

Magneto-Radiative Interactions on Boundary Layer Development: Heat Transfer and Skin Friction Analysis

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Abstract: This study presents a comprehensive numerical investigation into the boundary layer behavior of an electrically conducting fluid influenced by simultaneous magnetic and radiative effects over an exponentially stretching surface. The analysis focuses on the impacts of magnetic field intensity, thermal radiation, and Prandtl number on the velocity, temperature distribution, skin friction coefficient, and heat transfer rate. A system of coupled nonlinear differential equations governing the momentum and energy transport is formulated and solved using MATLAB's `bvp4c` solver. Results reveal that increasing magnetic field strength significantly reduces fluid velocity due to the Lorentz force, while enhancing the temperature profile through magnetic damping. Similarly, stronger thermal radiation increases fluid temperature and thickens the thermal boundary layer, though it reduces the Nusselt number by diminishing surface temperature gradients. In contrast, higher Prandtl numbers lead to lower temperatures and enhanced heat transfer without substantially affecting skin friction. These findings highlight the critical interplay of magnetic and radiative phenomena in modifying boundary layer characteristics, offering practical insights for the design and optimization of advanced thermal systems in materials processing, energy harvesting, and magnetic insulation technologies.

Keywords: Heat transfer, MHD, Boundary layer flow, Skin friction, exponentially stretching sheet.

I. Introduction

The study of boundary layer flows induced by exponentially stretching surfaces has gained substantial attention due to its applications in polymer processing, thermal extrusion, and magnetic insulation technologies. The interplay between magnetic fields and thermal radiation is particularly crucial in these processes as it strongly affects both the heat transfer rate and skin friction characteristics of the fluid [1].

The pioneering work of Sanjayanand and Khan [2] laid foundational insights into viscoelastic boundary layer flows influenced by exponential stretching and thermal diffusion. Subsequently, researchers such as Bidin and Nazar [3] emphasized the importance of thermal radiation in altering the thermal boundary layer thickness in similar configurations. Ishak [4] extended this framework by incorporating magnetohydrodynamic (MHD) effects alongside radiation, revealing their combined significance in energy transport. Likewise, Khan [5] provided analytical insight into viscoelastic fluid motion under exponential stretching, marking the beginning of many refined models.

Mukhopadhyay [6,7] investigated additional complexities including thermal stratification, velocity slip, suction/injection, and radiation effects on MHD flows, noting a significant influence on the thermal and momentum boundary layers. Eid [8] further extended these findings by analyzing chemical reaction effects within a two-phase nanofluid system subjected to exponential stretching and internal heat generation. Hayat et al. [9] applied similar principles to Oldroyd-B fluids and demonstrated the prominent role of stretching strength in controlling thermal distribution. Baag et al. [10] explored uniform heat sources and porous media influence in MHD flow, which was later expanded by Govindaraj et al.

[11] through the inclusion of variable viscosity and Prandtl number considerations. Rajput et al. [12] presented a comprehensive model of radiative MHD flow in porous domains, with particular focus on surface heating. Mushtaq et al. [13] studied variable fluid properties and their effects on flow and thermal fields, while Chaudhary et al. [14] highlighted the non-negligible impact of thermal radiation on boundary layer profiles. Goud et al. [15] reinforced these observations through numerical simulation of radiation-augmented MHD flow over exponential surfaces.

Konwar et al. [16] addressed mixed convection and porous media effects, incorporating temperature-dependent properties to reveal a more realistic thermal boundary layer evolution. Reddappa et al. [17] analyzed Jeffrey fluid dynamics and stratification, suggesting its importance in thermal regulation near exponentially stretching sheets. Raghunatha et al. [18] considered Casson nanofluid models and confirmed that MHD and buoyancy effects can be highly interactive in mixed convection settings.

Sharma [19,20] explored Darcy–Forchheimer resistance effects on nanofluid flows, underscoring the significant changes in heat and momentum transport. Tantry et al. [21] specifically examined the influence of Casson rheology and radiation, revealing its suppressive effect on the temperature gradient near the sheet. The work by Naveed Khan et al. [22] focused on chemically reactive nanofluid behavior at stagnation points, confirming the altered thermal boundary structure due to reaction kinetics. Aloliga et al.

