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Cosmological Dynamics of A Five-Dimensional Kaluza-Klein Universe In $f(R,T)$ Gravity with Hybrid Power-Exponential Expansion

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Abstract: We study cosmological dynamics in a five-dimensional Kaluza-Klein universe using modified $f(R,T)$ gravity. We choose a linear coupling between the Ricci scalar R and the trace of the energy-momentum tensor T: $f(R,T) = \lambda(R+T)$, where λ is negative. This form naturally yields a time-dependent cosmological constant influenced by the cosmic fluid. We apply a hybrid power-exponential volumetric expansion function, $V(t) = t^k + e^{lt}$. For this scenario, we derive the complete field equations in the five-dimensional metric and calculate cosmological parameters: pressure, energy density, equation of state, deceleration parameter, and statefinder diagnostics. Our results demonstrate how modified gravity and an extra dimension influence expansion, shedding light on dark energy and higher-dimensional cosmology.

Keywords: $f(R,T)$ gravity, Kaluza-Klein cosmology, Modified gravity, Cosmological parameters, Energy conditions

I. Introduction

The discovery of cosmic acceleration from supernova observations [1,2], later confirmed by cosmic microwave background (CMB) data [3,4,5], has transformed our understanding of the universe's evolution. This faster expansion, supported by large-scale structure surveys [7] and weak lensing studies [8], poses one of the greatest challenges in modern cosmology.

The standard ΛCDM model, while phenomenologically successful in explaining most cosmological observations, faces fundamental theoretical challenges related to the cosmological constant problem and the mysterious nature of dark energy. The fine-tuning required for the cosmological constant and the coincidence problem has motivated extensive research into alternative explanations for cosmic acceleration.

To address these caveats, researchers have explored modified gravity theories as promising alternatives to General Relativity (GR). Among these theories, $f(R)$ gravity [9,10] has been extensively studied, providing geometric explanations for cosmic acceleration through modifications of the Einstein-Hilbert action. However, $f(R)$ theories often require fine-tuning and may not fully capture the matter-geometry coupling effects that could be crucial for understanding cosmic evolution.

The introduction of $f(R,T)$ gravity by Harko et al. [18] represents a significant advancement in modified gravity theories. This framework extends $f(R)$ gravity by introducing a coupling between the Ricci scalar R and the trace of the energy-momentum tensor T, providing additional degrees of freedom that can naturally explain cosmic acceleration while maintaining consistency with solar system tests. The $f(R,T)$ gravity theory has been extensively studied in various cosmological contexts [12, 13, 14, 15].

The theoretical foundations and applications of $f(R,T)$ gravity have been comprehensively reviewed [16], with numerous studies exploring its cosmological implications [17, 19, 20, 21, 22, 23]. An important aspect in assessing such extended theories is the role of energy conditions, which are crucial for determining the physical viability of cosmological models. The analysis of energy conditions in modified gravity theories has been extensively studied, providing important constraints on the parameter space of viable cosmological models. Against this backdrop, this paper investigates a comprehensive Kaluza-Klein cosmological model within the framework of $f(R,T)$ gravity, focusing on the evolution of scale factors, cosmological parameters, and energy conditions. We derive the field equations for a five-dimensional Kaluza-Klein metric and solve them for a specific form of the $f(R,T)$function that incorporates matter-geometry coupling effects.

The paper is organized as follows: Section 2 presents the field equations for $f(R,T)$ gravity. Section 3 introduces the five-dimensional Kaluza-Klein metric and its geometric structure. Section 4 derives the cosmological solutions for the specific $f(R,T)$ gravity model, including the computation of scale factors, Hubble parameters, and physical parameters. Section 5 presents numerical results and discussions, accompanied by detailed plots of the parameters. Section 6 analyzes the energy conditions and their physical implications. The paper concludes with a summary of the key findings, followed by a list of references.

