Impact of the Peclet Number on Numerical Solutions of Modified Convection-Diffusion Equations Using Berger's Equation

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ABSTRACT:

The study investigates the influence of the Peclet number on the numerical solutions of modified convection-diffusion equations, specifically utilizing Berger's equation as the governing model. Finite difference methods has been employed to solve Convection diffusion equation under different Peclet numbers, analyzing the resulting numerical behavior, including solution profiles, error propagation, and computational efficiency. The findings reveal that as the Peclet number increases, the dominance of convective terms introduces numerical instabilities, such as oscillations and excessive diffusion, necessitating the implementation of specialized discretization techniques or stabilization methods. The study also explores the effectiveness of upwind schemes and adaptive mesh refinement in mitigating these challenges.

Keywords: Peclet number, numerical solution, modified Convection-Diffusion Equation, Burger’s Equation, Finite Difference Schemes, and Stability Conditions.

INTRODUCTION:

The study of convection-diffusion equations is fundamental in various fields of science and engineering, including fluid dynamics, heat transfer, and environmental modeling. These equations describe the transport of physical quantities, such as mass, heat, or momentum, due to the combined effects of convection (advection) and diffusion. The Peclet number (Pe), a dimensionless parameter, plays a critical role in determining the relative importance of these two processes. It is defined as the ratio of convective to diffusive transport rates and is given by:

$$Pe = \frac{Convection\ transport\ rate}{Diffusion\ rate} = \frac{uL}{D}$$

where u is the characteristic velocity, L is the characteristic length scale, and D is the diffusivity. High Peclet numbers indicate dominance of convection, while low Peclet numbers signify diffusion-dominated regimes.

In many practical applications, the standard convection-diffusion equation is modified to account for additional physical phenomena, such as source terms, nonlinearities, or boundary layer effects. Berger's equation, a specific form of the modified convection-diffusion equation, has been widely used to model such scenarios. However, the numerical solution of these equations is often challenging, particularly when the Peclet number is large, as it can lead to numerical instabilities, oscillations, or excessive diffusion if not handled properly.

The impact of the Peclet number on numerical solutions has been extensively studied in the context of standard convection-diffusion equations [1], [2], [3]. However, its influence on modified equations, such as Berger's equation, remains less explored. Understanding this relationship is crucial for developing robust numerical methods that can accurately capture the physical behavior of the system across a wide range of Peclet numbers.

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Many researchers have already been working on it.

Changjun and Shuwen [3] made a numerical simulation on river water pollution by using grey differential model. They corrected the model in finding the truncation error and found that the obtained results from the grey model are excellent and reasonable.

Atul Kumar, Dilip Kumar Jaiswal and Naveen Kumar [5] presented an analytical solution of the one-dimensional ADE by reducing the original ADE into a diffusion equation by using Laplace transformation technique. Mehdi Dehghan [6] presented the solution of the one-dimensional convection diffusion equation with constant coefficient by using several finite difference schemes.

This work investigates the effect of the Peclet number on the numerical solutions of modified convection-diffusion equations using Berger's equation as a case study. By analyzing the performance of various numerical schemes under different Peclet regimes, we aim to provide insights into the challenges and strategies for achieving accurate and stable solutions. The findings are expected to contribute to the development of more efficient numerical methods for solving complex convection-diffusion problems in engineering and scientific applications.