[23] also addressed non-Newtonian effects, identifying reductions in wall shear stress and enhanced heat transfer under strong magnetic fields. Arshad et al. [24] compared various nanostructured materials and their heat transfer capabilities, suggesting that nanoparticle choice can significantly modify thermal boundary layer performance. Additionally, Sulemana et al. [25] examined chemically reactive flows with magnetic influence in unsteady porous media, providing further evidence of the complex interactions governing thermal and skin friction characteristics. Recent studies by Zuberi et al. [26-28] have explored the thermofluidic behavior

of blood infused with various nanoparticles, including gold, silver, copper, alumina, and carbon nanotubes, highlighting their impact on heat transfer, flow dynamics, and biomedical applications under complex physiological and electromagnetic conditions. Zainal et al. [29] investigated mixed convection flow of a couple-stress hybrid nanofluid past a vertical shrinking plate, revealing enhanced thermal transport characteristics.

Given these extensive developments, the present study sets out to perform a comprehensive investigation of magneto-radiative interactions in boundary layer flow induced by an exponentially stretching surface, a configuration that continues to be of paramount relevance in cutting-edge thermal engineering processes such as polymer extrusion, thermal coating, nanoscale energy harvesting, and advanced material manufacturing. While numerous studies have explored either magnetic or radiative effects in isolation, the simultaneous influence of both on critical flow characteristics such as skin friction and heat transfer rate remains underexplored, particularly under exponential stretching conditions which more accurately represent practical industrial scenarios. The novelty of this research lies in its unified treatment of magnetic field effects, thermal radiation, and nonlinear stretching dynamics, which together govern the energy and momentum exchange within the boundary layer. To solve the resulting system of highly nonlinear boundary layer equations, the study employs the MATLAB *bvp4c* module—a robust and powerful boundary value problem solver based on collocation methods and error control. Despite its potential for precise and stable solutions, *bvp4c* has not been widely utilized in this class of magneto-radiative boundary layer problems, and its application here introduces a new dimension of numerical reliability and computational efficiency to the analysis. Thus, this study not only advances the fundamental understanding of magnetohydrodynamic (MHD) radiative boundary layer flows, but also serves as a strategic contribution to the growing body of research aiming to engineer next-generation thermal-fluid systems. It provides a computationally efficient, accurate, and practically relevant methodology for addressing challenging heat and momentum transfer problems encountered in a wide range of real-world industrial and environmental applications.

II. Method of Solution

To investigate the combined effects of magnetic field and thermal radiation on the boundary layer flow over an exponentially stretching surface, the governing equations of momentum and energy, derived using similarity transformations and boundary layer approximations, form a coupled system of nonlinear ordinary differential equations. These equations, which incorporate the magnetic parameter M , the radiation parameter K , and the Prandtl number Pr , are subject to nonlinear boundary conditions and exhibit strong interdependence due to the coupling of momentum and thermal effects.

To solve this system, the MATLAB built-in solver *bvp4c* is employed. This solver is based on collocation methods with adaptive mesh refinement and residual control, making it well-suited for two-point boundary value problems involving stiff and nonlinear equations. The equations are first rewritten as a first-order system by introducing appropriate substitutions for the second-order derivatives. Initial guesses for the solution profiles are carefully selected to ensure convergence and stability of the numerical scheme.

The *bvp4c* function iteratively adjusts the solution until the residuals of the differential equations and boundary conditions fall within a specified tolerance. The solver provides smooth and accurate solutions for the velocity and temperature profiles, from which the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ are computed directly. These quantities are then analyzed with respect to variations in M , K , and Pr to assess their influence on wall shear stress and heat transfer rate. The numerical approach offers high reliability and precision, capturing the complex interactions between electromagnetic damping, radiative heating, and thermal diffusion within the boundary layer.

III. Results and Discussion

In this study, the steady boundary layer flow of an electrically conducting fluid over an exponentially stretching surface was analyzed under the combined effects of a magnetic field and thermal radiation. The governing nonlinear differential equations, derived using appropriate similarity transformations under standard boundary layer assumptions, were solved numerically using MATLAB's *bvp4c* solver. The resulting velocity and temperature profiles, along with skin friction and Nusselt number, are graphically illustrated and discussed in detail to highlight the impact of key parameters.