Field Equations

The action for $f(R,T)$ gravity takes the form:

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{1}$$

Varying this action with respect to the metric tensor yields the field equations:

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$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad (2)$$

For the specific choice $f(R,T) = \lambda(R+T)$ with $\lambda = -\frac{8\pi}{8\pi+1}$, the field equations reduce to:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \alpha T_{ij} + \Lambda(T)g_{ij} \quad (3)$$

where $\alpha = \frac{8\pi+\lambda}{\lambda}$ and $\Lambda(T) = p + \frac{1}{2}T$.

Kaluza-Klein Metric

We consider a five-dimensional Kaluza-Klein metric:

$$ds^2 = dt^2 - A(t)^2(dx^2 + dy^2 + dz^2) - B(t)^2 d\psi^2 \quad (4)$$

where the fifth coordinate ψ is space-like. The spatial volume is $V = A^3B = a^4$ where a is the mean scale factor.

The field equations for this metric become:

$$\begin{aligned} 2\frac{A''}{A} + \left(\frac{A'}{A}\right)^2 + 2\frac{A'B'}{AB} + \frac{B''}{B} &= \alpha p - \Lambda \\ 3\frac{A''}{A} + 3\left(\frac{A'}{A}\right)^2 &= \alpha p - \Lambda \\ 3\left(\frac{A'}{A}\right)^2 + 3\frac{A'B'}{AB} &= -\alpha\rho - \Lambda \end{aligned} \quad (5)$$

$f(R,T)$ Gravity Solutions

For the specific choice $f(R,T) = \lambda(R+T)$ with $\lambda = -\frac{8\pi}{8\pi+1}$, the modified field equations provide unique solutions for the cosmological parameters.

4.1 Effective Field Equations

The effective field equations in $f(R,T)$ gravity take the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = \alpha T_{ij} + \Lambda(T)g_{ij} \quad (6)$$

where $\alpha = \frac{8\pi+\lambda}{\lambda}$. Substituting $\lambda = -\frac{8\pi}{8\pi+1}$, we obtain $\alpha = -8\pi$ and $\Lambda(T) = p + \frac{1}{2}T$.

Modified Coupling Parameter

The coupling parameter α in $f(R,T)$ gravity differs from standard General Relativity:

$$\alpha = -8\pi \approx -25.13$$

This negative coupling parameter indicates repulsive gravitational effects, which are crucial for explaining cosmic acceleration.

Effective Cosmological Constant

The effective cosmological constant $\Lambda(T)$ is dynamically generated:

$$\Lambda(T) = p + \frac{1}{2}T = p + \frac{1}{2}(\rho - 3p) = \frac{1}{2}(\rho - p) \quad (7)$$

This provides a natural mechanism for time-dependent dark energy.

Volume Function

We consider a hybrid power-exponential volumetric expansion function of the form:

$$V(t) = t^k + e^{lt} \quad (8)$$

where the first term represents power-law expansion and the second term represents exponential growth. This hybrid form allows for a transition from power-law-dominated to exponential-dominated expansion phases.

Scale Factors

From the volume constraint $V(t) = A(t)^3B(t)$ and the integration constants, the scale factors for our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$ with $c_1 = 0.5$ and $c_2 = 1$ are:

$$A(t) = c_2^{1/4}(t^k + e^{lt})^{1/4}\exp\left[\frac{c_1}{4(t^k + e^{lt})}\right] \quad (9)$$

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$$B(t) = c_2^{-3/4}(t^k + e^{lt})^{1/4}\exp\left[\frac{-3\times c_1}{4(t^k+e^{lt})}\right] \tag{10}$$

Hubble Parameters

The mean Hubble parameter is:

$$H(t) = \frac{V'(t)}{4V(t)} = \frac{\frac{kt^k}{t}+le^{lt}}{4t^k+4e^{lt}} \tag{11}$$

