

# An Exploration of The Volatility Clustering Present in Bitcoin's Price Data, Comparing The GARCH, EGARCH And GJR-GARCH Models.

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## ABSTRACT

This study looks at the dynamics of volatility clustering in Bitcoin's price data by comparing the performance of three econometric models namely, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Exponential GARCH (EGARCH), and Glosten Jagannathan Runkle GARCH (GJR-GARCH) models. Using daily closing price data, the analysis explores the sensitivity of Bitcoin's conditional variance to market shocks and the persistence of these effects over time. To achieve this, the data is cleaned and the series is differenced to achieve stationarity, after which the conditional mean and variance equations are estimated to model time-varying volatility. Model adequacy and comparative performance are assessed using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and log-likelihood values, while out-of-sample forecast accuracy is evaluated through Root Mean Squared Error (RMSE) and Mean Squared Error (MSE) metrics. The study examines the real-world relevance of volatility modeling by integrating the best-performing model into volatility-adjusted trading strategies and comparing their risk-adjusted returns measured by the Sharpe ratio with those of the conventional buy-and-hold strategy. The empirical results obtained confirm the presence of significant volatility clustering in Bitcoin's price and indicate that the EGARCH (3,2) model most effectively captures volatility responses to shocks in the market. The EGARCH (3,2) model also demonstrates higher forecasting performance relative to the others. When implemented within a volatility-based position sizing framework, it yields higher risk-adjusted returns than the buy and hold strategy. These findings show the value of advanced conditional heteroskedasticity models in enhancing predictive accuracy and informing more efficient cryptocurrency trading and risk management strategies.

**Keywords:** Volatility clustering, Models, Forecasting, Cryptocurrency, Portfolio

## INTRODUCTION

The emergence of Bitcoin and the broader cryptocurrency ecosystem, originating with the publication of the Bitcoin: A Peer-to-Peer Electronic Cash System white paper by Nakamoto (2008), has profoundly transformed the global financial landscape. With the goal of establishing a decentralized digitalized currency, Bitcoin introduced a trustless and distributed system of value exchange independent of traditional financial intermediaries. This innovation quickly spurred the development of a vast and rapidly evolving cryptocurrency market characterized by speculative activity, high liquidity, and extreme price fluctuations.

A unique feature of this market is its pronounced volatility, this often manifests as volatility clustering, that is, the empirical tendency for periods of large price movements to be followed by further large movements, and small changes to be followed by similarly small changes (Xu & Zhu, 2022). Such behavior is consistent with

findings in traditional financial markets but tends to be more pronounced in cryptocurrencies due to their sensitivity to speculative demand, technological developments, and regulatory changes (Katsiampa, 2017). Understanding and modeling this volatility is essential for risk management, derivative pricing, and the design of profitable trading strategies (Jiang et al., 2023).

While traditional asset classes have been extensively modeled using frameworks such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its variants, their relative performance and suitability for capturing Bitcoin’s unique volatility dynamics remain underexplored. In particular, asymmetric extensions like the Exponential GARCH (EGARCH) (Nelson, 1991) and the Glosten–Jagannathan–Runkle GARCH (GJR-GARCH) (Glosten et al., 1993) are designed to account for the leverage effects where negative shocks exert a larger impact on volatility than positive shocks of equal magnitude. However, limited comparative evidence exists regarding their effectiveness in modeling Bitcoin’s returns and their applicability in trading and portfolio risk management contexts.

## RESEARCH METHODOLOGY

The empirical analysis utilized 3,724 daily closing price observations of Bitcoin in USD obtained from the Binance API. This spanned from January 2015 to March 2025. The data was cleaned and transformed to obtain a format suitable for Time series analysis. Time series analysis was conducted and the best models were chosen and incorporated in real-world trading strategies to determine its utility in trading and portfolio management. All time series analysis, verification and visualization was implemented in Python 3.9 and R version 4.3.3, utilizing a range of specialized libraries such as numpy, pandas, plotly, rugarch and ggplot2.

### Data Analysis Results And Discussion

The cleaned data was tested for stationarity in accordance with Time series analysis which requires data stationarity.

