Parallel Machine Scheduling with Constraints in a Labelling Industry Case Study

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ABSTRACT

This paper would present a case study on solving a parallel machine scheduling problem in the printing department of a global leader in labeling solutions. Motivated by low-season conditions with below-capacity order volumes, the study prioritizes accelerating job completions to reduce resource idle time in downstream processes. It would focus on minimizing the total weighted completion time while addressing real-world constraints including machine-job compatibility, shift boundaries, and resource limitations in color matching processes. The problem was formulated as a three parallel machines with different speeds model and used Weighted Shortest Processing Time dispatching rule to find out solutions. The scheduling objective is to minimize total weighted completion time. Our approach progressively incorporates operational constraints and achieves around 11.11% improvement in objective value compared to the current manual scheduling method. Some discussions on implementation of research results, limitations, and future optimization opportunities and real-time production data integration would be mentioned.

Keywords— Parallel machine scheduling, Production constraints, Total weighted completion time, Dispatching rules, Weighted Shortest Processing Time rule.

INTRODUCTION

Production scheduling plays a crucial role in manufacturing environments, impacting resource utilization, operational costs, and customer satisfaction through delivery reliability. Efficient scheduling is particularly important in the printing industry, where machines with different capabilities must process various job types while respecting operational constraints.

Scheduling is defined as the allocation of resources (machines, labor, materials) over time to achieve specific production goals [1]. Manufacturing scheduling objectives typically include reducing makespan, optimizing inventory, adhering to deadlines, and improving resource utilization. Common challenges include balancing multiple conflicting objectives, accommodating complex production constraints, and adapting to dynamic changes in production conditions [2, 3].

This paper examines a real-world scheduling problem at AD label printing department. The current scheduling process relies on manual assignment by three separate planners using Excel spreadsheets, taking 30-45 minutes daily. This process is inefficient, lacks systematic dispatching rule application, and results in poor resource utilization.

This research would aim to improve this process by developing and implementing a systematic scheduling approach that minimizes the total weighted completion time while respecting key operational constraints. Our contributions include:

- A practical implementation of the Weighted Shortest Processing Time rule (WSPT rule) in constrained parallel machines with different speeds environment

- Progressive incorporation of operational constraints into the scheduling model

- Quantitative analysis of performance improvements over the manual method

- Identification of further optimization opportunities

The rest of this paper is organized as follows: Section II provides a literature review of relevant scheduling concepts. Section III describes the problem definition and mathematical formulation. Section IV presents the methodology and implementation approach. Section V discusses the results and performance comparison. Section VI concludes the paper and outlines future research directions.

LITERATURE REVIEW

Parallel machine scheduling problems have been extensively studied in operations research literature. The problem of minimizing total weighted completion time ($\sum w_i C_i$) on parallel machines is known to be NP-hard problems in the strong sense [4].

For identical parallel machines ($P_3 || \sum w_i C_i$), Smith [5] proved that the Weighted Shortest Processing Time (WSPT) rule provides an optimal solution. This rule sequences jobs in non-decreasing order of p_j/w_j, where p_j is the processing time and w_j is the weight of job j.

For uniform parallel machines ($Q_3 || \sum w_i C_i$), where machines have different speeds, the problem becomes more complex. Horowitz and Sahni [6] showed that WSPT is not generally optimal but can provide good approximations.

When additional constraints are introduced, such as machine eligibility constraints, release dates, or sequence-dependent setup times, heuristic approaches are typically used. Dispatching rules like WSPT are often modified to accommodate these constraints [7].

In practical applications, Kochhar and Morris [8] demonstrated that WSPT-based heuristics can significantly improve scheduling performance in manufacturing environments. Similarly, Weng et al. [9] applied weighted dispatching rules to semiconductor manufacturing with machine eligibility constraints.

This study would extend these approaches by applying WSPT in a printing production environment with multiple real-world constraints, including shift boundaries and resource limitations.

