Mathematical Modelling of The Dynamics of Poverty, Crime and Imprisonment

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Abstract: This study explores the evolution and application of mathematical modelling to complex social issues such as poverty, crime, and terrorism. Traditionally rooted in epidemiology, compartmental models have been successfully adopted in criminology and public health to capture the dynamics of addiction, ideological radicalization, and recidivism. The model was derived from a five–compartment representation from which a set of five ordinary differential equations (ODEs) was developed to capture the dynamism of poverty, crime, and imprisonment in a deterministic SCJ_1J_2R compartmental model using the next generation matrix to obtain the reproduction number R_0. This is used to analyze the local stability of the crime-free equilibrium of the SCJ_1J_2R model. The crime-free equilibrium and the local stability of the endemic equilibrium show that R_0 is asymptotically stable if $R_0 < 1$. We use hypothetical data to simulate the sensitivity of the parameters of the basic reproduction number (R_0) so as to obtain $R_0 < 1$ for a crime-free society. The SCJ_1J_2R compartmental model differentiates incarceration based on criminal evidence and accounts for both natural and crime–induced mortality as well as reintegration processes. This review highlights the growing role of mathematical approaches in policy-relevant analysis of community safety, systemic intervention, and stability of the models.

Keywords: Reproduction Number (R_0), Crime, Poverty, Incarceration, ODEs, SCJ_1J_2R Model,

I. Introduction

Mathematical modelling of the dynamics of poverty, crime, and imprisonment involves creating systems of equations using ordinary differential equations (ODEs) to represent these variables and parameters' interaction and change over time in the enclosed system.

There is a strong relationship between poverty and crime. Some of the reasons of addiction are directly or indirectly related to poverty such as: ignorance, unhealthy social environment, inability to deal with life and stress, unavailability of social and psychological help, family and social damages, family troubles and strained relationships, inability to complete education, etc. (Sakib et al., 2017). Crime is one among the most challenging problems in most developing countries in which poverty, unemployment, etc. is among the causes. This paper is intended to contribute to the eradication of poverty-related crimes in the developing countries by proposing a deterministic mathematical model. The model can help to analyze the impact of various intervention programs such as poverty alleviation programs, law enforcement, and rehabilitation programs (Mataru et al., 2023).

Crime has become an epidemic disease in our society. As such, epidemiologists and scholars are finding possible solutions to curb the crime menace through the formulation of mathematical modelling. Modelling in criminology and public health has evolved from the classical epidemic framework into sophisticated multi-layered representation of criminal behaviour, terrorism, and drug-related activities in our communities (McMillon, 2014; Njagarah, 2013). White and Comiskey (2017) used ordinary differential equations (ODEs) to embed treatment and relapse, foregrounding the cyclical patterns that the model would generate.

Mushayabasa et al. (2014) extended this paradigm by explicitly incorporating awareness and rehabilitation control, highlighting how prevention and treatment can reshape the qualitative behaviour of the enclosed system. Parallel development in the mathematical modelling of terrorism and radicalization recognized that ideology can diffuse through populations with epidemic–like mechanisms. Considering a unified framework that simultaneously captures drug crime and terrorist activity remains limited. The research of Malonza and Bonyo (2022) advanced the field with a dual-crime model for illicit trade and armed conflict in East Africa, which did not fully differentiate incarceration pathways or reintegration processes. Responding to this gap, this paper proposes an SCJ_1J_2R compartmental model, explicitly separating incarceration with or without crime involvement (J_1 and J_2), integrating both natural and crime–induced mortality, and allowing recovery and reintegration dynamics that can feed back into the susceptible class (S) in the enclosed system.

Recent mathematical modelling research has increasingly integrated structural socio-economic determinants. Khan et al. (2023) constructed a deterministic framework for urban drug trafficking that couples law enforcement with socio-economic stressors, while Aluko and Olayemi (2022) examined organized crime with recidivism and rehabilitation, emphasizing the "revolving door'' between incarceration and criminal relapse.