**Table1.** ADF Test Before And After First Differencing

Series	ADF-statistic	P-value	Lag Order
Bitcoin’s Price	-2.83	0.226	15
Bitcoin’s Price data After first Differencing	-14.82	<0.01	15

The Augmented Dicky Fuller (ADF) test was conducted and the data was found to be non-stationary with a p-value of 0.226 from Table 1. To achieve stationarity, the time series was transformed using first differencing represented as  $\Delta P_t = P_t - P_{t-1}$ . The ADF was then applied on the differenced series and that resulted in a p-value of less than 0.01, confirming the successful removal of the stochastic trend component. Following stationarity, the mean component of the time series was modelled using the Autoregressive Integrated Moving Average (ARIMA) framework. This was done comparatively across different orders. ARIMA (3,1,3) came out on top, having the lowest Akaike Information Criterion (AIC). Based on this, it was selected to filter out the linear dependencies leaving the residual dynamics primarily attributed to conditional volatility. The requirement for higher-order lags (3,3) in the mean structure suggested complex short-term dependencies in Bitcoin price changes, indicative of a rich autocorrelation structure than commonly observed in traditional financial assets.

### Volatility Clustering and ARCH Effects

To confirm the usage of the GARCH-family models, the existence of conditional heteroskedasticity in the ARIMA (3,1,3) residuals ( $\epsilon_t$ ) was formally tested. The ARCH-LagRange(ARCH-LM) test was performed on the ARIMA residuals. The results obtained rejected the null hypothesis of no ARCH effects at all tested lags, with p-values less than 0.01. This provided a statistically robust confirmation, that justified the use of the conditional

volatility modeling techniques. The persistence of autocorrelation in the squared residuals is what these GARCH models are designed to capture, as volatility is not static but time-dependent.

### GARCH-Family Model Specifications

Three primary models employed with many variants were evaluated.

The GARCH model specifies the conditional variance as:

$$\sigma_{\square}^2 = \omega + \alpha \varepsilon_{\square-1}^2 + \beta \sigma_{\square-1}^2$$

(3.1)

where  $\omega$  is the constant term,  $\alpha$  captures the impact of recent shocks (ARCH effect)  $\beta$  represents the persistence of volatility (GARCH effect).

The constraints  $\omega > 0$ ,  $\alpha \geq 0$ ,  $\beta \geq 0$ , and  $\alpha + \beta < 1$  ensure a positive, finite conditional variance and covariance stationarity.

The GJR-GARCH model extends the standard GARCH by his model captures conditional variance using the threshold term  $\square_{\{\square-1\}}$

$$\sigma_{\square}^2 = \omega + \alpha \varepsilon_{\square-1}^2 + \gamma \square_{\{\square-1\}} \varepsilon_{\square-1}^2 + \beta \sigma_{\square-1}^2$$

(3.2)

where  $\square_{\{\square-1\}} = 1$ ; if  $\square - 1 < 0$  and 0 otherwise.

EGARCH(p,q):

The Exponential GARCH (EGARCH) model expresses the conditional variance in a logarithmic form:

$$\ln(\sigma_{\square}^2) = \omega + \alpha \left( \frac{|\varepsilon_{\square-1}|}{\sigma_{\square-1}} - \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\varepsilon_{\square-1}}{\sigma_{\square-1}} + \beta \ln(\sigma_{\square-1}^2)$$

(3.3)

The EGARCH model explicitly incorporates the asymmetric effect via the  $\gamma$

parameter, which, if positive, indicates that negative shocks amplify volatility more than positive shocks

### Empirical Analysis and Model Diagnostics

The three models, GARCH, EGARCH and GJR-GARCH were evaluated to see which of them best modeled the in-sample data before those were chosen and compared to each other. Now this was done by optimizing a combination of ARIMA (3,1,3) with the various GARCH Family of models then the best forming model is chosen and that model caters for the volatility left after trend is removed. For both AIC and BIC, the lower the value the better. For loglikelihood on the other hand, the larger the figure the better.