Governing Equation and Numerical Schemes

Governing Equation

In this paper, we consider variable advection velocity u(t, x), so that the PDE reads as convection-diffusion equation (CDE) $\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}$ where, we have two unknowns c(t, x) and u(t, x). Therefore, we have to solve another equation and we select the viscous Burger's equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$ to compute the variable velocity u(t, x). Our problem is thus to solve the following system of PDE's simultaneously as an IBVP

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \qquad a < x < b, \quad t > 0, \tag{1a}$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \qquad a < x < b, \quad t > 0, \tag{1b}$$

Appended with initial condition

$$u(x,0) = f(x); \qquad c(x,0) = f(x) \qquad\qquad a \le x < b$$

and Neumann boundary conditions

$$\frac{\partial}{\partial x}u(t,a) = u_a(t)\,; \qquad \frac{\partial}{\partial x}u(t,b) = u_b(t) \qquad\qquad t_0 \le t \le T$$

$$\frac{\partial}{\partial x}c(t,a) = c_a(t)\,; \qquad \frac{\partial}{\partial x}c(t,b) = c_b(t) \qquad\qquad t_0 \le t \le T$$

where ca, cb, ua, ub are constant concentration values.

Explicit Upwind Difference Scheme (FTBSCS)

We consider our second order system of PDE's simultaneously

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \tag{2a}$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} \tag{2b}$$

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Let the solution $u(x_i, t_n)$ be denoted by U_i^n and its approximate value by u_i^n.

The discretization of $\frac{\partial u}{\partial t}$ is obtained by first order forward difference in time

$$\frac{\partial u}{\partial t} \approx \frac{U_i^{n+1} - U_i^n}{\Delta t} + O(\Delta t)$$

The discretization of $\frac{\partial u}{\partial x}$ is obtained by first order backward difference in space

$$\frac{\partial u}{\partial x} \approx \frac{U_i^n - U_{i-1}^n}{\Delta x} + O(\Delta x)$$

Discretization of $\frac{\partial^2 u}{\partial x^2}$ is obtain from second order centered difference in space

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{U_{i-1}^n - 2U_i^n + U_{i+1}^n}{\Delta x^2} + O(\Delta x^2)$$

The simplest numerical discretization of (2a) is

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{u_i^n - u_{i-1}^n}{\Delta x} = \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2},$$

We get, $u_i^{n+1} = (\gamma + pe)u_{i-1}^n + (1 - \gamma - 2\mathrm{pe})u_i^n + \mathrm{pe}u_{i+1}^n,$ (3a)

$$\text{where,} \gamma = \frac{\Delta t}{\Delta x} u_i^n \,, \qquad r = pe = \frac{\nu \Delta t}{\Delta x^2}$$

Let the solution $c(x_i, t_n)$ be denoted by C_i^n and its approximate value by c_i^n.

The discretization of $\frac{\partial c}{\partial t}$ is obtained by first order forward difference in time

$$\frac{\partial c}{\partial t} \approx \frac{C_i^{n+1} - C_i^n}{\Delta t} + O(\Delta t)$$

The discretization of $\frac{\partial c}{\partial x}$ is obtained by first order backward difference in space

$$\frac{\partial c}{\partial x} \approx \frac{C_i^n - C_{i-1}^n}{\Delta x} + O(\Delta x)$$

Discretization of $\frac{\partial^2 c}{\partial x^2}$ is obtain from second order centered difference in space

$$\frac{\partial^2 c}{\partial x^2} \approx \frac{C_{i-1}^n - 2C_i^n + C_{i+1}^n}{\Delta x^2} + O(\Delta x^2)$$

The simplest numerical discretization of (2b) is

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + u_i^n \frac{c_i^n - c_{i-1}^n}{\Delta x} = D \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta x^2},$$

we get, $c_i^{n+1} = (\gamma + pe)c_{i-1}^n + (1 - \gamma - 2pe\lambda)c_i^n + pec_{i+1}^n,$ (3b)

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$$\text{where,} \gamma = \frac{\Delta t}{\Delta x} u_i^n , \qquad \lambda = pe = \frac{D\Delta t}{\Delta x^2}$$

Which is the explicit upwind difference scheme and it is also known as FTBSCS techniques. The stability condition is controlled by

$$\gamma = \frac{\Delta t}{\Delta x} u_i^n , \; r = pe = \frac{\nu \Delta t}{\Delta x^2} \text{ and } \gamma = \frac{\Delta t}{\Delta x} u_i^n , \; \lambda = pe = \frac{D\Delta t}{\Delta x^2}$$

It is seen that the truncation errors for the forward and backward differences are of first order; whereas the centered difference yields a second order truncation error (using by Taylor Series expansions). Therefore, the scheme outlined above is consistent.