PROBLEM DEFINITION

Company background

AD is a global leader in labeling solutions and functional materials. The company's customer portfolio consists of 81 brands divided into three segments: Market Leading Accounts (MLA, 12 major brands), Key Retailers (medium priority), and Others (flexible-schedule). Their production involves seven distinct product families following a flow-shop manufacturing process in series as Printing, Inspection, and Packing.

The production facility houses 30 printing machines distributed across three areas: 14 machines at area 3A, 10 machines at area 3B, and 6 machines at area 1A. These machines are categorized as either camera-equipped (for quality-critical products) or non-camera (for standard products).

As mentioned above, the scheduling process at AD was inefficiently handled by three separate planners. Job assignments follow a priority hierarchy based on customer segmentation, with machine capability constraints (camera vs. non-camera) also considered when matching products to appropriate machines.

This fragmented approach leads to several operational challenges, including highly repetitive manual scheduling tasks, limited holistic planning effectiveness, inconsistent application of dispatching rules, and inflexible

prioritization logic. These issues result in poor utilization of labor and machinery resources, as the current system cannot effectively reflect actual constraints and shifting priorities over time.

Problem discription

To evaluate the effectiveness of applying dispatching rules compared to the current manual scheduling method, a demonstration using a scaled-down model reflecting the actual proportion of machine types in the HTL department was conducted. The demo focused on 3 machines (2 non-camera machines and 1 camera-equipped machine), which maintains the same ratio of machine types as the full production environment of 30 machines (18 non-camera and 12 camera-equipped).

In addition to testing the basic dispatching rules, realistic operational constraints that exist in the actual production environment were incorporated. These include shift boundary limitations, where jobs must start and finish within the same shift (Shift 1: 06:00-14:00; Shift 2: 14:00-22:00), and resource constraints related to color matching processes, which limit the number of jobs that can undergo color matching simultaneously due to limited colorist availability.

In the case study, parallel machines could be clustered into relative groups, in which the scheduling problem would be solved similarly. Therefore, in this study, the scheduling problem focused on assigning 25 jobs to 3 parallel machines (1 camera-equipped and 2 non-camera) while minimizing the total weighted completion time. Key characteristics of this problem include as followings, where Table 1 shows the machines characteristics, and Table 2 shows the priorities and parameters.

Machine characteristics (Table 1):

- Camera machine (Sakurai 1): Speed of 900 IMP/hr, better scrap rate, can process both YY-01 and HD-1096 products

- Non-camera machines (Sakurai 2 and 3): Speed of 1200 IMP/hr, higher scrap rate, can only process HD-1096 products

Table I. Machine Characteristics

i	Machine	Group	Conversed speed	YY-01	HD-1096
1	Sakurai 1	Camera	0.75	✓	✓
2	Sakurai 2	Non-camera	1		✓
3	Sakurai 3	Non-camera	1		✓

Table II. Job List

No.	Job_ID	Needed IMP	Product	Non camera	RB	wj	Setup time	Color matching
1	J1	450	HD-1096	Yes	Decathlon	5	0.2	TRUE
2	J2	900	YY-01	No	Adidas	8	0.2	TRUE
3	J3	1800	HD-1096	Yes	Walmart	1	0.2	FALSE
4	J4	300	HD-1096	Yes	Nike	10	0.2	TRUE
5	J5	600	YY-01	No	Nike	10	0.2	TRUE
6	J6	900	HD-1096	Yes	Walmart	1	0.2	FALSE
7	J7	3300	HD-1096	Yes	GAP	3	0.2	TRUE
8	J8	1200	HD-1096	Yes	Jako	4	0.2	FALSE
9	J9	600	HD-1096	Yes	Decathlon	5	0.2	TRUE
10	J10	2400	YY-01	No	Puma	7	0.2	FALSE
J11 --> J21								
22	J22	300	HD-1096	Yes	GAP	3	0.2	FALSE
23	J23	300	HD-1096	Yes	Decathlon	5	0.2	TRUE
24	J24	900	HD-1096	Yes	Adidas	8	0.2	TRUE
25	J25	1200	YY-01	No	Puma	7	0.2	TRUE

Job priorities:

- High priority: Market Leading Accounts (w = 8-10)

- Medium priority: Key Retailers (w = 4-7)