**Table 2.** Model Selection Criteria

Model	AIC	BIC	Log-likelihood
EGARCH (3,2)	13.226609	13.252705	-19678.4205

EGARCH (1,1)	13.230546	13.258840	-19689.2837
EGARCH (1,2)	13.234404	13.258577	-19694.0275
EGARCH (2,2)	13.237983	13.266185	-19697.356
EGARCH (2,3)	13.255673	13.285889	-19722.6967
EGARCH (1,3)	13.261874	13.288061	-19733.9300
EGARCH (3,1)	13.274240	13.304457	-19750.3437
EGARCH (2,1)	13.274385	13.300573	-19752.5595
GJR-GARCH (3,2)	13.290759	13.316947	-19776.9405
GJR-GARCH (2,2)	13.294256	13.326487	-19779.1469
GJR-GARCH (2,3)	13.298702	13.326904	-19787.7666
GARCH (2,3)	13.301167	13.331384	-19790.4378
GARCH (1,1)	13.305886	13.326031	-19802.4648
GJR-GARCH (1,2)	13.311228	13.335402	-19808.4192
GARCH (1,2)	13.312510	13.334669	-19811.3269

In-sample parameter estimates were obtained to draw further insights

**Table 3.** In-sample Parameter Estimates

PARAMETER/STATISTIC	GARCH (2,3)	p-value	GJRGARCH (3,2)	p-value	EGARCH (3,2)	p-value
$\mu$ (constant)	0.5174	<0.01	0.4508	<0.01	0.7186	<0.01
$\phi$ (AR term)	0.5301	<0.01	0.7469	<0.01	0.4291	<0.01
$\phi_2$ (AR2)	0.3721	<0.01	-0.7450	<0.01	-0.5961	<0.01
$\phi_3$ (AR3)	-0.9430	<0.01	0.9876	<0.01	-0.0827	<0.01
$\theta_1$ (MA1)	-0.5292	<0.01	-0.7527	<0.01	-0.4861	<0.01
$\theta_2$ (MA2)	-0.3666	<0.01	0.7518	<0.01	0.5971	<0.01
$\theta_3$ (MA3)	0.9326	<0.01	-0.9983	<0.01	-0.0052	<0.01
$\omega$ (GARCH constant)	6.1868	<0.01	4.3562	<0.01	0.2263	<0.01
$\alpha_1$ (ARCH effect)	0.1914	<0.01	0.1594	<0.01	0.0273	<0.01
$\alpha_2$ (ARCH effect)	0.1187	<0.01	0.1705	<0.01	0.0196	<0.01
$\beta_1$ (GARCH effect)	0.000	-	0.0000	-	0.8657	<0.01
$\beta_2$ (GARCH effect)	0.3133	<0.01	0.7436	<0.01	0.9706	<0.01
$\beta_3$ (GARCH effect)	0.3756	<0.01	-	-	-	<0.01

$\gamma_1$ (Asymmetry)	-	-	-0.0456	<0.01	0.4220	<0.01
$\gamma_2$ (Asymmetry)	-	-	-0.0473	<0.01	0.4033	<0.01
$\gamma_3$ (Asymmetry)	-	-	-0.0560	<0.01	-0.0923	<0.01
Log-Likelihood	19776.94	<0.01	-19779.15	<0.01	-19731.35	<0.01
AIC	13.2908	-	13.2943	-	13.2622	-
BIC	13.3169	-	13.3265	-	13.2944	-