Explicit Centered Difference Scheme (FTCS)

We consider our second order system of PDE's simultaneously

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}, \qquad (4a)$$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} \qquad (4b)$$

Let the solution $u(x_i, t_n)$ be denoted by U_i^n and its approximate value by u_i^n.

The discretization of $\frac{\partial u}{\partial t}$ is obtained by first order forward difference in time

$$\frac{\partial u}{\partial t} \approx \frac{U_i^{n+1} - U_i^n}{\Delta t} + O(\Delta t)$$

The discretization of $\frac{\partial u}{\partial x}$ is obtained by first order centered difference in space

$$\frac{\partial u}{\partial x} \approx \frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} + O(\Delta x^2)$$

Discretization of $\frac{\partial^2 u}{\partial x^2}$ is obtain from second order centered difference in space

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{U_{i-1}^n - 2U_i^n + U_{i+1}^n}{\Delta x^2} + O(\Delta x^2)$$

The simplest numerical discretization of (4a) is

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2},$$

$$\Rightarrow u_i^{n+1} = u_i^n - \frac{\Delta t}{2\Delta x} u_i^n (u_{i+1}^n - u_{i-1}^n) + \frac{\nu \Delta t}{\Delta x^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n),$$

$$\Rightarrow u_i^{n+1} = u_i^n - \frac{\gamma}{2} (u_{i+1}^n - u_{i-1}^n) + r(u_{i+1}^n - 2u_i^n + u_{i-1}^n),$$

$$\text{we get,} u_i^{n+1} = (pe + \frac{\gamma}{2}) u_{i-1}^n + (1 - 2pe) u_i^n + \left(\text{pe} - \frac{\gamma}{2}\right) u_{i+1}^n, \qquad (5a)$$

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$$\text{where, } \gamma = \frac{\Delta t}{\Delta x} u_i^n \,, \qquad r = pe = \frac{\nu \Delta t}{\Delta x^2}$$

Let the solution $c(x_i, t_n)$ be denoted by C_i^n and its approximate value by c_i^n.

The discretization of $\frac{\partial c}{\partial t}$ is obtained by first order forward difference in time

$$\frac{\partial c}{\partial t} \approx \frac{C_i^{n+1} - C_i^n}{\Delta t} + O(\Delta t)$$

The discretization of $\frac{\partial c}{\partial x}$ is obtained by first order centered difference in space

$$\frac{\partial c}{\partial x} \approx \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x} + O(\Delta x^2)$$

Discretization of $\frac{\partial^2 c}{\partial x^2}$ is obtain from second order centered difference in space

$$\frac{\partial^2 c}{\partial x^2} \approx \frac{C_{i-1}^n - 2C_i^n + C_{i+1}^n}{\Delta x^2} + O(\Delta x^2)$$

The simplest numerical discretization of (4b) is

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + u_i^n \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta x} = D \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta x^2},$$

we get, $$c_i^{n+1} = \left(\frac{1}{pe} + \frac{\gamma}{2}\right) c_{i-1}^n + \left(1 - \frac{2}{pe}\right) c_i^n + \left(\frac{1}{pe} - \frac{\gamma}{2}\right) c_{i+1}^n, \qquad (5b)$$

$$\text{where, } \gamma = \frac{\Delta t}{\Delta x} u_i^n \,, \qquad \lambda = pe = \frac{D \Delta t}{\Delta x^2}$$

Which is the explicit centered difference scheme and it is also known as FTCS techniques. The stability condition is controlled by

$$\gamma = \frac{\Delta t}{\Delta x} u_i^n \,,\ r = pe = \frac{\nu \Delta t}{\Delta x^2} \ \text{ and } \ \gamma = \frac{\Delta t}{\Delta x} u_i^n \,,\ \lambda = pe = \frac{D \Delta t}{\Delta x^2}$$

It is seen that the truncation errors for the forward difference is of first order; whereas the centered difference yields a second order truncation error (using by Taylor Series expansions). Therefore, the scheme outlined above is consistent.