- Low priority: Others (w = 1-3)

Job parameters:

- Processing time (p_j)

- Weight value (w_j)

- Setup time ($S_{i'i}$): 0.2 hours for all job transitions

- Color matching requirement: 0.3 additional hours if required

Operational constraints:

- Machine-job compatibility constraints

- Shift boundary constraints: Jobs must start and finish within single shifts (Shift 1: 06:00–14:00; Shift 2: 14:00–22:00)

- Color matching resource constraint: Maximum 2 jobs can undergo color matching simultaneously

Mathematical Fomulation

The production process at AD follows a specific workflow as illustrated in the flowchart (Fig. 1). After receiving the job jacket from the Planning department and checking materials (ink, substrate), operators proceed with machine setup. For products requiring color matching, the process includes additional complex steps: the machine runs a trial and samples are submitted to the Colorist for checking. If the colors do not meet requirements, the ink formula is adjusted, and the trial process is repeated until standards are met. Only when colors are approved can mass printing begin. Upon completion, products are transferred to the Inspection station.

Color matching is a technical process requiring a Colorist's expertise to ensure color accuracy according to customer specifications. This process takes an additional 0.3 hours and requires specialized resources - each Colorist can only handle a maximum of two color matching jobs simultaneously, creating an important constraint in the scheduling process.

The total time to complete a job (Total Time - TTji) is calculated as the sum of: machine setup time (Setup Time - 0.2 hours) + color matching time (if required - 0.3 hours) + processing time (dependent on job volume). This calculation formula accurately reflects the actual time needed to complete each job in the production environment and serves as the basis for developing an optimized scheduling model.

A uniform parallel machine with different speeds scheduling problem ($Q_3||\sum w_i C_i$) with additional constraints was formulated. The mathematical model objective aims to minimize the total weighted completion time ($\sum w_i C_i$), where each job's weight reflects its relative priority in production. By minimizing $\sum w_i C_i$, the completion of high-priority jobs was accelerated. It could help reduce the overall work-in-process (WIP) inventory. This objective directly supports operational goals during the low season by enabling faster job transitions to subsequent stages, minimizing resource idle time, and improving overall throughput and labor productivity.

$$Objective = minimize \sum_{j=1}^{n} w_j C_j \quad (1)$$

Where:

- w_j = weight (priority) of job j

- C_j = completion time of job j

Subject to:

- Machine eligibility constraints

- Shift boundary constraints

- Color matching resource constraints

- Non-preemption constraints

The total time for processing job j on machine i (TT_{ji}) is calculated as:

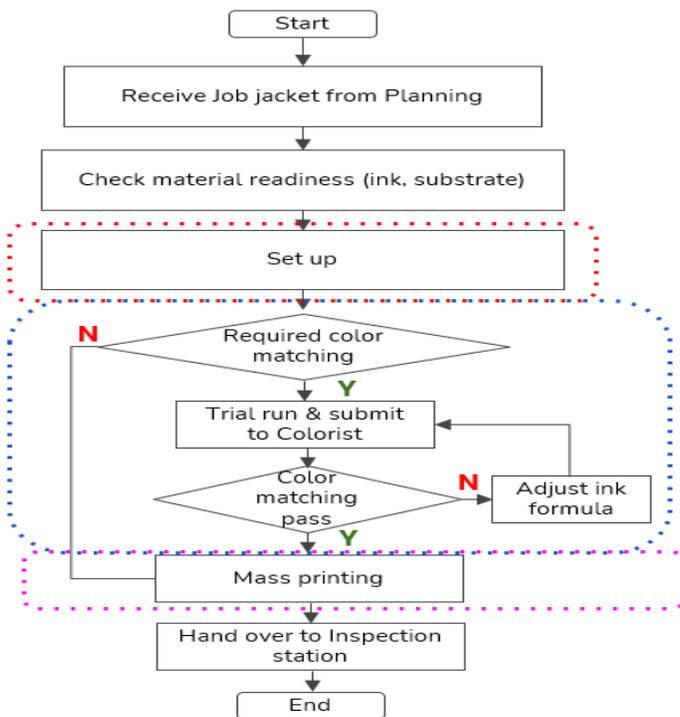$$TT_{ji} = Setup\ Time + Color\ Matching\ Time + Processing\ time \qquad (2)$$

Fig. 1. Flow chart

PROGRAME DEVELOPMENT AND IMPLEMENTATION

Data collection

Data was collected through:

- Real scheduling data from company records

- Interviews with planners and operational staff

The collected data included job specifications (processing times, setup times, priority weights) and machine characteristics (speed, capability profiles, shift availability).

