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A Study on $\hat{\mu}\beta$ Connectedness in Bitopological Spaces

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ABSTRACT

The objective of this paper is to study a special case of connectedness in bitopological spaces by considering $\tau_1\tau_2$ $\hat{\mu}\beta$ open sets and examining their relationships with $\tau_1\tau_2$□□connected space and $\tau_1\tau_2$ pre-connected space.

Key words: $\tau_1\tau_2$ $\hat{\mu}\beta$ open set , $\tau_1\tau_2\hat{\mu}\beta$ connected space .

INTRODUCTION

The study of bitopological spaces was initiated by Kelly, J .C [5] . A triple (X, τ_1, τ_2) □is called bitopological space if (X , τ_1) and (X , τ_1) □are two topological spaces . In 1997, Kumar Sampath S [6] introduced the concept of $\tau_1\tau_2$ - □□-open sets in bitopological spaces. In 1981, Bose S [1] introduced the notion of $\tau_1\tau_2$ - semi - open sets in bitopological spaces . In 1992, Kar A [4] have introduced the notion of $\tau_1\tau_2$ - pre- open sets in bitopological spaces . In 2012 , H . I Al-Rubaye Qaye [2] introduced the notion of $\tau_1\tau_2$ - semi -□□-open sets in bitopological spaces . In this paper, we study a special case of $\hat{\mu}\beta$ connectedness in bitopological spaces, and we prove several results by comparing them with similar cases in topological spaces.

Preliminaries

Throughout the paper, spaces always mean a bitopological spaces , the closure and the interior of any subset A of X with respect to τ_i□, will be denoted by $\tau_i cl$□A□, and $\tau_i$int□A□ respectively, for

i □1,2.

Definition 2.1 :

Let (X, τ_1, τ_2) □be a bitopological space , A□□X , A is said to be :

(i) $\tau_1\tau_2$ pre-open set [4] if A□ $\tau_1$int($\tau_2 cl(A)$).

(ii) $\tau_1\tau_2\hat{\mu}\beta$ closed set [1] if τ_2 $\hat{\mu}cl(A)$□□U , whenever A□□U, U is β open in τ_1.

(iii) $\tau_1\tau_2$□□open set [6] if A□ $\tau_1$int($\tau_2 cl(\tau_1$int(A))).

Remark 2.2 :

The family of $\tau_1\tau_2$pre-open (resp. $\tau_1\tau_2\hat{\mu}\beta$ open) sets of X is denoted by

$\tau_1\tau_2$ $PO(X)$ (resp. $\tau_1\tau_2$ $\hat{\mu}\beta O(X)$).

Example 2.3 :

Let X □{a, b, c, d}, □□ □□□X, □□□a},{b},{a, b},{a, b, c}} , and □□□□□□X,□□□a},{c},{a,c},{a,b,c}□ is a bitopological space . The family of all $\tau_1\tau_2\hat{\mu}\beta$ open sets of X is : $\tau_1\tau_2$ $\hat{\mu}\beta$O(X) = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}.

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Remark 2.4 :

It is clear by definition that in any bitopological space the following hold :

(i) Every τ_1 open set is $\tau_1\tau_2$ $\hat{\mu}\beta$ open , $\tau_1\tau_2$semi -open , $\tau_1\tau_2$ □□open set .

(ii) Every $\tau_1\tau_2$□□-open set is $\tau_1\tau_2$pre-open , $\tau_1\tau_2$semi open set .

(iii) The concept of $\tau_1\tau_2$pre-open and $\tau_1\tau_2$semi open sets are independent .

Proposition 2.5 :

A subset *A* of a bitopological space (X, τ_1, τ_2)□is $\tau_1\tau_2$□□open set if and only if there exists

an τ_1 open set□*U* , such that *U* □□*A* □ $\tau_1$int($\tau_2 cl(U)$) .

Proof :

This follows directly from the definition (2.1) (iii).

Proposition 2.6 : [7]

A subset *A* of a bitopological space (X, τ_1, τ_2)□is $\tau_1\tau_2$semi open set if and only if there

exists an $\tau_1$open set □*U*, such that *U* □□*A* □□$\tau_2 cl(U)$.