Etim et al. (2024) underscored the effect of poverty and unemployment in operation as upstream drivers of crime, motivating prevention strategies which act on macro-structural levers rather than purely punitive controls. A complementary methodological strand stems from the study of criminal hotspots using mathematical models grounded in reaction–diffusion PDEs and self-exciting processes. The research of Short et al. (2008, 2010) demonstrated how local crime attractiveness, repeat victimization, and offender movement can produce spatio-temporal clustering (hotspots) and how interventions may dissipate these criminal clusters.

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While these models are not compartmental in the traditional epidemiological sense, they underscore the value of mechanistic formulations that connect micro-level behaviour to macro-level emergent patterns. Hence, the SCJ_1J_2R model developed in this study contributes to this trend by offering a unified, policy-relevant structure that can quantify how the differentiated incarceration paths, mortality channels, and the recovery mechanisms interact to bring the community to safety and stability.

II. Formulation of the SCJ_1J_2R Model

The model is an SCJ_1J_2R model which analyses the dynamics of poverty, crime and imprisonment in the enclosed system. The population is divided into five compartment; the susceptible class(S), the criminal class(C), the jailed class due to crime(J_1), the jailed class without committing crime (J_2) and rehabilitation class(R). The total population at time (t) is given by

$T(t) = S(t) + C(t) + J_1(t) + J_2(t) + R(t)$ - (1)

The system of ODEs governing the model is

$$\frac{dS}{dt} = \pi + \emptyset R - (\beta C + \sigma + \mu)S \quad 2(a)$$

$$\frac{dC}{dt} = \beta SC - (\delta_3 + \mu + \epsilon + \emptyset)C \quad 2(b)$$

$$\frac{dJ_1}{dt} = \varphi C - (\mu + \delta_1)J_1 \quad 2(c) \qquad (2)$$

$$\frac{dJ_2}{dt} = \propto S - (\mu + \delta_2)J_2 \quad 2(d)$$

$$\frac{dR}{dt} = \delta_1 J_1 + \delta_2 J_2 + \delta_3 C - (\mu + \emptyset)R \quad 2(e)$$

In the following tables, the variable and parameters of the SCJ_1J_2R model are interpreted as this:

Table 1: The Model Variables

Variable	Descriptions
S(t)	Susceptible class
C(t)	Criminal class
$J_1(t)$	Jailed class due to crime
$J_2(t)$	Jailed class without committing crime
R(t)	Rehabilitation class

Table 2: The Model Parameters

Parameters	Descriptions
π	Recruitment rate
$\emptyset$	Rate of movement from rehabilitation class to susceptible class
β	Rate of movement from susceptible class to criminal class
σ	Rate of movement from susceptible class to jailed class without committing crime
μ	Natural death rate
ϵ	Induce death rate due to crime
δ_1	Rate of movement from jail due to crime to rehabilitation class
δ_2	Rate of movement from jail without committing crime to rehabilitation class
δ_3	Rate of movement from criminal class to rehabilitation class
φ	Rate of movement from criminal class to jailed class

