satisfy two principal design criteria, which are Ultimate limit state (ULS) and Serviceability limit state (SLS). All design process involves a number of assumptions, and all engineering design criteria have a common goal, which is ensuring a safe structure. A structure is deemed to satisfy the ultimate limit state, if factored bending, shear and tensile or compressive stresses are below the factored resistances calculated for the section under consideration.. Also, a structure may become unfit for use, when it violates the serviceability requirements of deflection, vibrations, cracks due to fatigue, corrosion and fire. Therefore, in addition to the ULS check, a service limit state (SLS) computational check must be performed. The aim is to prove that under the action of characteristic design loads (un-factored) and/or whilst applying certain (un-factored) magnitudes of imposed deformations, settlements or vibration or temperature gradients etc, the structural behavior complies with and does not exceed the SLS design criteria values, specified on the relevant engineering standard.

2.2: Stability theory addresses the solutions of differential equations (eg, $a = \frac{\partial^2 x}{\partial^2 t} \approx 0$), and of the trajectories (ie, a = f(x, t) of dynamical systems under small perturbations of initial conditions (Chen W F, 2000). Many parts of the qualitative theory of dynamical systems deal with asymptotic properties of solutions of associated differential equations and the trajectories with respect to time period. The simplest kind of behavior is exhibited by equilibrium points or fixed points and by periodic orbits. In the study of iterated functions, a periodic point, is a point which the system returns to after certain number of function iterations or certain amount of time.

Given a mapping f from a set of X into itself

. $f: X \rightarrow X$,

. a point x in X is called period point, if there exists an n > 0 so that $f_n(x) = x$

Where f_n is the nth iterate of f.

The smallest positive integer n satisfying the above is known as the prime period or least period of point x

Stability theory addresses the following 2 questions

Will a nearby orbit indefinitely stay close to a given orbit?
Will it converge to the given orbit?

In the former case, the orbit is called stable, while in the latter case it is called asymptotically stable and the given orbit is said to be attracting

An equilibrium solution to the autonomo9us system is known to be Stable (Pukdeboon C, 2011),

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if for every (small) $\epsilon > 0$, there exists a $\delta > 0$ such that every solution f(t) having initial conditions within distance δ, ie, $|f(t) - f_e| < \delta$, if the equilibrium remains within distance $\epsilon|\theta$, $|f(t) - f_e| < \epsilon$ for all $t \geq t_0$

Asymptotically stable if it is stable and in addition there exists $\delta_o > 0$

such that whenever $|f(t_o) - f_e| < \delta_o$ then $f(t) \to f_e$ as $t \to \alpha$

Dynamics of Sway Displacement

Lateral displacement (or sway) involve movement of building structure laterally as a result of the applied loading, such as wind load, seismic load, cranes etc, and sometimes secondary movement from vertical loads and actions (or, nonsymmetrical loading). Extent of side-sway limitation can have significant effect on stability and equilibrium of structural frames (Ashraf et al (2004). A non-sway frame, is a structure which, from the point of design and stability condition, can be considered to have relatively negligible inter-storey displacements. Storey displacement is the total displacement of the ith storey, with respect to the ground, and it is described as the maximum permissible limit for lateral displacement in the IS codes/International Standards for building. Storey drift is defined as, the ratio of displacement of two consecutive floor to the height of that floor (Maurice et al (2012).

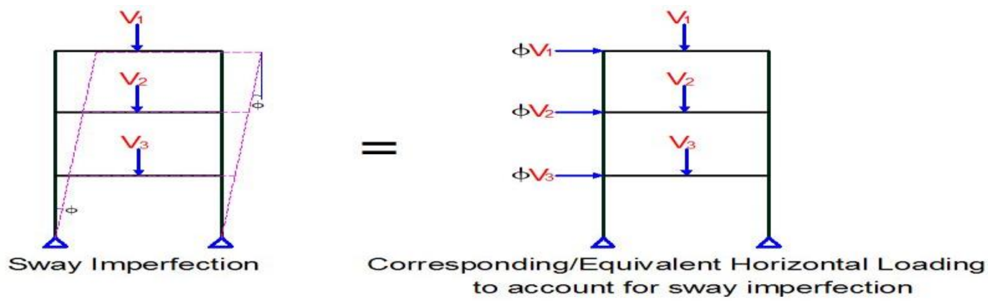

Fig. 1: Lateral Movement and Interstory Displacement

3.1*:* Structural dynamics, is the condition of motion, force and displacement, characterized with small lateral movement (Δ) and rotational displacement (θ) in frame structures (Erikson and Nordmark, 2019).

Displacement must be within permissible threshold for inter-storey drift and displacement, to ensure stability of the structure, and required static equilibrium condition. From the principle of conservation of energy for minimal or virtual displacement of frame structure,

ie, Sum of work done = 0 (for no displacement action)

Therefore, for the static equilibrium condition, $T\theta/2 = F\Delta/2$,

T is the torque or twisting moment that causes rotation about an axis, and which could account for the sway mechanism in frame structures, with an equivalent force F leading to a corresponding lateral displacement Δ.

Then, $F = T\theta/\Delta$, and Noting that $\tan\theta = \theta = \Delta/H$, for small angle displacement

hence, $F = T/H$,

Where, F= restoring force required for static equilibrium and, limiting or minima displacement, and H = storey height.

Drift in building frames, is a result of flexural and shear mode contributions, due to the column axial deformations, and to the diagonal and beam deformations respectively. In medium to high rise structures, the higher axial forces and deformations in the columns, and the accumulation of their effects over a greater height, cause the flexural component of displacement to become dominant, leading to sway.