Stability analysis of Convection Diffusion equation

We determine stability conditions of CDE for both the schemes as in the following two propositions.

Proposition-01

Statement: The stability conditions of CDE for the FTBSCS scheme are

$$0 \le pe \le 1 \text{ and } -pe \le \frac{\Delta t}{\Delta x} u_i^n \le 1 - 2pe$$

This is guaranteed by the simultaneous inequalities

$$0 \le pe \le 1 \ \text{ and } -\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$$

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Proposition-02

Statement: The stability conditions of CDE for the FTCS scheme are

$$0 \le pe \le \frac{1}{2} \text{ and } -2pe \le \frac{\Delta t}{\Delta x} u_i^n \le 2(1 - pe).$$

This is guaranteed by the conditions $0 \le pe \le \frac{1}{2}$ and $-\frac{2D}{\Delta x} \le \max(u_i^0) \le 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

Numerical Simulation and Results Discussions for CDE

Various finite difference equations can be used to represent the system of PDE's which is convection diffusion equation (1a), (1b). It is extremely important to experiment with the application of these numerical techniques. It is hoped that by writing computer codes and analyzing the results, additional insights into the solution procedures are gained. Therefore, this section proposes an example and presents solutions by the described schemes.

Verification of Stability Conditions of CDE

In this study, we assume that the length of spatial domain, l = 6 meters at all time, t = 1minute to t = 6 minutes with viscosity, ν = 0.01m2/s = 36 m^2/h and diffusion coefficient, D = 0.01m^2/s = 36 m^2/h.

The convection-diffusion equation for this problem is $\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2}$. Various values of spatial nodes size and time steps are to be used to investigate the numerical schemes and the effect of steps on stability.

An attempt is made to solve the stated problem subject to the imposed initial and Neumann boundary conditions by the following:

The FTBSCS and FTCS schemes with

Δx = 0.05   Δt = 0.033 s, nt = 3600, T = 60×2 sec $pe \ll 1$

Δx = 0.05   Δt = 0.067 s, nt = 3600, T = 60×4 sec $pe \ll 1$

Δx = 0.05   Δt = 0.100 s, nt = 3600, T = 60×6 sec $pe < 1$

Δx = 0.05   Δt = 0.1192 s, nt = 3600, T = 60×7.152 sec $pe \approx 1$

Δx = 0.05   Δt = 0.122 s, nt = 3600, T = 60×7.32 sec $Pe \approx 1$

Stability of CDE by FTBSCS and FTCS schemes:

Case I. When the step sizes are Δx = 0.05, Δt = 0.033, $pe \ll 1$

In this case, both the schemes are to be used as stated previously.

The stability conditions of FTBSCS is determined by proposition 3.1-01 as

$0 \le pe \le 1$ and $-pe \le \frac{\Delta t}{\Delta x} u_i^n \le 1 - 2pe$

This is guaranteed by the simultaneous inequalities

$0 \le pe \le 1$ and $-\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$

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The stability conditions of FTCS is determined by proposition 3.2-02 as

$0 \le pe \le \frac{1}{2}$ and $-2pe \le \frac{\Delta t}{\Delta x} u_i^n \le 2(1 - pe)$.

This is guaranteed by the conditions $0 \le pe \le \frac{1}{2}$ and $-\frac{2D}{\Delta x} \le \max(u_i^0) \le 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

where, $pe = \frac{D\Delta t}{\Delta x^2}$.