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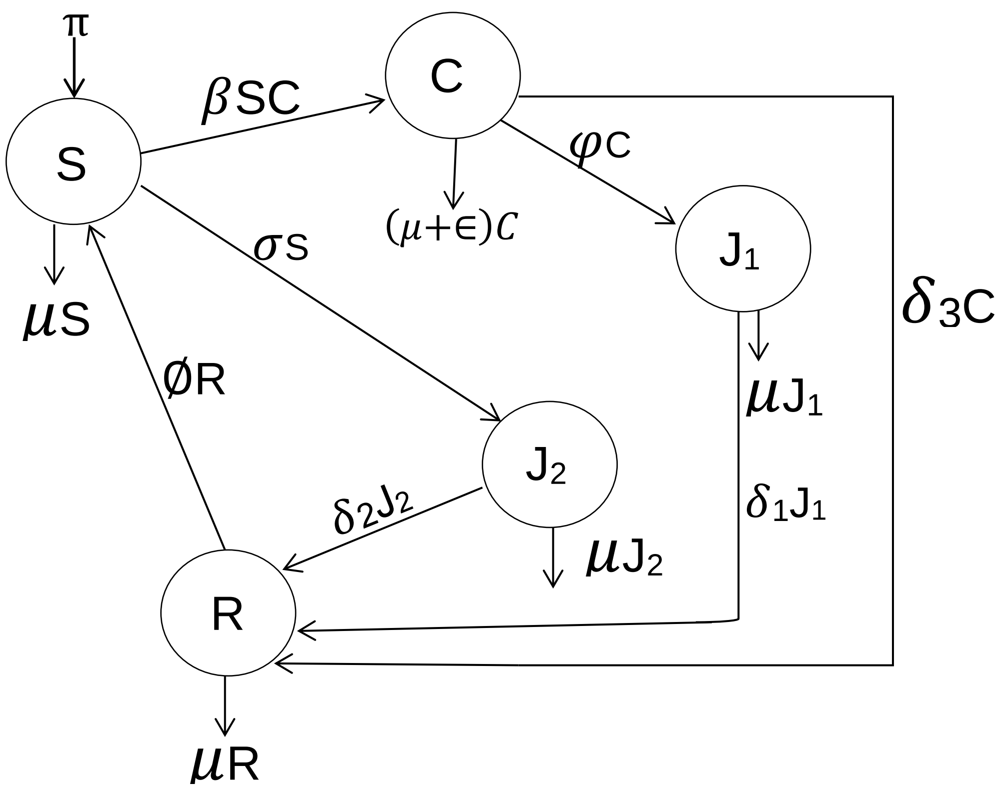
Figure 1: The flow chart of the SCJ_1J_2R model



III. The Model Properties

In this section, we shall discuss the existence and uniqueness of the model solution.

Existence and Uniqueness of Solution

If the system of equations in (2) has solution and it is unique, then we apply the Lipchitz condition to verify the existence and uniqueness solution of the system.

Theorem 3.1.1

Let D' denote the region R such that

$R = \{(x, y): |x - x_0| \leq a; |y - y_0| \leq b, (a, b > 0)\}$

Where x = x_1, x_2, x_3……..x_n, and y = y_1, y_2, y_3……..y_n,

Suppose f(t, x) satisfied the Lipchitz condition, then

$||f(t_1, x_1) - f(t_1, x_2)|| \leq k||x_1 - x_2||$

The pairs (t_1, x_1) and (t_1, x_2) belong to D^I, where k is a positive constant. There is a constant $\delta > 0$ such that there exists a unique solution which satisfied the requirement that $\frac{\partial fi}{\partial tj}, i, j = 1,2,3, \dots \dots n$ be continuous and bounded in D^I.

Hence by applying the Lipschitz condition, the system in (2) satisfies the conditions of Theorem 3.1.1, ensuring the existence and uniqueness of solutions.

IV. Mathematical Analysis of the SCJ_1J_2R model

In this section, we shall study and analyse the stability status of the equilibrium solutions of the system of equations in (2).

Crime-Free Equilibrium (CFE)

Here, we are interested in a population dynamics of the system where there is crime-free equilibrium (CFE). We assume the absence of crime in the society, hence C=0, which implies J_1 = 0.

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Setting all derivatives in (2) to zero:

$$\frac{dS}{dt} = \frac{dC}{dt} = \frac{dJ_1}{dt} = \frac{dJ_2}{dt} = \frac{dR}{dt} = 0 \quad (3)$$

Solving equations (3) and letting ϵ_0 denotes the crime-free equilibrium. Hence

$$\epsilon_0 = \left(\frac{\pi(\mu+\delta_2)(\mu+\emptyset)}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}, 0, 0, \frac{\sigma\pi(\mu+\emptyset)}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}, \frac{\delta_2\sigma\pi}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}\right) \quad (4)$$

$$Where\ S = \frac{\pi(\mu+\delta_2)(\mu+\emptyset)}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}; J_2 = \frac{\sigma\pi(\mu+\emptyset)}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}; R = \frac{\delta_2\sigma\pi}{(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}$$

Basic Reproduction Number of SCJ_1J_2R Model

Using the next generation matrix approach, Diekmann and Heesterbeck (2001),

$$R_0 = \rho\left(FV^{-1}\right) \quad (5)$$

After computation,

$$R_0 = \frac{\beta S}{\left(\delta_3 + \mu + \varepsilon + \phi\right)} \quad (6)$$