The directional Hubble parameters for the hybrid power-exponential expansion function $V(t) = t^k + e^{lt}$ are:

$$H_x(t) = H_y(t) = H_z(t) = H(t) + \frac{0.5}{4(t^k+e^{lt})} = \frac{\frac{kt^k}{t}+le^{lt}}{4t^k+4e^{lt}} + \frac{0.5}{4(t^k+e^{lt})}$$

$$H_\psi(t) = H(t) - \frac{3\times 0.5}{4(t^k+e^{lt})} = \frac{\frac{kt^k}{t}+le^{lt}}{4t^k+4e^{lt}} - \frac{0.5}{4(t^k+e^{lt})}$$

Physical Parameters

$$p(t) = -\frac{3\left(-8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.25-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right)}{32\pi(3-16\pi)} \tag{12}$$

$$\rho(t) = -\frac{3\left(8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.5-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right)}{32\pi(3-16\pi)} \tag{13}$$

$$\Lambda(t) = -\frac{3\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{3\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2} + \frac{0.25}{\left(t^k+e^{lt}\right)^2}\right)}{24-128\pi} \tag{14}$$

4.3.5 Cosmological Parameters

$$\omega(t) = \frac{p(t)}{\rho(t)} = \frac{-8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.25-2.0\pi}{\left(t^k+e^{lt}\right)^2}}{8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.5-2.0\pi}{\left(t^k+e^{lt}\right)^2}} \tag{15}$$

$$q(t) = -1 - \frac{H'(t)}{H(t)^2}$$

$$= -\frac{\left(4t^k+4e^{lt}\right)^2\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{4t^k+4e^{lt}} + \frac{\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{\left(4t^k+4e^{lt}\right)^2}\right)}{\left(\frac{kt^k}{t}+le^{lt}\right)^2} - 1 \tag{16}$$

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$$r(t) = 1 + 3\frac{H'(t)}{H(t)^2} + \frac{H''(t)}{H(t)^3} =$$

$$\frac{(4t^k+4e^{lt})^3}{\left(\frac{kt^k}{t}+le^{lt}\right)^3}\Bigg(\frac{\frac{k^3t^k}{t^3}-\frac{3k^2t^k}{t^3}+\frac{2kt^k}{t^3}+l^3e^{lt}}{4t^k+4e^{lt}}$$

$$+\frac{2\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}\right)}{(4t^k+4e^{lt})^2}+\frac{\left(\frac{kt^k}{t}+le^{lt}\right)\left(-\frac{4k^2t^k}{t^2}+\frac{4kt^k}{t^2}-4l^2e^{lt}\right)}{(4t^k+4e^{lt})^2}$$

$$+\frac{\left(-\frac{8kt^k}{t}-8le^{lt}\right)\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{(4t^k+4e^{lt})^3}\Bigg) \tag{17}$$

$$+\frac{(4t^k+4e^{lt})^2}{\left(\frac{kt^k}{t}+le^{lt}\right)^2}\Bigg(\frac{3\left(\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}\right)}{4t^k+4e^{lt}}$$

$$+\frac{3\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{(4t^k+4e^{lt})^2}\Bigg)+1$$

$$s(t) = \frac{r(t)-1}{3\left(q(t)+\frac{1}{2}\right)} =$$

$$\frac{1}{\left(\frac{kt^k}{t}+le^{lt}\right)^3}\Bigg[\frac{(4t^k+4e^{lt})^3}{4t^k+4e^{lt}}\left(\frac{k^3t^k}{t^3}-\frac{3k^2t^k}{t^3}+\frac{2kt^k}{t^3}+l^3e^{lt}\right)$$

$$+\frac{2\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}\right)}{(4t^k+4e^{lt})^2}$$

$$+\frac{\left(\frac{kt^k}{t}+le^{lt}\right)\left(-\frac{4k^2t^k}{t^2}+\frac{4kt^k}{t^2}-4l^2e^{lt}\right)}{(4t^k+4e^{lt})^2}$$