Proposition 2.7 : [3]

A subset *A* of a bitopological space (X, τ_1, τ_2)□is $\tau_1\tau_2$pre open set if and only if there exists

an τ_1 open set □*U* , such that *A*□*U* □□$\tau_2 cl(A)$.

Theorem 2.8 :

A subset *A* of a bitopological space (X, τ_1, τ_2)□is an $\tau_1\tau_2$□□open set if and only if *A* is $\tau_1\tau_2$semi open set and $\tau_1\tau_2$ pre open set .

Proof :

Follows from definition (2.1) and remark (2.5) .

Definition 2.9 : [4,6]

Let (X, τ_1, τ_2)□□be a bitopological space and *A*□□*X* ,the intersection of all $\tau_1\tau_2$□□ closed

(resp . $\tau_1\tau_2$pre closed) sets containing *A* is called $\tau_1\tau_2$□□ closure (resp . $\tau_1\tau_2$pre closure)of *A* , and is denoted by $\tau_1\tau_2$□$cl(A)$ (resp . $\tau_1\tau_2$ p$cl(A)$) ;

i.e $\tau_1\tau_2$□$cl(A)$ □□□{*B* □□*X* : *B* is $\tau_1\tau_2$□□ closed set , *A*□□*X*} and

$\tau_1\tau_2$p$cl(A)$ □□□{*B* □□*X* : *B* is $\tau_1\tau_2$pre closed set , *A*□□*X*} .

Remark 2.10 : [2]

(i) Every τ_1 open set □is $\tau_1\tau_2\hat{\mu}\beta$□-open set, but the converse need not be true.

(ii) If every τ_1 open set □is τ_1□closed □and every nowhere τ_1□dense set □is τ_1□closed □in any

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bitopological space , then every $\tau_1\tau_2\hat{\mu}\beta$ open set is an τ_1 open set.

Remark 2.11 : [2]

(i) Every $\tau_1\tau_2$□□open set is $\tau_1\tau_2\hat{\mu}\beta$ open set, but the converse is not true in general .

(ii) If every □τ_1 open set □is τ_1□closed □set in any bitopological space, then every $\tau_1\tau_2\hat{\mu}\beta$ open set is an $\tau_1\tau_2$□□open set .

Remark 2.12 : [2]

The concepts of $\tau_1\tau_2\hat{\mu}\beta$-open and $\tau_1\tau_2$ pre open sets are independent , as the following

example.

Example 2.13:

In example (2.3) , $\{b,c\}$is a $\tau_1\tau_2$- pre-open set but not $\tau_1\tau_2$ - semi -□□-open set .

Remark 2.14 : [2]

(i) It is clear that every $\tau_1\tau_2$ semi open and $\tau_1\tau_2$ pre open subsets of any bitopological space is

$\tau_1\tau_2\hat{\mu}\beta$ open set

(ii) An $\tau_1\tau_2\hat{\mu}\beta$ open set in any bitopological space (X, τ_1, τ_2)□is $\tau_1\tau_2$ pre open set if every τ_1 open subset □of X is τ_1□closed □set (from remark (2.17) (ii) and remark (2.5) (iii)) .

Definition 2.15 : [2]

The complement of $\tau_1\tau_2\hat{\mu}\beta$ open set is called $\tau_1\tau_2\hat{\mu}\beta$ closed set . Then family of all $\tau_1\tau_2\hat{\mu}\beta$ closed sets of X is denoted by $\tau_1\tau_2\hat{\mu}\beta C(X)$.

Definition 2.16 : [2]