Implementation Approach

A Python-based scheduling solution was developed following these steps:

Simple WSPT Implementation:

- Classify jobs based on product type compatibility with machines

- For each job j and eligible machine i, compute priority index = TT_{ji}/w_j

- Assign job j to machine i with lowest TT_{ji}/w_j

- Group jobs by assigned machine

- Sequence jobs on each machine in increasing order of TT_j/w_j

Shift Constraint Integration:

- Add shift boundary constraints (06:00-14:00 and 14:00-22:00)

- Check if job fits within current shift; if not, defer to next shift

Color Matching Constraint Implementation:

- Limit simultaneous color matching to maximum 2 jobs

- If more than 2 color-matching jobs are in queue:

  o Move up a job that does not require color matching

  o If no such job is available, delay the third color-matching job

The scheduling solution generates visualizations (Gantt charts) and numerical results for analysis.

RESULTS & DISCUSSIONS

Performance Comparison

Four scheduling scenarios were considered:

Manual scheduling: Currently used by planners

The current manual scheduling process at AD operates across two shifts: Shift 1 from 06:00 to 14:00 and Shift 2 from 14:00 to 22:00. Planners begin by reviewing jobs in order of priority, with highest weight (w_j) jobs considered first. For each job, in which they identify all compatible machines based on technical requirements. They then select the machine that will become available earliest from this eligible list. Before finalizing the assignment, planners verify that the machine's available time plus the job's total processing time does not exceed the shift end. If this condition is met, the job is assigned, and the machine's availability is updated accordingly. If the job cannot be completed within the remaining shift time, planners attempt to assign it to the next eligible machine. Any jobs that cannot be accommodated in Shift 1 are reconsidered for Shift 2 following the same prioritization and assignment rules. This manual scheduling method results (Fig. 2) in a total weighted completion time (objective function) of $\mathbf{C_0} = \sum w_i C_i = \mathbf{720.68}$.
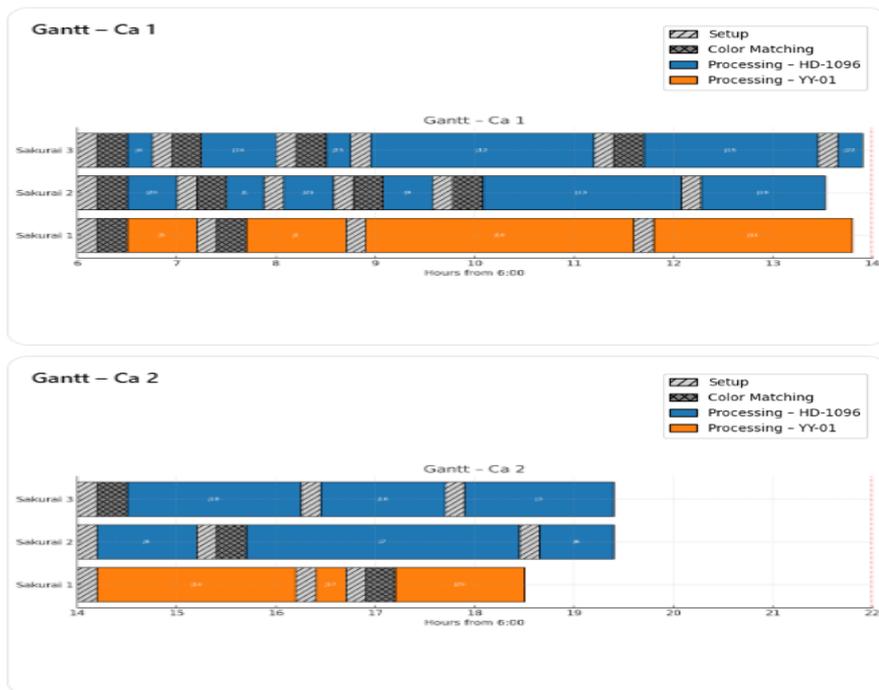
Fig. 2. Gantt chart – manual scheduling