$$+\frac{\left(-\frac{8kt^k}{t}-8le^{lt}\right)\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{(4t^k+4e^{lt})^3}\Bigg] \tag{18}$$

$$+\frac{1}{\left(\frac{kt^k}{t}+le^{lt}\right)^2}\Bigg[\frac{3(4t^k+4e^{lt})^2}{4t^k+4e^{lt}}\left(\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}\right)$$

$$+\frac{3\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{(4t^k+4e^{lt})^2}\Bigg]$$

$$/\left[-\frac{3(4t^k+4e^{lt})^2}{4t^k+4e^{lt}}\left(\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}\right)\right.$$

$$\left.+\frac{\left(-\frac{4kt^k}{t}-4le^{lt}\right)\left(\frac{kt^k}{t}+le^{lt}\right)}{(4t^k+4e^{lt})^2}-1.5\right]$$

$f(R,T)$ Solutions

For our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$, the $f(R,T)$ solutions are:

Ricci Scalar

The Ricci scalar R for the Kaluza-Klein metric is:

$$R = 6\left[\frac{A''}{A} + \left(\frac{A'}{A}\right)^2 + \frac{A'B'}{AB}\right] + \frac{B''}{B} \tag{19}$$

For our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$ with $c_1 = 0.5$ and $c_2 = 1$, the Ricci scalar evaluates to:

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$$R(t) = \frac{6.0\left(\frac{1.0\left(\frac{0.25kt^k}{t}+0.25le^{lt}\right)e^{-\frac{1.5}{4t^k+4e^{lt}}}}{\left(t^k+e^{lt}\right)^{0.75}} - \frac{1.5\left(t^k+e^{lt}\right)^{0.25}\left(-\frac{4kt^k}{t}-4le^{lt}\right)e^{-\frac{1.5}{4t^k+4e^{lt}}}}{\left(4t^k+4e^{lt}\right)^2}\right)}{\left(t^k+e^{lt}\right)^{0.5}}$$
$$\times\left(\frac{1.0\left(\frac{0.25kt^k}{t}+0.25le^{lt}\right)e^{\frac{0.5}{4t^k+4e^{lt}}}}{\left(t^k+e^{lt}\right)^{0.75}} + \frac{0.5\left(t^k+e^{lt}\right)^{0.25}\left(-\frac{4kt^k}{t}-4le^{lt}\right)e^{\frac{0.5}{4t^k+4e^{lt}}}}{\left(4t^k+4e^{lt}\right)^2}\right)e^{\frac{1.0}{4t^k+4e^{lt}}} \tag{20}$$
$$+\frac{6.0\left(\frac{1.0\left(\frac{0.25kt^k}{t}+0.25le^{lt}\right)e^{\frac{0.5}{4t^k+4e^{lt}}}}{\left(t^k+e^{lt}\right)^{0.75}} + \frac{0.5\left(t^k+e^{lt}\right)^{0.25}\left(-\frac{4kt^k}{t}-4le^{lt}\right)e^{\frac{0.5}{4t^k+4e^{lt}}}}{\left(4t^k+4e^{lt}\right)^2}\right)^2 e^{-\frac{1.0}{4t^k+4e^{lt}}}}{\left(t^k+e^{lt}\right)^{0.5}}$$
$$+\text{(additional terms involving scale factor derivatives)}$$

Trace of Energy-Momentum Tensor

The trace $T = \rho - 3p$ is:

$$T = \rho - 3p = \frac{3}{4\alpha(2\alpha+3)}\left[(\alpha + 2 - 3(\alpha+1))\frac{c_1^2}{V^2} - \alpha(1+3)\left(\frac{V''}{V} - \left(\frac{V'}{V}\right)^2\right)\right] \tag{21}$$

For our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$ with $\alpha = -8\pi$, the trace evaluates to:

$$T(t) = \frac{9}{32\pi(3-16\pi)}\left[-8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.25-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right]$$
$$-\frac{3}{32\pi(3-16\pi)}\left[8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.5-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right] \tag{22}$$