Figure 2 presents the variation in the velocity profile of the fluid with respect to changes in the magnetic parameter. It is clearly observed that as the magnetic parameter increases, the fluid velocity within the boundary layer decreases significantly. This behavior can be attributed to the presence of a transverse magnetic field acting on the electrically conducting fluid, which gives rise to the Lorentz force. The Lorentz force acts in a direction opposite to the fluid motion, introducing a resistive drag that suppresses the flow velocity. Physically, this results in a retardation of the fluid particles due to magnetic damping. As the magnetic strength intensifies, the resistive force becomes stronger, further inhibiting the motion of the fluid and causing a more pronounced deceleration throughout the boundary layer. This reduction in velocity leads to a thinner momentum boundary layer, as the fluid adheres more closely to the stretching sheet. The decrease in velocity gradient near the surface also implies a lower shear rate, which has direct implications for the skin friction experienced at the wall.

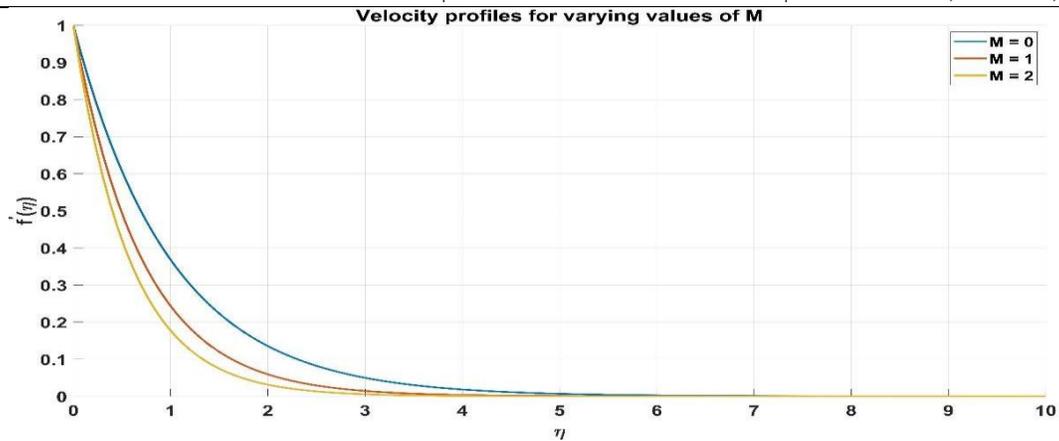


Fig. 2: Variations in velocity due to variations in magnetic parameter.

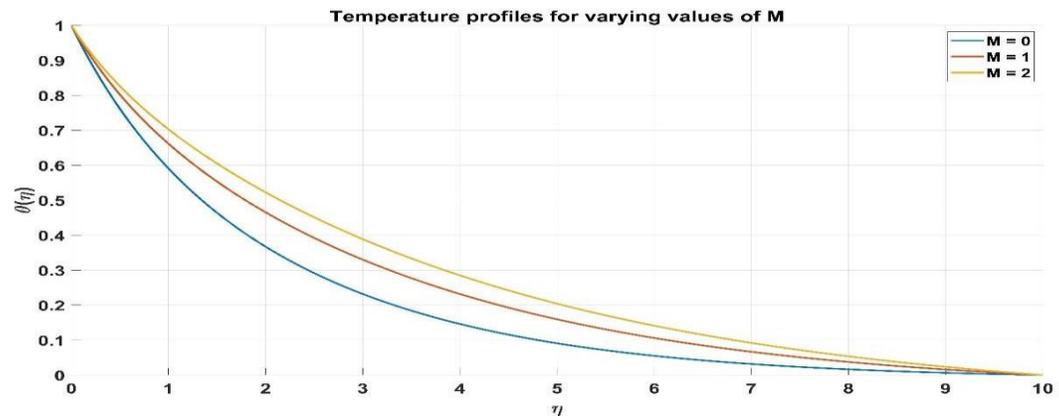


Fig. 3: Variations in temperature due to variations in magnetic parameter.