WSPT rule: Basic implementation with Job and Machine constraints

In proposed scheduling approach, first jobs are classified according to their product specifications. Products of type YY-01 must be processed exclusively on Sakurai 1 because of the only camera-equipped machine capable of handling these products. Products of both types YY-01 and HD-1096 can technically be processed on all three machines (Sakurai 1, 2, and 3), but HD-1096 products are preferred to assign to non-camera machines (Sakurai 2 or 3) whenever possible due to their faster processing speed.

For each job j that can be processed on machine i, the total processing time (TTji) is calculated as the sum of setup time, color matching time (if required), and the actual processing time. A priority index for each job-machine combination is then computed by dividing the total time by the job's weight (TT_{ji}/w_j). Jobs are assigned to machines that yield the lowest priority index value, effectively implementing the Weighted Shortest Processing Time (WSPT) rule. After assignment, jobs on each machine are sequenced in increasing order of their priority index as on Fig. 3. This optimized scheduling method achieves a total weighted completion time of $\mathbf{C_1} = \sum w_i C_i = \mathbf{612.93}$, demonstrating significant improvement over the manual approach.
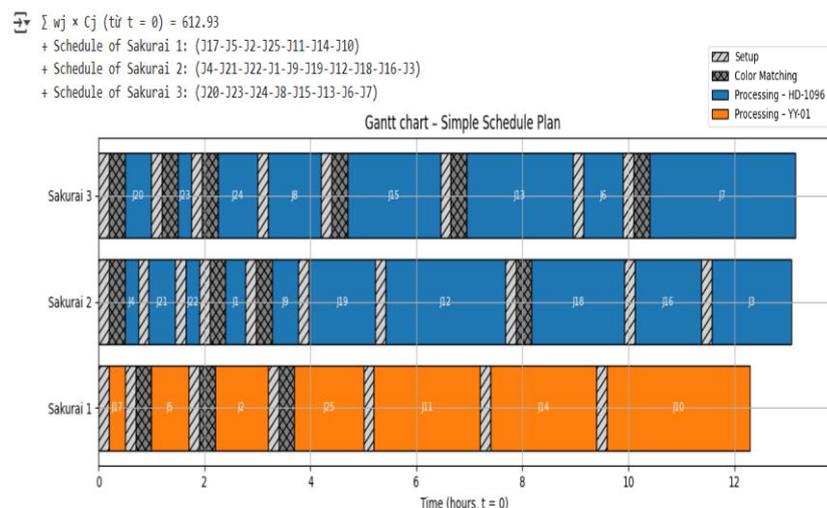

Fig. 3. Gantt chart – WSPT job & machine constraints

The Gantt chart displays a WSPT scheduling solution with three Sakurai machines running parallel job sequences, clearly showing setup periods (diagonal stripes), processing times (solid colors), and idle periods (crosshatched patterns). While the solution successfully demonstrates job scheduling with corresponding setup times and color matching requirements while adhering to technological constraints, it fails to account for shift end times—a critical practical consideration in manufacturing environments where shifts have defined boundaries and operators leave at scheduled times, making this solution theoretically sound but impractical for real-world implementation.