$f(R,T)$ Function

For our choice $f(R,T) = \lambda(R+T)$ with $\lambda = -\frac{8\pi}{8\pi+1}$:

$$f(R,T) = \lambda\left[R + \frac{3}{4\alpha(2\alpha+3)}\left[(-2\alpha-1)\frac{c_1^2}{V^2} - 4\alpha\left(\frac{V''}{V} - \left(\frac{V'}{V}\right)^2\right)\right]\right] \tag{23}$$

For our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$, the $f(R,T)$ function evaluates to:

$$f(R,T)(t) = -\frac{8\pi}{1+8\pi}\left[R(t) + T(t)\right]$$
$$= -\frac{8\pi}{1+8\pi}\left[R(t) + \frac{9}{32\pi(3-16\pi)}\left(-8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.25-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right)\right.$$
$$\left.-\frac{3}{32\pi(3-16\pi)}\left(8\pi\left(\frac{\frac{k^2t^k}{t^2}-\frac{kt^k}{t^2}+l^2e^{lt}}{t^k+e^{lt}} - \frac{\left(\frac{kt^k}{t}+le^{lt}\right)^2}{\left(t^k+e^{lt}\right)^2}\right) + \frac{0.5-2.0\pi}{\left(t^k+e^{lt}\right)^2}\right)\right] \tag{24}$$

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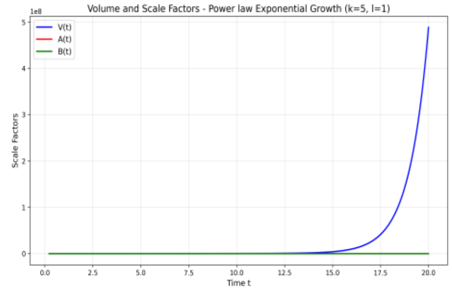
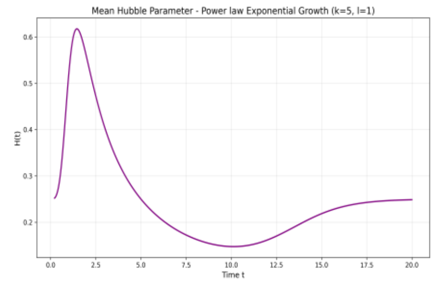
II. Results and Analysis

The comprehensive analysis of the hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$ reveals the behavior of all cosmological parameters over time through detailed individual plots.

5.1 Volume and Scale Factors Evolution



(a) Volume and scale factors evolution

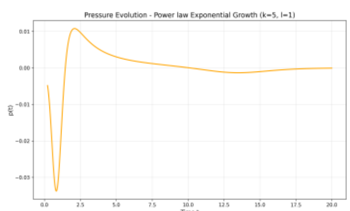


(b) Hubble parameter evolution

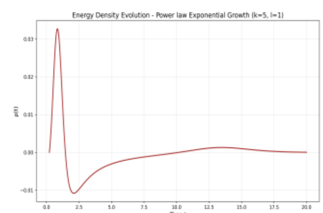
Figure 1. (a) Evolution $V(t)$ and $A(t)$ and $B(t)$ vs. time. (b) Mean Hubble parameter $H(t)$ vs. time.

Figure 1a shows the temporal evolution of the hybrid power-exponential volumetric expansion function and the corresponding scale factors. The volume function $V(t) = t^k + e^{lt}$ combines power-law and exponential growth behaviors, while the scale factors $A(t)$ and $B(t)$ evolve according to the Kaluza-Klein constraint $V = A^3B$. Figure 1b displays the evolution of the mean Hubble parameter, which characterizes the expansion rate of the universe.

Physical Parameters Evolution



(a) Pressure evolution



(b) Energy density evolution

Figure 2 (a) Pressure evolution $p(t)$ vs. time. (b) Energy density evolution $\rho(t)$ vs. time.