Figure 3 shows the effect of varying the magnetic parameter on the temperature distribution within the boundary layer. It is evident that as the magnetic parameter increases, the temperature of the fluid also increases throughout the boundary layer region. This rise in temperature can be physically explained by the influence of the Lorentz force generated by the applied magnetic field, which not only resists the fluid motion but also induces internal friction within the fluid. This resistive action reduces the convective transport of heat away from the surface, effectively trapping thermal energy near the stretching sheet. As a result, the thermal boundary layer becomes thicker, and the fluid retains more heat, leading to an overall increase in temperature. Moreover, the suppression of fluid velocity due to magnetic effects reduces the efficiency of momentum transfer, causing the fluid particles to linger longer near the heated surface and absorb more thermal energy. This behavior illustrates how the application of a magnetic field can enhance the thermal insulation effect in magnetohydrodynamic flows, thereby significantly influencing the heat transfer characteristics in systems involving radiatively and magnetically affected fluids.

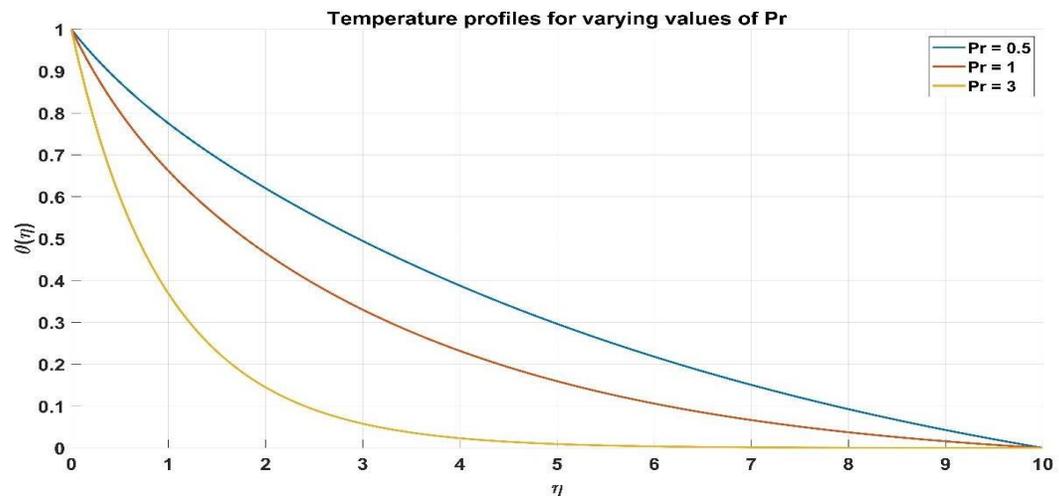


Fig. 4: Variations in temperature due to variations in Prandtl number.

Figure 4 illustrates the effect of varying the Prandtl number on the temperature distribution in the boundary layer over an exponentially stretching sheet under the influence of magneto-radiative interactions. As the Prandtl number increases, the temperature of the fluid decreases throughout the boundary layer. This occurs because higher Prandtl numbers correspond to fluids with lower thermal diffusivity, which limits the ability of heat to spread through the fluid. Consequently, less heat is transferred from the surface into the fluid, resulting in a thinner thermal boundary layer and lower temperature values at each point away from the surface. Despite the presence of magnetic and radiative effects that generally act to increase thermal energy retention, the dominant influence of the Prandtl number leads to reduced temperature profiles. Therefore, as the Prandtl number rises, the fluid becomes less thermally conductive, and the temperature within the boundary layer declines accordingly.