WSPT rule with shift constraints: No job printed between two shifts

When implementing shift constraints into the scheduling model, two 8-hour shifts per day were enforced, in which Shift 1 operates from 06:00 to 14:00 and Shift 2 operates from 14:00 to 22:00. A critical constraint was added ensuring that no job could span across shifts—each job must start and finish within a single shift. This was achieved by checking if the remaining time in a shift was sufficient to complete a job; specifically, if the shift end time minus the latest job completion time was less than the total processing time (TT_{ij}) of the new job, that job would be deferred to the next available shift. The results (Fig. 4) show that our objective value for this shift-constrained schedule is $C_2 = \sum w_i C_i = \textbf{630.70}$, with a total idle time of 9.48 hours across all machines. In Shift 1, a total weighted completion time of 328.75 with 1.73 hours of idle time, distributing jobs optimally across the three machines (Sakurai 1: J17-J5-J2-J25-J11; Sakurai 2: J4-J21-J23-J1-J8-J15-J6; Sakurai 3: J20-J22-J24-J9-J19-J12) was determined. Shift 2 had a weighted completion time of 301.95 with a higher idle time of 7.75 hours, with jobs scheduled as Sakurai 1: J14-J10; Sakurai 2: J13-J16-J3; and Sakurai 3: J18-J7. This approach represents a practical improvement over the baseline model while respecting real-world operational constraints.
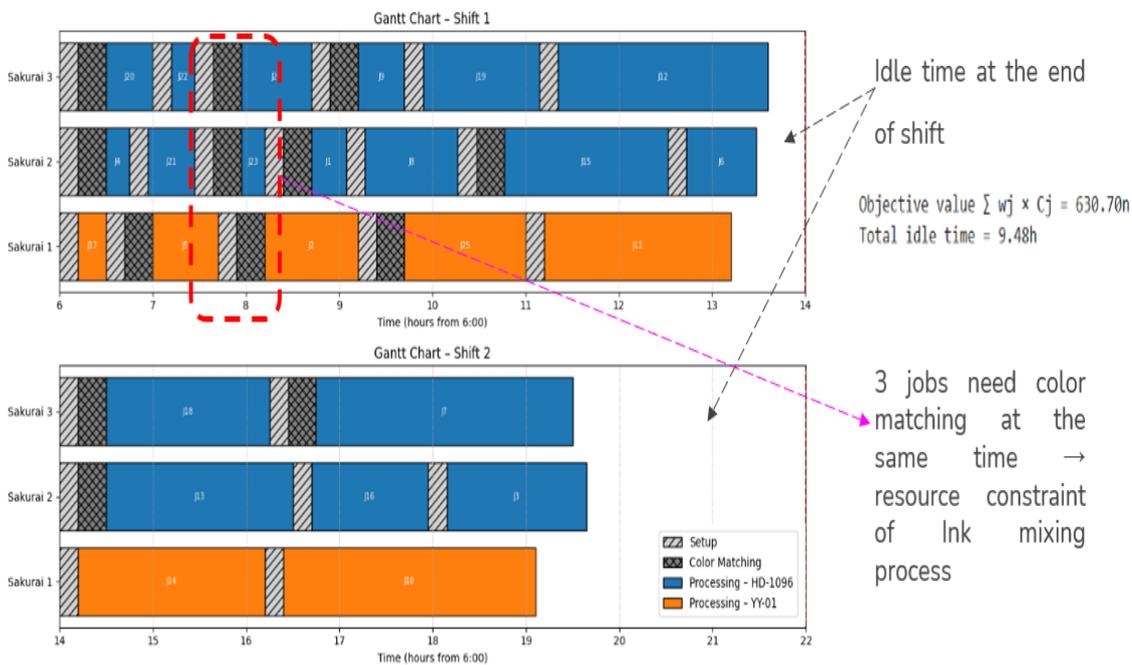

Fig. 4. Gantt chart – WSPT shift constraints

The WSPT scheduling approach shown in both Gantt charts achieves an objective value of 630.70 with 9.48 hours of total idle time, divided across two shifts. The solution successfully implements dispatching rules and technical constraints while strictly enforcing the requirement that no job can run across shifts (Shift 1: 06:00-14:00 and Shift 2: 14:00-22:00). Notable observations include the presence of unavoidable idle time at the end of shifts when remaining time is insufficient to complete the next job, requiring deferral to the next available shift. The charts also highlight a resource constraint situation where three machines require color matching simultaneously, demonstrating the handling of resource limitations in the ink mixing process. The scheduling effectively balances workload with Shift 1 showing 1.73 hours idle time and Shift 2 showing 7.75 hours idle time, while maintaining adherence to all operational constraints.