From Table 2 and Table 3, the GARCH (2,3), GJR-GARCH (3,2), and EGARCH (3,2) models revealed important features of Bitcoin’s volatility behavior. All models showed statistically significant ARCH and GARCH effects ( $p < 0.05$ ), confirming the presence of volatility clustering where high-volatility periods persist over time. The sum of the ARCH and GARCH coefficients being close to one in the GJR-GARCH (3,2) and EGARCH (3,2) models indicated strong volatility persistence, showing shocks to volatility decayed slowly. Both models also captured asymmetric or leverage effects, where negative price shocks have a greater impact on future volatility than positive ones which is consistent with how bad news tends to increase market uncertainty. Specifically, the GJR-GARCH model showed a negative asymmetry term ( $\gamma_1 = -0.0496$ ), while the EGARCH model shows a positive  $\gamma$  (0.4220), both confirming this effect. Among the three, the EGARCH (3,2) model outperformed the others with the highest log-likelihood (19731.35) and the lowest AIC (13.2622) and BIC (13.2944), indicating the best fit and forecasting efficiency. The GJR-GARCH (3,2) followed as the next best, while the standard GARCH (2,3) ranked lowest. Overall, the results highlighted the necessity of modeling asymmetric effects to better explain Bitcoin’s volatility dynamics, with EGARCH (3,2) providing the most accurate representation.

### Out of sample Volatility Forecast Evaluation

To assess the model’s predictive performance, we conducted an out-of-sample test of volatility overcast against mean volatility using the remaining 30% of the data. Table 4 presents the forecast accuracy metrics.

**Table 4.** Out of Sample Model Evaluation

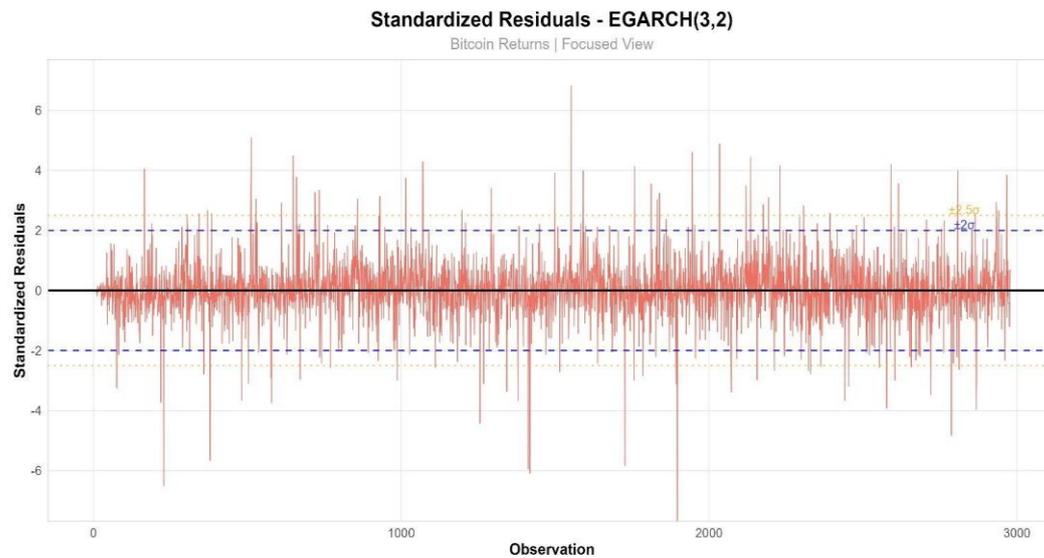
MODEL	MSE	RMSE	MAE	IMPROVEMEN T %
ARIMA (3,1,3)- GARCH (2,3)	1,189,454	1090.62	823.45	3.8%
ARIMA(3,1,3)GJR- GARCH (3,2)	1,156,823	1075.56	812.30	5.1%
ARIMA(3,1,3)- EGARCH (3,2)	1,098,234	1048.44	788.95	7.8%
NAÏVE BENCHMARK	-	-	855.70	0.0%

From Table 4, The out-of-sample evaluation revealed that the EGARCH (3,2) model slightly demonstrated superior performance compared to the other models with respect to forecast accuracy, with the lowest MSE value of 1,098,234, an RMSE of 1048.44, and MAE value of 788.95. Also, we compared the volatility forecast with a naïve mean volatility model and realized that the EGARCH (3,2) component provided a 7.8% better result in terms of having a lower error rate compared to the naïve mean absolute error.