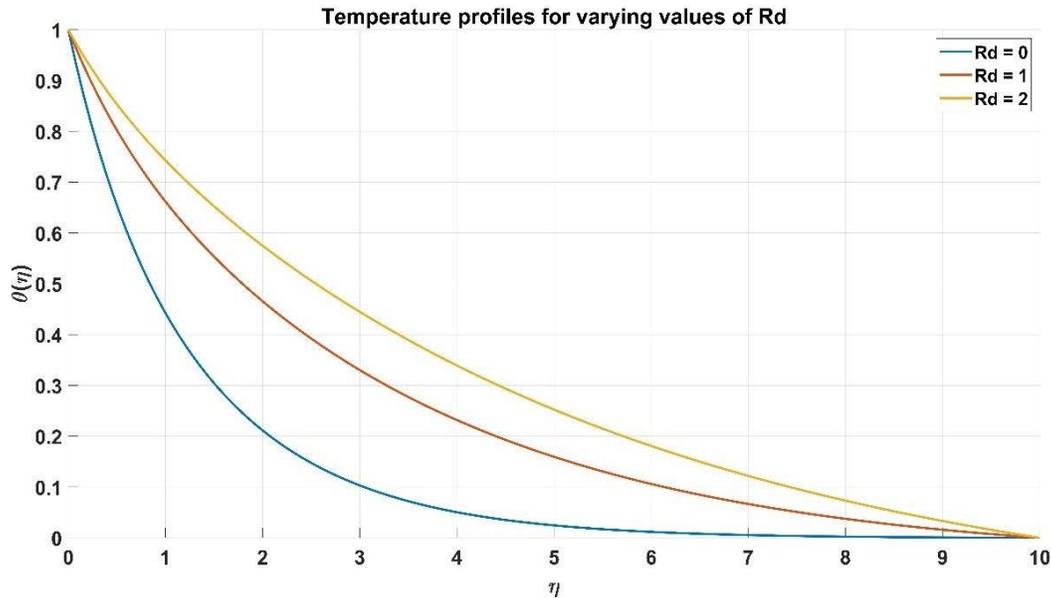


Fig. 5: Variations in temperature due to variations in thermal radiation parameter.

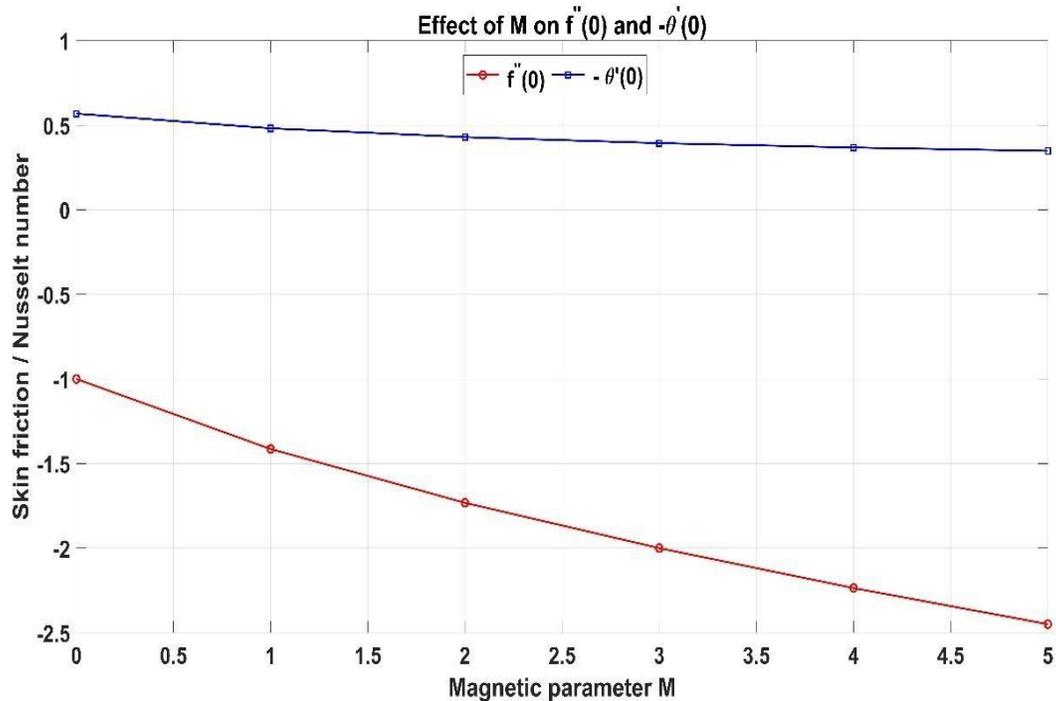


Fig. 6: Variations in Skin friction/Nusselt number due to variations in magnetic parameter.