WSPT with shift and color matching constraints

The solution (Fig. 5) shows scheduling data with an objective value of 640.60 and total idle time of 9.48 hours, divided between two cases. Case 1 has a lower idle time of 1.73 hours with an objective value of 338.65, featuring three Sakurai machine schedules with job sequences (J17-J5-J11-J2-J25), (J4-J21-J23-J1-J8-J15-J6), and (J20-J22-J24-J9-J19-J12). Case 2 shows significantly higher idle time at 7.75 hours with an objective value of 301.95, containing three different Sakurai machine schedules: (J14-J10), (J13-J16-J3), and (J18-J7). The scheduling problem includes important constraints where no jobs can be printed between shifts and a maximum of two jobs can undergo color matching simultaneously. When more than two color-matching jobs appear next in the queue, either a non-color-matching job is moved up to fill the gap, or if none are available, the third color-matching job is delayed until one of the two running color-matching jobs completes. This scheduling approach achieves an overall objective value of $C_3 = \sum w_i C_i = \mathbf{640.60}$.
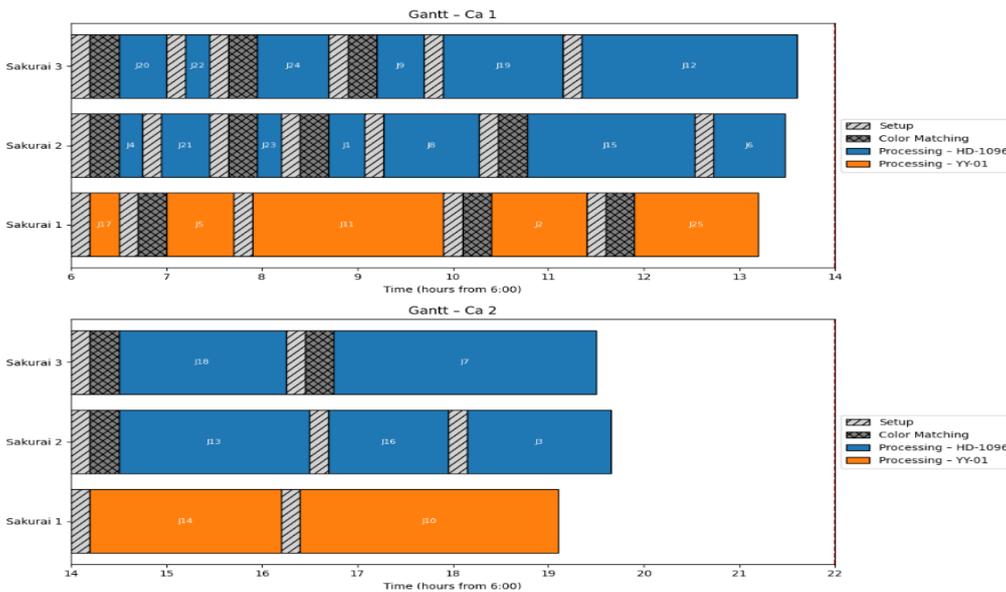

Fig. 5. Gantt chart – WSPT shift and color-matching constraints

The WSPT Gantt charts demonstrate a successful job scheduling implementation with an objective value of 640.60 hours and total idle time of 9.48 hours across two shifts. This solution effectively addresses the color-matching constraint, ensuring no more than two jobs undergo color matching simultaneously as required. When examining the charts, it's clear that the solution properly handles situations where three potential color-matching jobs might have occurred at the same time by either advancing non-color-matching jobs or delaying the third color-matching job until resources become available. However, opportunities for further optimization remain evident, particularly in the arrangement of jobs across machines. Future improvements could explore applying local search techniques (such as Tabu Search or Simulated Annealing algorithms) to better distribute jobs from Sakurai 2 and 3 to Sakurai 1, potentially reducing idle time. Additionally, the solution could benefit from strategies to insert appropriate jobs into idle periods near shift ends, which would require a larger data pool to properly implement. These refinements could further enhance the scheduling efficiency beyond the current solution.