For this application,

The value of $\max(u_i^0) = 0.02$ which satisfies the guaranteed inequality in both the schemes.

$\gamma = \frac{\Delta t}{\Delta x} \max(u_i^0) = \frac{0.033}{0.05} \times 0.02 = 0.0132$ and $pe = \frac{D\Delta t}{\Delta x^2} = \frac{0.01 \times 0.033}{(0.05)^2} = 0.132$

FTBSCS$\Rightarrow$ $0 \le 0.132 \le 1$ and $-0.132 \le 0.0132 \le 1 - 2 \times 0.132 = 0.736$

and

FTCS$\Rightarrow$ $0 \le 0.132 \le \frac{1}{2}$ and $-0.264 \le 0.0132 \le 1.736$

Therefore, the stability conditions for both the schemes are satisfied and a stable solution is expected when $pe \ll 1$, convection is dominating the process. The concentration profiles are shown in Figure 4.1.

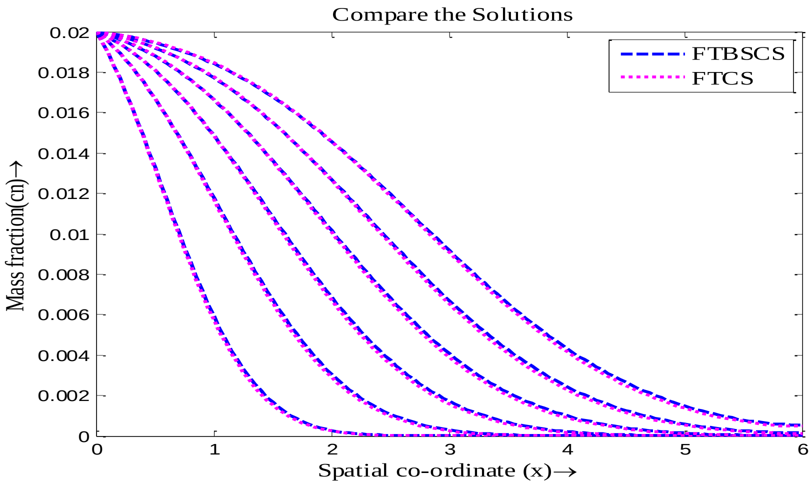

Figure 4.1: Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.033, $pe \ll 1$

Case II. When the step sizes are Δx = 0.05, Δt = 0.067, $pe \ll 1$.

In this case, both the schemes are to be used as stated previously.

The stability conditions of FTBSCS is determined by proposition-01 as

$0 \le pe \le 1$ and $-pe \le \frac{\Delta t}{\Delta x} u_i^n \le 1 - 2pe$

This is guaranteed by the simultaneous inequalities

$0 \le pe \le 1$ and $-\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$

The stability conditions of FTCS is determined by proposition-02 as

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$0 \le pe \le \frac{1}{2}$ and $-2pe \le \frac{\Delta t}{\Delta x} u_i^n \le 2(1 - pe)$.

This is guaranteed by the conditions $0 \le pe \le \frac{1}{2}$ and $-\frac{2D}{\Delta x} \le \max(u_i^0) \le 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

where, $pe = \frac{D\Delta t}{\Delta x^2}$.

For this application,

The value of $\max(u_i^0) = 0.02$ which satisfies the guaranteed inequality in both the schemes.

$\gamma = \frac{\Delta t}{\Delta x} \max(u_i^0) = \frac{0.067}{0.05} \times 0.02 = 0.0268$ and $pe = \frac{D\Delta t}{\Delta x^2} = \frac{0.01 \times 0.067}{(0.05)^2} = 0.268$

FTBSCS $\Rightarrow 0 \le 0.268 \le 1$ and $-0.268 \le 0.0268 \le 1 - 2 \times 0.268 = 0.464$

and

FTCS $\Rightarrow 0 \le 0.268 \le \frac{1}{2}$ and $-0.536 \le 0.0268 \le 1.464$

Therefore, the stability conditions for both the schemes are satisfied and a stable solution is expected. when pe≪1, the convection is dominating the process. The concentration profiles are shown in Figure 4.2.