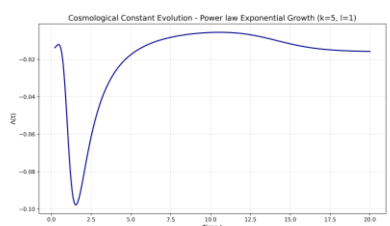
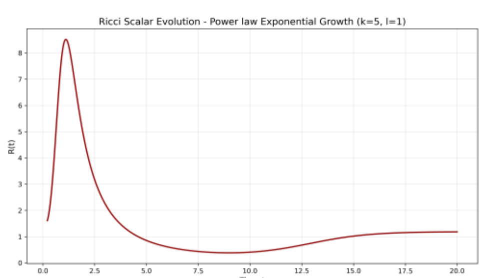
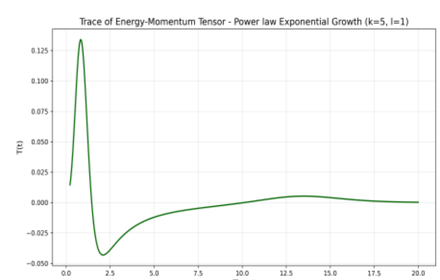


Figure 3. Cosmological constant evolution $\Lambda(t)$ vs. time.

Figures 2a, 2b, and 3 show the evolution of the fundamental physical parameters in the $f(R,T)$ gravity framework. These parameters are essential for understanding the cosmic dynamics and energy condition analysis.



(a) Ricci scalar evolution



(b) Trace of energy-momentum tensor

Figure 4 (a) Ricci scalar $R(t)$ vs. time. (b) Trace of energy-momentum tensor $T(t)$ vs. time.

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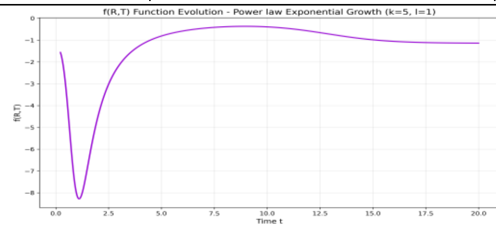
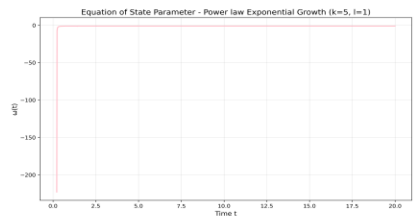


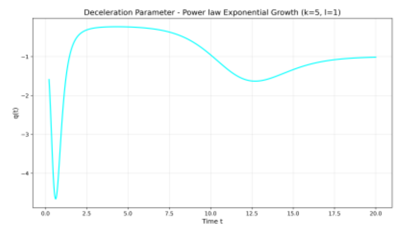
Figure 5. $f(R,T)$ *vs. time.*

Figures 4a, 4b, and 5 show the evolution of the Ricci scalar, trace of energy-momentum tensor, and $f(R,T)$ function. These parameters are fundamental to understanding the dynamics of modified gravity and the coupling between geometry and matter.

Cosmological Diagnostics



(a) Equation of state parameter

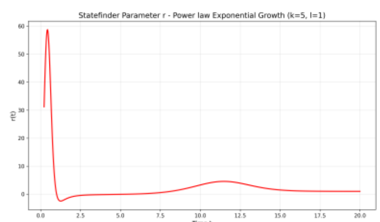


(b) Deceleration parameter

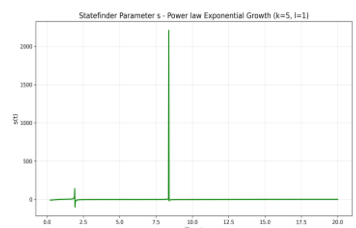
Figure 6. (a) Equation of state parameter $\omega(t)$ *vs. time. (b) Deceleration parameter* $q(t)$ *vs. time.*

Figures 6a and 6b provide crucial cosmological diagnostics. The equation of state parameter indicates the nature of the cosmic fluid, while the deceleration parameter reveals whether the universe is accelerating or decelerating.