### Model Validation and Diagnostic Testing

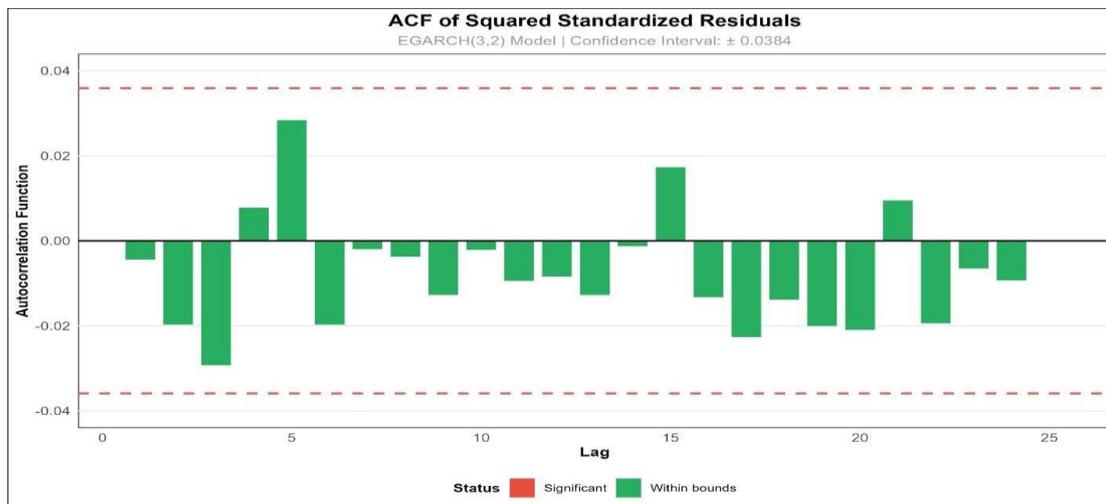
To validate the adequacy of the selected EGARCH (3,2) model, we a comprehensive diagnostic test was conducted on the standardized residuals.

**Figure 1** Time series plot of standardized residuals



From Figure 1, The residuals were seen to fluctuate randomly around zero with no clear pattern, indicating that the model effectively removed most autocorrelation from the price data. However, occasional large spikes suggest the presence of extreme values not fully captured by the model, warranting additional diagnostic tests like autocorrelation and normality checks to confirm its adequacy

**Figure 2:** ACF of Squared Standardized Residuals



From figure 2, the plot indicated that the squared standardized residuals exhibited no significant autocorrelation for all lags. The absence of autocorrelation in the squared residuals confirms that the model has effectively captured the volatility dynamics.

**The Ljung-Box Test on standardized and squared standardized residuals**

The Ljung-Box Test, which is an empirical test to determine if there is significant autocorrelation or dependence in the data given was also performed.

Null Hypothesis  $H_0$ : The data are independently distributed (no significant autocorrelation in the residuals up to lag  $m$ ).

Alternative Hypothesis  $H_1$ : The data are not independently distributed (there is significant autocorrelation in the residuals up to lag  $m$ ). Decision Rule ( $\alpha = 0.05$ ):

**Table 5:** Ljung-Box Q-Test Results

LAGS	Q-STATISTIC (Standardized residuals)	P-VALUE	Q-STATISITC (Squared Standardized residuals)	P-VALUE
10	35.40	0.0001	4.80	0.9039
15	38.74	0.0007	5.91	0.9812
20	44.01	0.0015	8.36	0.9892

From Table 5, The standardized residuals showed evidence of some autocorrelation at all lags, however, for the squared standardized residuals there was no evidence of autocorrelation. This implied that the mean equation has some room for improvement but the conditional variance has been completely captured by the model.

**ARCH-LM test on Squared standardized Residuals**

The ARCH-LM test on the squared standardized residuals was also done to validate whether the model was sufficient in capturing all ARCH effects.