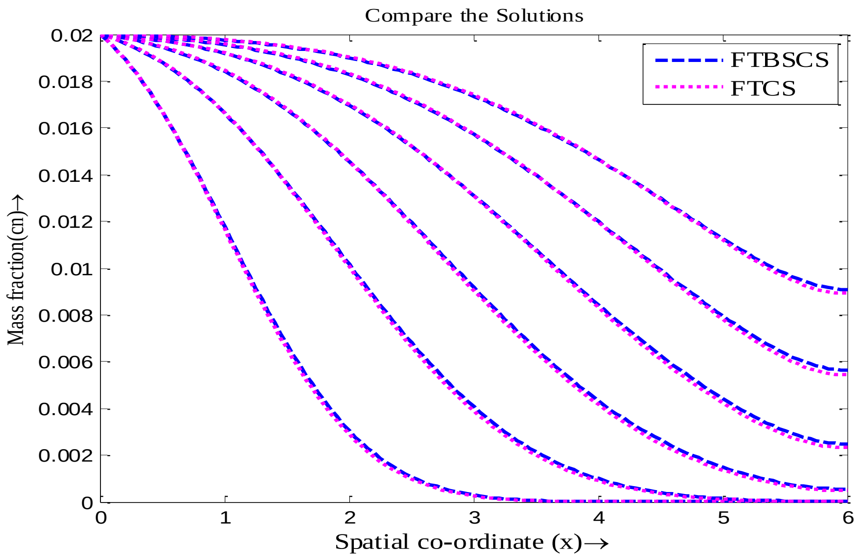

Figure 4.2: Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.067 $pe \ll 1$

Case III. When the step sizes are Δx = 0.05, Δt = 0.1, $pe \ll 1$.

In this case, both the schemes are to be used as stated previously.

The stability conditions of FTBSCS is determined by proposition-01 as

$0 \le pe \le 1$ and $-pe \le \frac{\Delta t}{\Delta x} u_i^n \le 1 - 2pe$

This is guaranteed by the simultaneous inequalities

$0 \le pe \le 1$ and $-\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$

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The stability conditions of FTCS is determined by proposition-02 as

$0 \leq pe \leq \frac{1}{2}$ and $-2pe \leq \frac{\Delta t}{\Delta x} u_i^n \leq 2(1 - pe)$.

This is guaranteed by the conditions $0 \leq pe \leq \frac{1}{2}$ and $-\frac{2D}{\Delta x} \leq \max(u_i^0) \leq 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

where, $pe = \frac{D\Delta t}{\Delta x^2}$.

For this application,

The value of $\max(u_i^0) = 0.02$ which satisfies the guaranteed inequality in both the schemes.

$\gamma = \frac{\Delta t}{\Delta x} \max(u_i^0) = \frac{0.1}{0.05} \times 0.02 = 0.04$ and $pe = \frac{D\Delta t}{\Delta x^2} = \frac{0.01 \times 0.1}{(0.05)^2} = 0.4$

FTBSCS $\Rightarrow$ $0 \leq 0.4 \leq 1$ and $-0.4 \leq 0.04 \leq 1 - 2 \times 0.4 = 0.2$

and

FTCS $\Rightarrow 0 \leq 0.4 \leq \frac{1}{2}$ and $-0.8 \leq 0.04 \leq 1.2$

Therefore, the stability conditions for both the schemes are satisfied and a stable solution is expected where $pe < 1$, the process is balanced convection and diffusion. The concentration profiles are shown in Figure 4.3.