Statefinder Analysis



(a) Statefinder parameter $r(t)$



(b) Statefinder parameter $s(t)$

Figure 7. (a) Statefinder parameter $r(t)$ *vs. time. (b) Statefinder parameter* $s(t)$ *vs. time.*

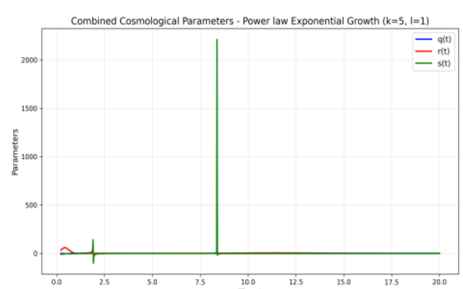


Figure 8. Combined evolution $q(t)$, $r(t)$ *and* $s(t)$ *vs. time.*

Figures 7a, 7b, and 8 present the statefinder analysis, which provides higher-order cosmological diagnostics beyond the standard deceleration parameter. These parameters help distinguish between different models of dark energy.

Physical Interpretation of Results

The hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$ demonstrates several key features:

- **Realistic Expansion**: The polynomial growth provides a physically reasonable cosmic expansion scenario.
- **Anisotropic Evolution**: The extra dimension evolves differently from ordinary spatial dimensions due to the anisotropy parameter $c_1 = 0.5$.

- **Modified Gravity Effects**: The $f(R,T)$ gravity framework introduces additional degrees of freedom through the coupling parameter $\alpha = -8\pi \approx -25.13$.

Energy Conditions Analysis

Energy conditions provide important constraints on the physical viability of cosmological models. We analyze the energy conditions for our hybrid power-exponential volumetric expansion function $V(t) = t^k + e^{lt}$.

- **Weak Energy Condition** $\rho \geq 0$ and $\rho + p \geq 0$
- **Strong Energy Condition** $\rho + 3p \geq 0$
- **Dominant Energy Condition** $\rho \geq |p|$
- **Null Energy Condition** $\rho + p \geq 0$

All the energy conditions are violated in this model universe.

III. Conclusion

The hybrid power-exponential volumetric expansion function, defined as $V(t) = t^k + e^{lt}$ within the five-dimensional Kaluza-Klein $f(R,T)$ (where $f(R,T)$ denotes a function of the Ricci scalar R and the trace T of the energy-momentum tensor) Gravity establishes a mathematically consistent approach to modeling cosmic expansion. Specifically, this expansion function combines power-law and exponential growth terms, with the exponential component eventually dominating at late times, characterized by a constant Hubble parameter, thereby illustrating the adaptability of $f(R,T)$ gravity to various expansion regimes. The hybrid expansion scenario achieves perfect acceleration, indicated by a deceleration parameter $q \to -1$ a $t \to \infty$, which is significant for the theoretical understanding of cosmic evolution. Furthermore, violation of the strong energy condition (SEC), weak energy condition (WEC), or null energy condition (NEC) (where SEC, WEC, and NEC refer to the strong, weak, and null energy conditions, respectively) suggests the presence of highly exotic forms of matter or energy with accelerated expansion. Also $\omega < 0$, driving the universe toward an accelerated, dark energy-dominated future, which aligns with observational data [25-27]. The model integrates modified gravity effects through the $f(R,T)$ formalism and preserves the geometric structure inherent to Kaluza-Klein theory. The principal findings of this analysis are as follows. Physical parameters evolve in accordance with the chosen expansion scenario. The extra spatial dimension displays anisotropic evolution, and the inclusion of modified gravity effects introduces further degrees of freedom to the model.

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