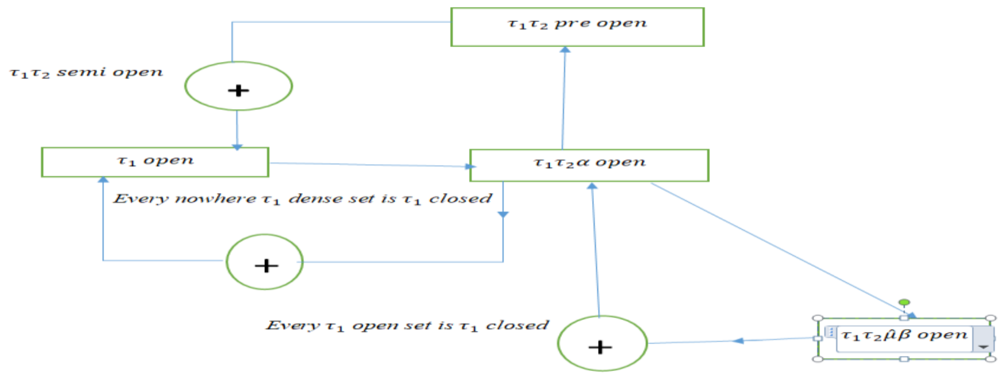
Let (X, τ_1, τ_2)□be a bitopological space and A□□X ,the intersection of all $\tau_1\tau_2\hat{\mu}\beta$ closed sets containing A is called $\tau_1\tau_2\hat{\mu}\beta$ closure of A , and is denoted by $\tau_1\tau_2\hat{\mu}\beta cl(A)$□;

i.e $\tau_1\tau_2\hat{\mu}\beta cl(A)$ □□□$\{B$ □□X : B is $\tau_1\tau_2\hat{\mu}\beta$ closed set , A□□$X\}$.

Remark 2.17 : [2]

The following diagram shows the relations among the different types of weakly open sets

that were studied in this section:



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$\tau_1\tau_2\hat{\mu}\beta$ Connectedness in Bitopological Spaces :

In this section the notion of $\tau_1\tau_2\hat{\mu}\beta$ connected space is introduced in bitopological spaces and their relationships with $\tau_1\tau_2$□□ connected space and $\tau_1\tau_2$ pre connected space are studied.

Definition 3.1 :

Let (X, τ_1, τ_2)□□be a bitopological space , two non-empty subsets *A* and *B* of *X* are said to be $\tau_1\tau_2\hat{\mu}\beta$ separated if □*A* □$\tau_1\tau_2\hat{\mu}\beta cl(B)$ □□□and $\tau_1\tau_2\hat{\mu}\beta cl(A)$ □□*B* □□□.

Definition 3.2 :

A bitopological space (X, τ_1, τ_2)□is called $\tau_1\tau_2\hat{\mu}\beta$ connected if it is not the union of two non empty $\tau_1\tau_2\hat{\mu}\beta$ separated $\tau_1\tau_2\hat{\mu}\beta$ opensets .

A subset *B* □□*X* is $\tau_1\tau_2\hat{\mu}\beta$ connected if it is $\tau_1\tau_2\hat{\mu}\beta$ connected as a subspace of *X*.

An $\tau_1\tau_2\hat{\mu}\beta$ disconnection of *X* is a pair of complement, non-empty, $\tau_1\tau_2\hat{\mu}\beta$ open $\tau_1\tau_2\hat{\mu}\beta$ closed

subsets.

Remark 3.3 :

The only $\tau_1\tau_2$ $\hat{\mu}\beta$ open $\tau_1\tau_2\hat{\mu}\beta$ closed subsets in $\tau_1\tau_2\hat{\mu}\beta$ connected space *X* are *X* and □□.

Remark 3.4 :

Every $\tau_1\tau_2\hat{\mu}\beta$ connected space is τ_1 connected, but the converse is not true.

Proof :

Suppose that *X* is not τ_1 connected, then there exist an □two non-empty *A* , *B* are τ_1□open such that □*A* □□*B* □□□and *A* □□*B* □□*X* . we have *A* , *B* are $\tau_1\tau_2\hat{\mu}\beta$ open sets , *A* □□*B* □□*X* and

A □□*B* □□□, hence *X* is not $\tau_1\tau_2\hat{\mu}\beta$ connected which is a contradiction. Thus, *X* is τ_1 connected.

But the converse is not true as in the following example.

Example 3.5 :

Let *X* □{*a*, *b*, *c*, d}, □□ □□□*X*, □□□a},{b},{a, b},{a, b, c}} , and □□□□□□*X*,□□□a},{c},{a,c},{a,b,c}□ is a bitopological space . The family of all $\tau_1\tau_2\hat{\mu}\beta$ open sets of *X* is : $\tau_1\tau_2$ $\hat{\mu}\beta$O(X) = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}.

Then *X* is □□ connected space, but *X* is not $\tau_1\tau_2\hat{\mu}\beta$ connected.