At CFE:

$$R_0 = \frac{\beta\pi\left(\mu+\delta_2\right)\left(\mu+\phi\right)}{\left(\delta_3+\mu+\varepsilon+\varphi\right)\left(\mu+\phi\right)\left(\sigma+\mu\right)\left(\mu+\delta_2\right)-\phi\delta_2} \quad (7)$$

Local Stability of Crime-Free Equilibrium of the Model

Theorem 3.1.3: The crime-free equilibrium is locally asymptotically stable if $R_0<1$ and asymptotically unstable if $R_0>1$.

Hence;

$$\frac{\beta\pi(\mu+\delta_2)(\mu+\emptyset)}{(\delta_3+\mu+\in+\varphi)(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2} < 1 \quad (8)$$

$R_0 < 1$

$\lambda_3 = -(\mu + \delta_1) < 0$

$\lambda_4 = -(\mu + \delta_2) < 0$

$\lambda_5 = -(\mu + \emptyset) < 0$

Therefore for

$P(\lambda) < 0$

Then, $R_0 = \frac{\beta\pi(\mu+\delta_2)(\mu+\emptyset)}{(\delta_3+\mu+\in+\varphi)(\mu+\emptyset)(\sigma+\mu)(\mu+\delta_2)-\emptyset\delta_2}$ (9)

By implication

$\lambda_3 = \frac{1}{R_0} < 0, R_0 = \frac{1}{\lambda_3} < 1$

Numerical Simulation

Numerical evaluation of the basic reproduction number (R_0) using the parameter data supplied in the generated data indicates strong parameter sensitivity. For the parameter combinations in Table 4, the computed R_0 values lie between 0.07 and 0.59, and all ten combinations satisfy $R_0 < 1$. Consequently, under the Table 4 parameter regime the Crime-Free Equilibrium is locally asymptotically stable and criminality is predicted to die out. By contrast, parameter combinations in Table 5 produce $R_0 > 1$ (range 1.08–6.52), predicting persistence of criminal behaviour.

Table 3 contains a mix of outcomes (one of ten parameter sets yields $R_0 < 1$; this highlights that targeted changes in parameters such as the recruitment rate into the criminal class (β) or increased rehabilitation (φ terms) can move the system from an endemic to a crime-free state.

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Table 3

Entry	π	μ	σ	δ_1	β	ϵ	φ	δ_2	∅	ρ	δ_3	R_0	$1/R_0$ <1
1	55	0.12	0.2	0.22	0.14	0.04	0.04	0.06	0.22	0.27	0.025	0.9	1.1111
2	55	0.12	0.25	0.22	0.2	0.04	0.06	0.08	0.26	0.29	0.03	1.38	0.7246
3	55	0.12	0.3	0.22	0.26	0.04	0.08	0.1	0.3	0.31	0.035	1.86	0.5376
4	55	0.12	0.35	0.22	0.32	0.04	0.1	0.12	0.34	0.33	0.04	2.34	0.4274
5	55	0.12	0.4	0.22	0.38	0.04	0.12	0.14	0.38	0.35	0.045	2.81	0.3559
6	55	0.12	0.45	0.22	0.44	0.04	0.14	0.16	0.42	0.37	0.05	3.27	0.3058
7	55	0.12	0.5	0.22	0.5	0.04	0.16	0.18	0.46	0.39	0.055	3.7	0.2703
8	55	0.12	0.55	0.22	0.56	0.04	0.18	0.2	0.5	0.41	0.06	4.12	0.2427
9	55	0.12	0.6	0.22	0.62	0.04	0.2	0.22	0.54	0.43	0.065	4.53	0.2208
10	55	0.12	0.65	0.22	0.68	0.04	0.22	0.24	0.58	0.45	0.07	4.91	0.2037