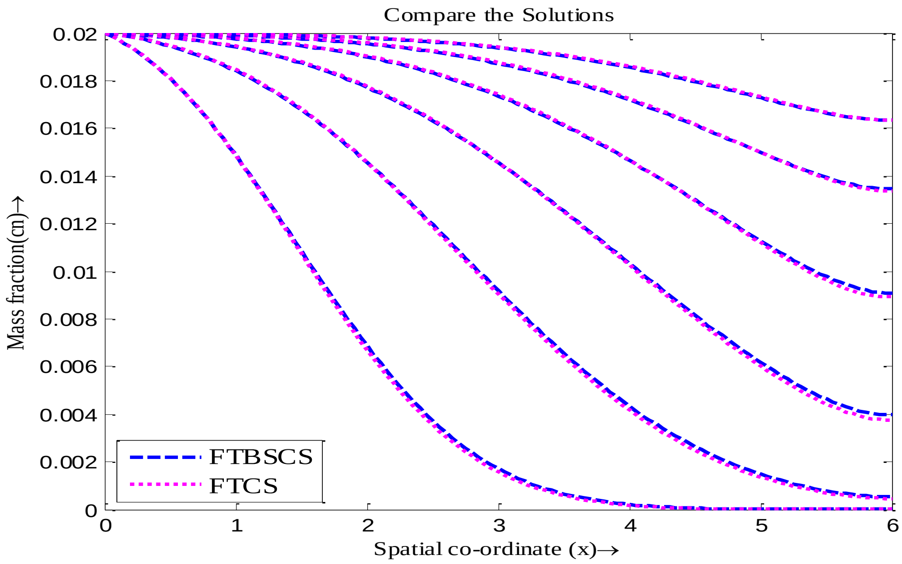

Figure 4.3: Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.1 $pe < 1$

Instability of CDE by FTBSCS and FTCS schemes:

Case IV. When the step sizes are increased to Δx = 0.05, Δt = 0.1192, $pe \approx 1$.

The stability conditions of FTBSCS is determined by proposition-01 as

$$0 \leq pe \leq 1 \text{ and } -pe \leq \frac{\Delta t}{\Delta x} u_i^n \leq 1 - 2pe$$

This is guaranteed by the simultaneous inequalities

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$$0 \le pe \le 1 \text{ and } -\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$$

The stability conditions of FTCS is determined by proposition-02 as

$$0 \le pe \le \frac{1}{2} \text{ and } -2pe \le \frac{\Delta t}{\Delta x} u_i^n \le 2(1-pe) \ .$$

This is guaranteed by the conditions $0 \le pe \le \frac{1}{2}$ and $-\frac{2D}{\Delta x} \le \max(u_i^0) \le 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

$$\text{where, } pe = \frac{D\Delta t}{\Delta x^2}.$$

For this application,

The value of $\max(u_i^0) = 0.02$ which satisfies the guaranteed inequality in both the schemes.

$$\gamma = \frac{\Delta t}{\Delta x}\max(u_i^0) = \frac{0.1192}{0.05} \times 0.02 = 0.04768 \text{ and } pe = \frac{D\Delta t}{\Delta x^2} = \frac{0.01 \times 0.1192}{(0.05)^2} = 0.4768$$

$$\text{FTBSCS} \Rightarrow 0 \le 0.4768 \le 1 \text{ and } -0.4768 \le 0.04768 \le 0.0464,$$

which does not satisfy the stability condition of FTBSCS scheme,

and

$$\text{FTCS} \Rightarrow 0 \le 0.4768 \le \frac{1}{2} \text{ and } -0.9536 \le 0.04768 \le 1.0464$$

In this case, FTBSCS of the CDE shows an instability. The concentration profiles are shown in the following Figure 4.4.