Remark 3.6 :

If every τ_1 open set□□is τ_1 closed □and every nowhere τ_1□dense set □is τ_1□closed □in any

bitopological space , then every τ_1 connected space is $\tau_1\tau_2\hat{\mu}\beta$ connected .

Proof :

Follows from remark (2.16) (ii) .

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Definition 3.7 :

A function f : (X, τ_1, τ_2)□□□ (X, σ_1, σ_2)□□is called $\tau_1\tau_2\hat{\mu}\beta$ open if for each τ_1 open set *U* of *X* ,

f (*U*) is $\tau_1\tau_2\hat{\mu}\beta$ open in *Y*.

Definition 3.8 :

A function f : (X, τ_1, τ_2)□□□ (X, σ_1, σ_2)□is called $\tau_1\tau_2\hat{\mu}\beta$ continuous if and only if the inverse image of each τ_1□open subset of *Y* is $\tau_1\tau_2\hat{\mu}\beta$ open subset of *X*.

Proposition 3.9 :

Every τ_1□continuous function is $\tau_1\tau_2\hat{\mu}\beta$ continuous.

Proof :

Follows from remark (2.16) (i) .

Definition 3.10 :

A function f : (X, τ_1, τ_2)□□□ (X, σ_1, σ_2)□□is called $\tau_1\tau_2\hat{\mu}\beta$ irresolute if and only if the inverse image of each $\tau_1\tau_2\hat{\mu}\beta$ open subset of *Y* is $\tau_1\tau_2\hat{\mu}\beta$ open subset of *X* .

Proposition 3.11 :

Every $\tau_1\tau_2\hat{\mu}\beta$ irresolute function is $\tau_1\tau_2\hat{\mu}\beta$ continuous.

Proof :

Let *A* be any σ_1 open set □in *Y*. Then we have *A* is an open set in $\tau_1\tau_2\hat{\mu}\beta$ open in Y [from remark (2.16) (i)] . Since *f* is $\tau_1\tau_2\hat{\mu}\beta$ irresolute function, then f^{-1}(A) □is $\tau_1\tau_2\hat{\mu}\beta$ open set in *X* . Therefore *f* is $\tau_1\tau_2\hat{\mu}\beta$ continuous.
□

Proposition 3.12 :

Let f : (X, τ_1, τ_2)□□□ (X, σ_1, σ_2)□□be two bitopological spaces . If *X* is $\tau_1\tau_2\hat{\mu}\beta$ connected and *f* is $\tau_1\tau_2\hat{\mu}\beta$ continuous function from □(X, τ_1, τ_2) onto (X, σ_1, σ_2)□, then *Y* is τ_1 connected.

Proof :

Suppose that *A* is an τ_1□open τ_1□closed subset of *Y*, then f^{-1}(A) □is $\tau_1\tau_2\hat{\mu}\beta$ open $\tau_1\tau_2\hat{\mu}\beta$ closed in *X* . Hence f^{-1}(A)□□is □□or *X* , but *X* is $\tau_1\tau_2\hat{\mu}\beta$ connected . So *A* is □□or *Y*. Hence *Y* is τ_1 connected.

Proposition 3.13 :

An $\tau_1\tau_2\hat{\mu}\beta$ irresolute image of any $\tau_1\tau_2\hat{\mu}\beta$ connected bitopological space is $\tau_1\tau_2\hat{\mu}\beta$ connected.

Proof :

Follows directly from proposition (3.12) .

Definition 3.14 :

Let (X, τ_1, τ_2)□be a bitopological space , two non-empty subsets *A* and *B* of *X* are said to

be $\tau_1\tau_2$□□separated (resp. $\tau_1\tau_2$ pre separated) if *A* □$\tau_1\tau_2$□*cl*(*B*) □□□(resp. *A* □$\tau_1\tau_2 pcl(B)$ □□□□)

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and $\tau_1\tau_2$□$cl(A)$ □□B □□□□(resp . $\tau_1\tau_2$ p$cl(A)$ □□B □□□□) .

Definition 3.15 :

A bitopological space (X, τ_1, τ_2)□is called $\tau_1\tau_2$□□ connected (resp. $\tau_1\tau_2$ pre connected) space

if it is not the union of two non-empty $\tau_1\tau_2$□□ separated (resp. $\tau_1\tau_2$ pre separated) $\tau_1\tau_2$□□ open

(resp. $\tau_1\tau_2$ pre open) sets.