Figure 5 shows the influence of the thermal radiation parameter on the temperature distribution in the boundary layer over an exponentially stretching sheet in the presence of magneto-radiative interactions. As the thermal radiation parameter increases, the temperature within the boundary layer also increases. This is because stronger thermal radiation enhances the radiative heat flux in the system, which contributes additional thermal energy to the fluid. The absorbed radiative energy raises the internal energy of fluid particles, resulting in an overall rise in temperature throughout the boundary layer. Additionally, the thermal boundary layer becomes thicker with increasing radiation parameter, as more heat is retained near the surface and diffused into the fluid. This effect

is especially prominent in the presence of exponential stretching, which already intensifies the temperature gradients. Hence, with an increase in the thermal radiation parameter, the fluid experiences higher temperatures and a broader thermal boundary layer due to the enhanced radiative energy input.

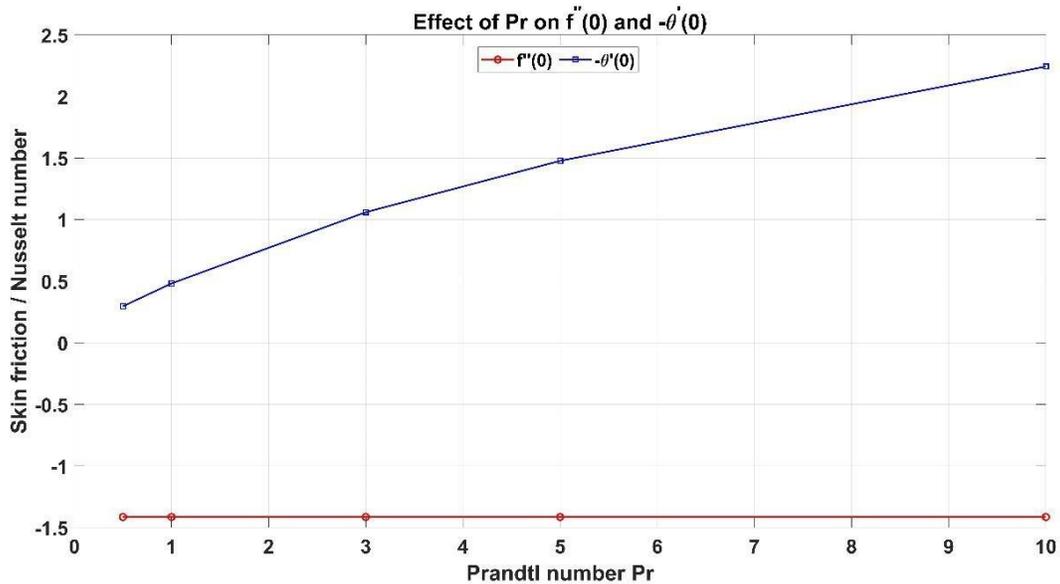


Fig. 7: Variations in Skin friction/Nusselt number due to variations in Prandtl number.

Figure 6 presents how the skin friction coefficient and Nusselt number respond to changes in the magnetic parameter within the framework of magneto-radiative boundary layer flow over an exponentially stretching surface. With an increase in the magnetic parameter, the skin friction coefficient shows a rising trend, while the Nusselt number exhibits a decline. The enhanced magnetic field generates a stronger Lorentz force that resists the motion of the conducting fluid, thereby increasing shear stress at the wall and elevating skin friction. Conversely, this magnetic resistance slows down the fluid flow, weakening the convective transport of heat from the surface. As a result, the thermal gradient at the wall decreases, leading to a reduction in the Nusselt number. Thus, the application of a stronger magnetic field intensifies momentum resistance while simultaneously impeding thermal energy transfer from the stretching surface.