Table 4

Entry	π	μ	ϵ	δ_1	β	∅	φ	δ_2	δ	ρ	δ_3	R_0	$1/R_0$ <1
1	55	0.11	0.011	0.212	0.19	0.056	0.07	0.1	0.31	0.31	0.04	0.59	1.6949
2	55	0.11	0.0101	0.224	0.26	0.062	0.1	0.14	0.42	0.4	0.05	0.49	2.0408
3	55	0.11	0.0092	0.236	0.33	0.068	0.13	0.18	0.53	0.49	0.06	0.4	2.5
4	55	0.11	0.0083	0.248	0.4	0.074	0.16	0.22	0.64	0.58	0.07	0.33	3.0303
5	55	0.11	0.0074	0.26	0.47	0.08	0.19	0.26	0.75	0.67	0.08	0.27	3.7037
6	55	0.11	0.0065	0.272	0.54	0.086	0.22	0.3	0.86	0.76	0.09	0.22	4.5455
7	55	0.11	0.0056	0.284	0.61	0.092	0.25	0.34	0.97	0.85	0.1	0.17	5.8824
8	55	0.11	0.0047	0.296	0.68	0.098	0.28	0.38	1.08	0.94	0.11	0.14	7.1429
9	55	0.11	0.0038	0.308	0.75	0.104	0.31	0.42	1.19	1.03	0.12	0.1	10.0
10	55	0.11	0.0029	0.32	0.82	0.11	0.34	0.46	1.3	1.12	0.13	0.07	14.2857

Table 5

Entry	π	μ	ϵ	δ_1	β	∅	φ	δ_2	δ	P	δ_3	R_0
1	110	0.55	0.12	0.09	0.15	0.24	0.025	0.1	0.48	0.4	0.045	1.08
2	110	0.5	0.15	0.11	0.21	0.24	0.035	0.14	0.48	0.58	0.065	1.47
3	110	0.45	0.18	0.13	0.27	0.24	0.045	0.18	0.48	0.76	0.085	1.89
4	110	0.4	0.21	0.15	0.33	0.24	0.055	0.22	0.48	0.94	0.105	2.36

5	110	0.35	0.24	0.17	0.39	0.24	0.065	0.26	0.48	1.12	0.125	2.87
6	110	0.3	0.27	0.19	0.45	0.24	0.075	0.3	0.48	1.3	0.145	3.45
7	110	0.25	0.3	0.21	0.51	0.24	0.085	0.34	0.48	1.48	0.165	4.08
8	110	0.2	0.33	0.23	0.57	0.24	0.095	0.38	0.48	1.66	0.185	4.8
9	110	0.15	0.36	0.25	0.63	0.24	0.105	0.42	0.48	1.84	0.205	5.6
10	110	0.1	0.39	0.27	0.69	0.24	0.115	0.46	0.48	2.02	0.225	6.52

Empirical Analysis of the Basic Reproduction Number (R_0)