Null Hypothesis ( $H_0$ ): There is no ARCH effect

Alternative Hypothesis ( $H_1$ ): There is an ARCH effect Decision Rule ( $\alpha = 0.05$ ):

**Table 6** ARCH-LM test on squared standardized residuals

LAG	LM-STATISTIC	p-VALUE	INTEPRETATION
1	0.0448	0.8324	No ARCH effects
5	9.1606	0.1028	No ARCH effects
10	10.0131	0.4393	No ARCH effects
15	11.1102	0.7447	No ARCH effects
20	14.2708	0.8165	No ARCH effects

From Table 6, the test on squared standardized residuals showed no significant autocorrelation ( $p$ -values  $> 0.05$ ), demonstrating that the EGARCH (3,2) model had successfully captured all volatility clustering and heteroskedastic patterns. Most importantly, the ARCH-LM tests confirmed the absence of remaining ARCH effects (all  $p$ -values  $> 0.05$ ), validating that the model had fully captured Bitcoin's volatility dynamics. The

failure to reject the null hypothesis in volatility-related tests indicated that the EGARCH (3,2) specification was well-suited for modeling Bitcoin's conditional variance, with only minor mean equation adjustments potentially needed.

**Normality tests**

Normality test checks are done to check whether the residuals come from a normal distribution.

For the Jarque–Bera (JB) Test:

Null Hypothesis  $H_0$ : Data is normally distributed.

Alternative Hypothesis  $H_1$ : Data is not normally distributed.

Decision Rule ( $\alpha = 0.05$ ):

For the Shapiro–Wilk Test,

Null Hypothesis  $H_0$ : Standardized residuals come from a normal distribution.

Alternative Hypothesis  $H_1$ : Standardized residuals do not come from a normal distribution.

Decision Rule ( $\alpha = 0.05$ ):

**Table 7.** Normality test of Standardized residuals

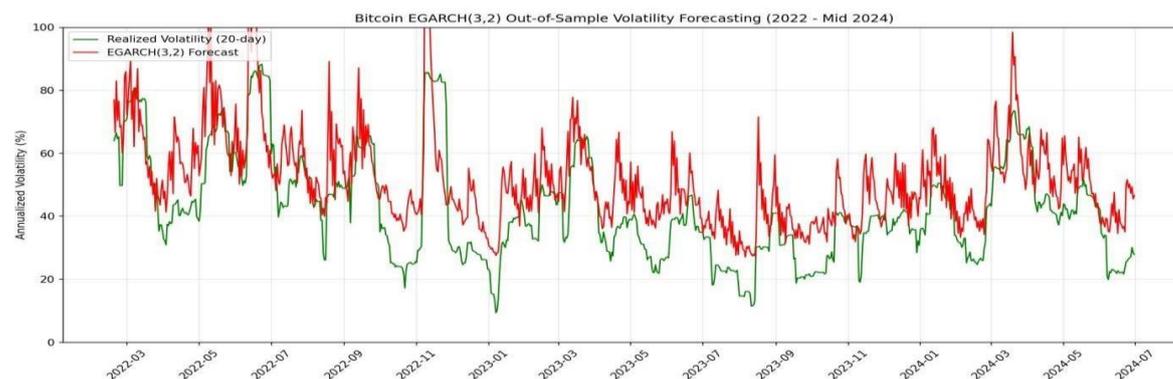
TEST	STATISTIC	P-VALUE
Jarque-Bera	187.34	0.001<
Shapiro-Wilk	0.96	0.001<

From Table 7, The normality tests strongly reject the null hypothesis of normally distributed standardized residuals, as both the Shapiro Wilk test and the Jarque-Berra tests have p-values < 0.01. This finding is consistent with the stylized facts of financial data, which often exhibit fat tails even after accounting for time-varying volatility.

**Volatility Forecasting and Practical Applications**

In this part of the analysis, the selected model EGARCH (3,2) was used to forecast the volatility of Bitcoin’s price to see if it has good predictive value.

**Figure 3:** Forecasted vs. Realized Volatility on out-of-sample data



From figure 3, It can be seen that forecasts track the general pattern of realized volatility reasonably well, capturing major volatility spikes and calm periods. However, the model tends to underestimate extreme volatility events and may overestimate volatility during calm periods.