Maximum storey drift corresponding to the design lateral force, including displacement due to vertical deformation of the isolated system shall not exceed the following limits.

The maximum storey drift of the structure above the isolation system calculated by response spectrum analysis shall not exceed 0.015h.

The maximum storey drift of the isolation system calculated by response history analysis based on the force-deflection characteristics of non-linear elements of the lateral force-resisting system shall not exceed 0.02h.

Where h = height of a shear wall, measured as the maximum clear height from top of foundation to bottom of diaphragm framing. Or, h = the maximum clear height from top of diaphragm to bottom of diaphragm framing.

Virtual displacement, is infinitesimal change of the system coordinates, occurring while time t is held constant. It is called ‘virtual’ rather than ‘real’, since no actual displacement can take place without the passage of time (Allen, 1991). Also, virtual displacement are spatial displacements, and exclusively, time is fixed while they occur, and when computing virtual differentials of quantities

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that are functions of space and time coordinates, no dependence of time is considered. Work-done by a force on a particle through virtual displacement is known as virtual work, since the displacement δs of a point, is any arbitrary infinitesimal change in the position of the point consistent with the constraints imposed on the motion path. The principle of virtual work can be used in the solution of equilibrium problems,

Assuming, **$W = F\,\delta s \approx 0$,** since, $\delta s \approx 0$

The principle states that, considering all necessary conditions for equilibrium of a particle, the virtual work done by all forces acting on the body during any virtual displacement is zero (Silva et al, 2020).

Concept of Quantum Displacement

The knowledge of quantum state, together with system performance under service load, can be used with the wave function to evaluate the system behavior, which is a mathematical description of the quantum state of an engineering system, that indicate the degrees of freedom corresponding to some maximal set of commutative observables. A quantum, is the minimum amount of any physical entity (or, property) involved in an action, such as where the action or momentum of a system is quantized, since the displacement is to be relatively small, and negligible.

Quantum mechanics (or matrix mechanics), is a fundamental theory that describes “nature” at the smallest level of atoms and subatomic particles. It describes the state of a system at a given time by a complex wave function known as “state vector” in a complex vector space. A vector space (or linear space), is a collection of objects called vectors, which may be added together and multiplied (scaled) by numbers called scalars. Euclidean vectors are example of vector space, which represent physical quantities such as forces, and similarly, in the geometric sense, vectors representing displacements in a plane or in three-dimensional space are “vector spaces”.

4.1: Minimum total potential energy principle, indicate that, a body (ie, structure), shall deform or displace to a position that locally minimizes the total “potential energy”, with the lost in potential energy being converted to kinetic energy for possible motion and displacement, etc. The Potential energy is associated with forces, that act on a body in a way that the total work done by these forces on the body depends only on displacement, that is, the initial and final position of the body in space. These forces known as, conservative forces, can be represented at every point in space by vectors expressed as gradients of a certain scalar function called potential.

Total Potential Energy (π), is the sum of the elastic strain energy, U, stored in the deformed body and the potential energy, V, associated to the applied forces.

Ie, $\pi = U + V$

This energy, is at a stationary position, when an infinitesimal variation from such position involves no change in energy,

and, $\Delta \pi = \delta U + \delta V = 0$

In Engineering, variations are small changes in mathematical functions and functionals, used to determine the extreme value (ie, maxima and minima) of functional, or mappings from a set of functions to the real numbers.

4.2: Conservation Law, suggest that the total energy of an isolated system remains constant and considered to be conserved assuming no additional acts on the system. The static equilibrium state is a conserved criterion for load application since it is expected that work done by the system is negligible to maintain the equilibrium or minimal displacement

Castigliano’s Theorem can be used to determining the displacements of a linear-elastic system based on the partial derivatives of the energy, since a change in energy is equal to the causing force times the resulting displacement. Therefore, the partial derivatives are necessary to relate causing forces and resulting displacements to the change in energy.

Castigliano’s First Theorem (for, Forces acting on an elastic Structure), state that, If the strain energy of an elastic structure can be expressed as a function of generalized displacement (Z_i), then the partial derivative of the strain energy with respect to generalized force (P_i) is defined,

Ie, $\mathbf{P_i = \delta U / \delta Z_i}$

Castigliano’s Second Theorem (for displacements of an elastic structure), state that, if the strain energy of a linearly elastic structure can be expressed as a function of generalized force P_i, then the partial derivative of the strain energy with respect to generalized displacement Z_i, in the direction of P_i, is defined thus,

Ie**,** $\mathbf{Z_i = \delta U / \delta P_i}$

III. Conclusion

Structural actions, such as lateral displacements, are attributes of the dynamics of a physical systems, from which the equation of motion of the system can be determined (eg, F = ma). Also, a particle is in static equilibrium if the net force on that particle is zero, and also when the momentum at specific position, is constant in all direction.

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The action principle can be extended to obtain the equation of motion for fields, such as the trajectory of sway displacement, and using the principle of stationary or least action, which indicates that, a different action, must be minimized or maximized. Thus, Constraints are parameters necessary to restraint the lateral movement of frame structures, hence made possible the opportunities of reducing lateral movement by optimizing the constraint and shear wall resistance.

Sway is a lateral displacement, and the action can be minimized by limiting the degree of movement, to prevent such displacement, that could cause dynamic instability of the rigid body. A degree of freedom of a "physical system", is an independent parameter that is necessary to characterized the position and the state of physical systems.The basic assumption in structural analysis, is that, Structure must remain "firm and fit" in the position of load application, which indicates that every particles and components constituting the rigid body system must remain in the permissible static equilibrium, during the serviceability period. Otherwise, the system will be subjected to disequilibrium, instability, disintegration, and with high probability of failure and structural Collapse.

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