Here is a comprehensive comparison of different scheduling methods under various operational constraints. The Table III presents four scheduling scenarios, including the manual method and three WSPT-based approaches with progressively increasing constraints. Each scenario is evaluated based on the objective function value ($\sum w_j C_j$) and practical realism. The baseline WSPT approach (Scenario 1) achieves the lowest objective value of 612.93 but has low practical realism as it ignores real-world constraints. Scenario 2 incorporates shift constraints, resulting in a slightly higher objective value of 630.7 with medium practical realism. The final model (Scenario 3) implements both shift and color-matching constraints, achieving an objective value of 640.6, which represents an 11.11% improvement over the manual method (720.68) while maintaining high practical realism by addressing all relevant operational constraints.

Table III. Performance Comparison of Scheduling Methods

Scenario	Scheduling Conditions			Objective Function ($\sum wjCj$)	Practical Realism	Reason for $\sum w_j C_j$ increase
	WSPT	Shift constraint	Color-matching constraint			
Manual		☑		720.68		
1	☑			612.93	Low	Baseline → no constraints, ideal conditions
2	☑	☑		630.7	Medium (similar to Manual)	Jobs must fit within shifts → idle time near shift end
3	☑	☑	☑	640.6 ⇓11.11% compared to Manual method	High (better than Manual)	Jobs delayed to satisfy color matching limit → higher completion time

Analysis of Constraints Impact

Shift Constraints Impact:

The implementation of shift constraints significantly impacted the scheduling solution, increasing the objective function from 612.93 to 630.70, representing a 2.9% increase. This constraint created unavoidable idle time periods at shift endpoints when insufficient time remained to complete scheduled jobs. Despite this increase in the objective function value, the incorporation of shift constraints substantially enhanced the practical applicability of the scheduling model by respecting the organizational shift structure that exists in real manufacturing environments.

Color Matching Constraints Impact:

Additionally, the introduction of color matching constraints further increased the objective function to 640.60, an additional 1.6% increase. This constraint necessitated job resequencing to ensure compliance with the limitation of maximum two simultaneous color-matching operations at any given time. While this constraint further increased the completion time, it successfully eliminated resource constraint violations that were present in previous models, resulting in a practically implementable solution that balances efficiency with operational feasibility. The final model with both constraints still achieved an 11.11% improvement over the manual scheduling method.

Implementation Considerations

The proposed scheduling approach offers immediate practical benefits:

- Systematic application of dispatching rules

- Automatic consideration of all operational constraints

- Reduced scheduling time (from 30-45 minutes to seconds)

- Improved objective function (11.11% reduction in total weighted completion time)

However, several limitations and improvement opportunities are identified:

Optimization Techniques:

- Apply Local Search algorithms to find better solutions beyond standard dispatching rules

- Implement local search to optimize job sequence on each machine

Resource Utilization:

- Fill end-of-shift idle time with short jobs

- Consider job splitting for better utilization (if operationally feasible)

Organizational Improvements:

- Centralize scheduling data with a single planner having comprehensive access

- Implement real-time integration with production signals for dynamic adjustments

CONCLUSION AND FUTURE WORK

This paper presented a case study on improving scheduling efficiency at AD printing department through the application of the WSPT dispatching rule with real-world constraints. The proposed approach achieved an 11.11% improvement in total weighted completion time compared to the current manual method.

The key contributions include:

- Successful adaptation of the WSPT rule to a constrained printing environment

- Quantification of the impact of different operational constraints on scheduling performance

- Development of a practical scheduling solution that respects all production requirements

Future research directions should focus on:

- Implementing heuristics algorithms to further optimize job sequences

- Investigating dynamic scheduling approaches that can adapt to production disruptions

- Integrating real-time production data for adaptive scheduling

- Expanding the model to include all 30 machines across the three production areas

This case study demonstrates that even with the application of relatively simple dispatching rules, significant improvements can be achieved when operational constraints are properly incorporated into the scheduling process.

ACKNOWLEDGEMENTS

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