Proposition 3.16 :

Every $\tau_1\tau_2$ pre connected space is $\tau_1\tau_2$□□ connected.

Proof :

Follows from remark (2.5) (ii).

Proposition 3.17 :

If every $\tau_1\tau_2$ pre open set in a bitopological space (X, τ_1, τ_2)□is $\tau_1\tau_2$ semi open set , then X is

$\tau_1\tau_2$ pre connected space , whenever it is an $\tau_1\tau_2$□□connected space .

Proof :

Follows from theorem (2.9).

Proposition 3.18 :

Every $\tau_1\tau_2$ pre connected space is τ_1 connected.

Proof :

Follows from remark (2.5) (i).

Remark 3.19 :

Every $\tau_1\tau_2\hat{\mu}\beta$ connected space is $\tau_1\tau_2$□□ connected.

Proof :

Follows from remark (2.17) (i).

Proposition 3.20 :

In a bitopological space (X, τ_1, τ_2) if every τ_1□open subset □of X is τ_1□closed □set , then X is $\tau_1\tau_2\hat{\mu}\beta$ connected space , whenever it is an $\tau_1\tau_2$□□ connected space.

Proof : Follows from remark (2.17) (ii).

Remark 3.21 :

The concepts of $\tau_1\tau_2$ pre connected space and $\tau_1\tau_2\hat{\mu}\beta$ connected space are independent.

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Proposition 3.22 :

If every τ_1 open set in a bitopological space (X, τ_1, τ_2) is τ_1 closed, then X is $\tau_1\tau_2\hat{\mu}\beta$ connected , whenever it is $\tau_1\tau_2$ pre connected.

Proof :

Follows from propositions (3.16) and (3.20).

Proposition 3.23 :

If every $\tau_1\tau_2$ pre open set in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ semi open set , then X is

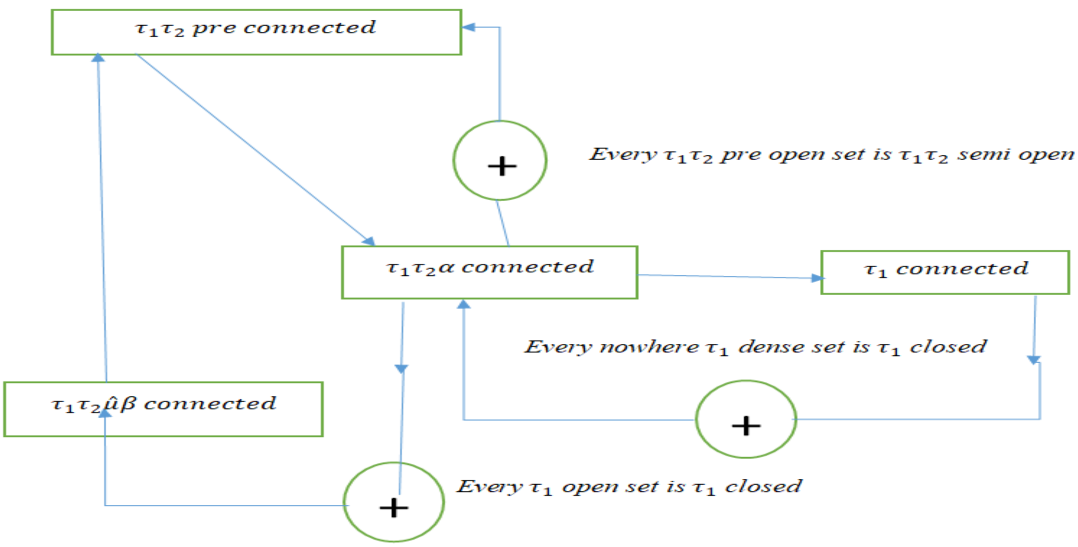
$\tau_1\tau_2$ pre connected space, whenever it is an $\tau_1\tau_2\hat{\mu}\beta$ connected space.

Proof :

Follows from remark (3.19) and proposition (3.20).

Remark 3.24 :

The following diagram shows the relations among the different types of connectedness:



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