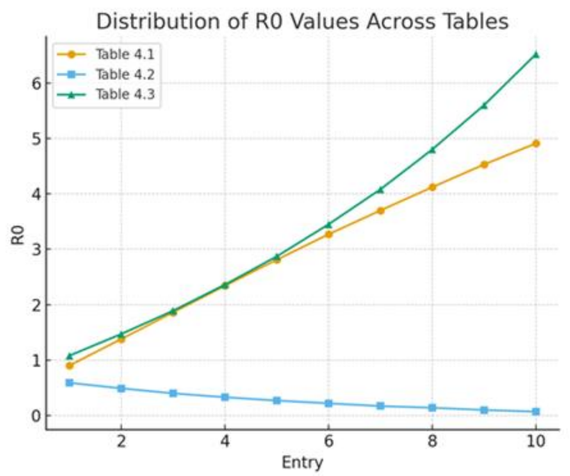

Figure 2: Distribution of R_0 values across parameter tables

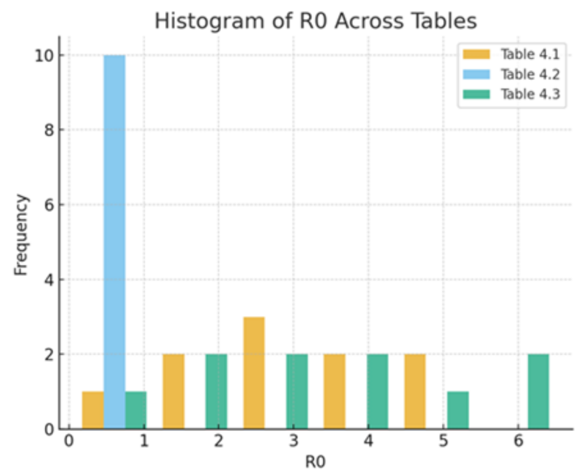

Figure 3: Histogram of R_0 across Tables 3–5

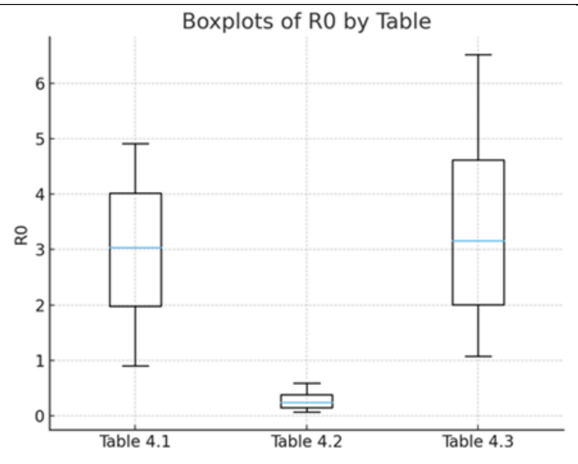

Figure 4: Boxplots of R_0 by Table 3, 4 & 5

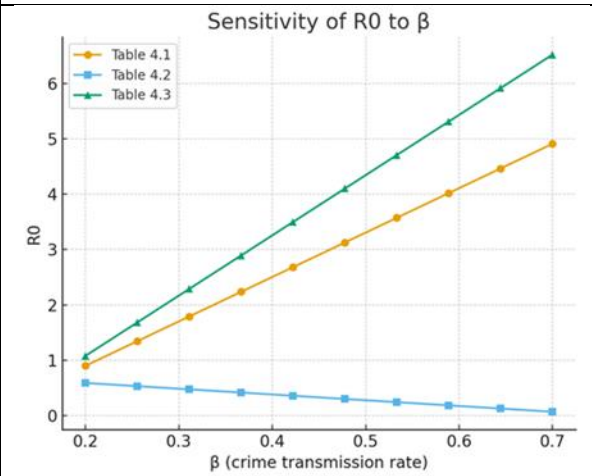

Figure 5: Sensitivity of R_0 to β

Empirical plots of R_0 across parameters. Fig 2- Distribution of R_0 values across parameter tables, Fig 3- Histogram of R_0 across Tables 3–5, Fig 4- Boxplots of R_0 by Table, Fig 5- Sensitivity of R_0 to β.

V. Results and Discussion

The four empirical plots collectively highlight the threshold-driven behaviour of the basic reproduction number (R_0):

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1. Separation across Parameter Regimes: Both the distribution and histogram demonstrate a clear distinction between parameter tables. Table 4 values cluster below unity, indicating crime-free stability, while Tables 3 and 5 lie above unity, predicting persistence of criminal activity.

2. Variability of Outcomes: The boxplots reveal that Table 4 values are tightly bounded and always sub-threshold ($R_0 \prec 1$), while Tables 3 and 5 show wide variability with medians above one, underscoring their unstable regimes.

3. Sensitivity to Crime Transmission (β): The sensitivity plot shows divergent effects of β:Table 3 shows a moderate upward trend with β; Table 4 shows a decreasing trend with β, reflecting stabilizing dynamics; Table 5 shows sharp escalation with β, underscoring its destabilizing effect.

4. Threshold Property of R_0: Table 4 always produces $R_0 \prec 1$, predicting crime eradication. Table 5 consistently produces $R_0 \succ 1$, predicting persistence. Table 3 straddles the threshold, reflecting sensitivity to parameter tuning.

VI. Conclusion

The combined plots reinforce the threshold nature of R0: sub-threshold values ($R_0 \prec 1$) predict eradication of crime, while super-threshold values ($R_0 \succ 1$) predict persistence. Strategic interventions, particularly reducing recruitment (β) and strengthening rehabilitation (φ, δ2), can transition the system from endemic to crime-free states.

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