**Forecast of Bitcoin’s percentage volatility**

Bitcoin’s percentage volatility was forecasted with the chosen model across different time horizons and presented

**Table 8.** Forecast of the percentage volatility of Bitcoin’s price across multiple Horizons.

HORIZON/DAYS	MSE	RMSE	MAE	ACCURACY
1	4.23	2.06	1.58	72.4%
5	8.47	2.91	2.24	58.7%
10	12.85	3.58	2.87	51.2%
30	19.42	4.41	3.65	36.5%

From Table 8, the forecast accuracy started high and deteriorated as the time horizon increased, which is expected for volatility forecasting.

**Trading strategy Implementation**

To assess the practical utility of the EGARCH (3,2) volatility forecasts, we implemented and back tested three volatility-based trading strategies:

1. Volatility-Based Position Sizing Strategy
2. Volatility Threshold Strategy
3. Value-at-Risk (VaR) Based Position Sizing Strategy

**Trading Strategy Performance**

The performance of the various trading strategies as compared to the buy and hold strategy as explored in this section and summarized below:

**Table 9:** Trading Strategy Performance Metrics (2017-2023)

METRIC	BUY AND HOLD	VOLATILITY-BASED POSITION SIZING	VOLATILITY THRESHOLD STRATEGY	VALUE AT RISK BASED POSITION SIZING
Annualized Return (%)	42.87	31.24	28.76	32.18
Annualized	78.59	42.31	39.87	43.65

Volatility (%)				
Sharpe Ratio	0.55	0.74	0.72	0.74
Maximum Drawdown (%)	72.34	43.21	41.87	44.32
Calmar Ratio	0.59	0.72	0.69	0.73
Win Rate (%)	53.21	54.87	52.34	55.12

Table 9, presents the performance comparison of volatility-based trading strategies relative to the buy-and-hold benchmark. The EGARCH (3,2) model’s strong ability to capture Bitcoin’s asymmetric and persistent volatility makes it well suited for practical trading applications. Its conditional volatility forecasts were incorporated into three dynamic position-sizing frameworks each designed to manage risk adaptively according to forecasted market conditions. The volatility-based strategy achieved an annualized return of 31.24%, annualized volatility of 42.31%, and a Sharpe ratio of 0.74, demonstrating a notable improvement in risk-adjusted performance. The volatility-threshold approach, which categorizes markets into volatility regimes, recorded the lowest volatility (39.87%) and drawdown (41.87%), making it ideal for risk-averse investors, while the VaR-based strategy produced the highest annualized return (32.18%) and a Calmar ratio of 0.73, indicating strong downside risk control. Across all three strategies, volatility and drawdowns were reduced by approximately 46–49% and 40–42%, respectively, relative to buy-and-hold, with Sharpe ratios improving to 0.72–0.74. Overall, the findings highlight that volatility forecasts derived from the EGARCH (3,2) model can meaningfully enhance portfolio performance and stability, underscoring the practical value of advanced volatility modeling for cryptocurrency investment and risk management.

## CONCLUSION

The empirical analysis confirms that Bitcoin exhibits significant volatility clustering, characterized by periods of high price fluctuations followed by sustained volatility. This behavior validates the suitability of GARCH-family models for modeling the conditional variance of Bitcoin’s price data. Among the competing models, the Exponential GARCH (EGARCH) model demonstrates superior performance in capturing asymmetric volatility responses, reflecting the differing impacts of positive and negative market shocks that are typical of cryptocurrency trading environments. Specifically, the EGARCH (3,2) model yields the best in-sample fit and out-of-sample forecast accuracy, indicating its robustness in modeling Bitcoin’s complex volatility dynamics.

The study’s application of the EGARCH model within volatility-based trading strategies reveals its practical relevance, as it improves risk-adjusted returns compared to the traditional buy-and-hold approach. This finding suggests that incorporating volatility forecasting into trading and portfolio management can significantly improve performance in the highly volatile cryptocurrency market.

In summary, the results demonstrate the importance of using asymmetric volatility models when analyzing cryptocurrency markets, where nonlinear and shock-sensitive behaviors prevail. Future research may extend this analysis by incorporating high-frequency data, exploring multivariate volatility models, or applying deep learning augmented GARCH frameworks to further enhance predictive accuracy and trading efficiency.

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