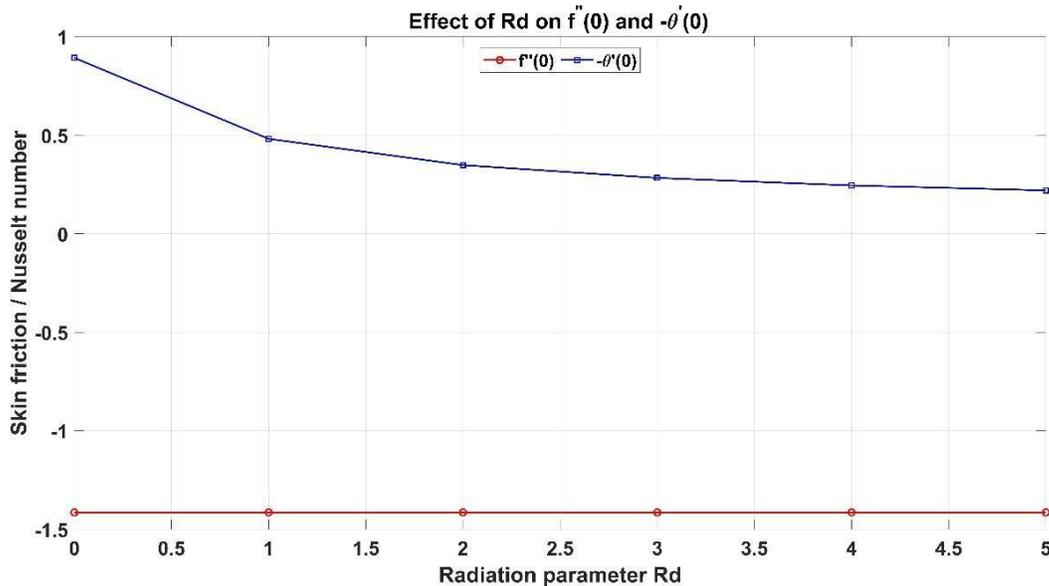


Fig. 8: Variations in Skin friction/Nusselt number due to variations in thermal radiation parameter.

Figure 7 depicts the impact of the Prandtl number on the skin friction coefficient and Nusselt number in the boundary layer flow influenced by magneto-radiative effects over an exponentially stretching sheet. As the Prandtl number increases, the Nusselt number increases, while the skin friction coefficient remains nearly unchanged or shows only slight variation. The rise in the Nusselt number with increasing Prandtl number is due to the reduced thermal diffusivity of the fluid, which enhances the temperature gradient at the surface and results in a higher rate of heat transfer. Since the Prandtl number primarily governs thermal behavior rather than momentum diffusion, its influence on skin friction is minimal. Therefore, the figure demonstrates that higher Prandtl numbers

improve heat transfer efficiency without significantly altering the wall shear stress. Figure 8 illustrates the effect of the thermal radiation parameter on the skin friction coefficient and Nusselt number for the boundary layer flow over an exponentially stretching sheet in the presence of magneto-radiative interactions. As the thermal radiation parameter increases, the Nusselt number decreases, while the skin friction coefficient remains nearly constant with minimal variation. The decline in the Nusselt number is due to the enhanced radiative heat flux, which elevates the fluid temperature and reduces the temperature gradient at the wall, thereby lowering the heat transfer rate. On the other hand, since radiation primarily influences the thermal field and has little direct impact on the momentum boundary layer, the skin friction coefficient is largely unaffected. This indicates that increasing thermal radiation weakens the surface heat transfer without significantly influencing the wall shear stress.

IV. Conclusions

This study investigated the boundary layer development of an electrically conducting fluid influenced by magneto-radiative effects over an exponentially stretching sheet. The main focus was on understanding how magnetic field strength, thermal radiation, and fluid properties like the Prandtl number affect the heat transfer and skin friction characteristics of the flow. The system of governing nonlinear ordinary differential equations derived from the momentum and energy equations was numerically solved using MATLAB's `bvp4c` solver, which efficiently handles two-point boundary value problems with high accuracy. The major findings of the analysis are as follows:

- Increasing the magnetic parameter leads to a decrease in fluid velocity and an increase in fluid temperature, due to the enhanced Lorentz force that opposes fluid motion and induces resistive heating.
- As the magnetic parameter increases, the skin friction coefficient rises, indicating stronger resistance to fluid motion near the wall, while the Nusselt number decreases due to reduced thermal gradients at the surface.
- Higher Prandtl number results in a lower temperature profile, owing to decreased thermal diffusivity, and enhances the Nusselt number, reflecting more efficient heat transfer. However, its influence on skin friction is minimal.
- The temperature distribution increases with an increase in the thermal radiation parameter, as radiative heat flux raises thermal energy within the boundary layer and thickens the thermal layer.
- The Nusselt number decreases with rising thermal radiation due to the reduction in surface temperature gradients, while skin friction remains nearly unchanged, confirming the dominant thermal nature of radiative effects.

These results offer valuable insights for optimizing thermal management and material processing applications involving magnetohydrodynamic and radiative boundary layer flows over stretching surfaces.

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