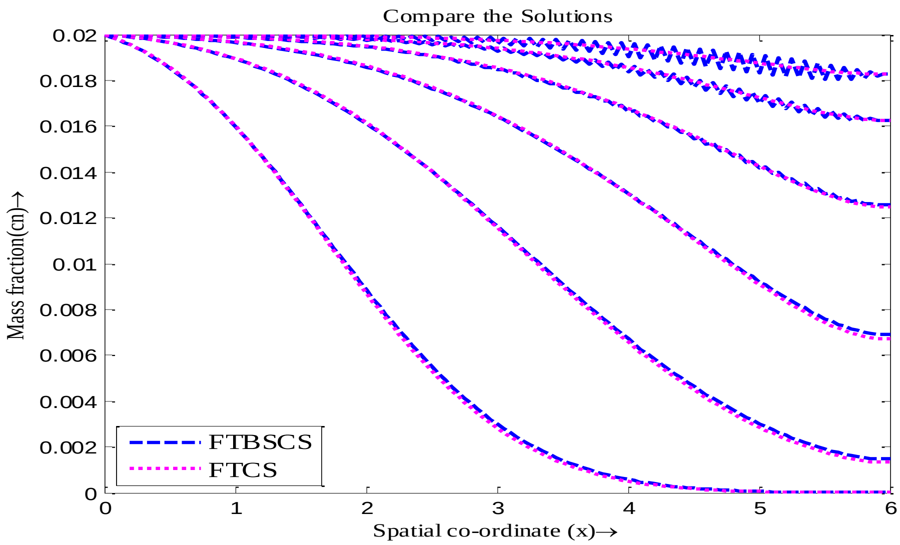

Figure 4.4: Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.1192, $pe \approx 1$

Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.1192 is presented. Here, the stability conditions for the scheme FTBSCS of CDE are not satisfied at Δt = 0.122 whereas the stability conditions for the scheme FTCS are satisfied. Since $pe \approx 1$, the transport is spreading out for the FTBSCS scheme.

Case V. When the step sizes are increased to Δx = 0.05, Δt = 0.122, $Pe \approx 1$ which is only a fraction of an increase over preceding cases.

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The stability conditions of FTBSCS is determined by proposition-01 as

$$0 \le pe \le 1 \text{ and } -pe \le \frac{\Delta t}{\Delta x} u_i^n \le 1 - 2pe$$

This is guaranteed by the simultaneous inequalities

$$0 \le pe \le 1 \text{ and } -\frac{D}{\Delta x} \le \max(u_i^0) \le \frac{\Delta x}{\Delta t} - 2\frac{D}{\Delta x}$$

The stability conditions of FTCS is determined by proposition-02 as

$$0 \le pe \le \frac{1}{2} \text{ and } -2pe \le \frac{\Delta t}{\Delta x} u_i^n \le 2(1 - pe).$$

This is guaranteed by the conditions $0 \le pe \le \frac{1}{2}$ and $-\frac{2D}{\Delta x} \le \max(u_i^0) \le 2\left(\frac{\Delta x}{\Delta t} - \frac{D}{\Delta x}\right)$.

$$\text{where, } pe = \frac{D\Delta t}{\Delta x^2}.$$

For this application,

The value of $\max(u_i^0) = 0.02$ which satisfies the guaranteed inequality in both the schemes.

$$\gamma = \frac{\Delta t}{\Delta x}\max(u_i^0) = \frac{0.122}{0.05} \times 0.02 = 0.0488 \text{ and } pe = \frac{D\Delta t}{\Delta x^2} = \frac{0.01 \times 0.1192}{(0.05)^2} = 0.976$$

$$\text{FTBSCS} \Rightarrow 0 \le 0.976 \le 1 \text{ and } -0.976 \le 0.0488 \le -0.0952,$$

which does not satisfy the stability condition of FTBSCS scheme, and

FTCS $\Rightarrow 0 \le 0.976 \le \frac{1}{2}$ and $-1.952 \le 0.0488 \le 0.024$ which does not satisfy the stability condition of FTCS scheme.

In this case, both FTBSCS and FTCS schemes of the CDE show an instability. The concentration profiles are shown in the following Figure 4.5.

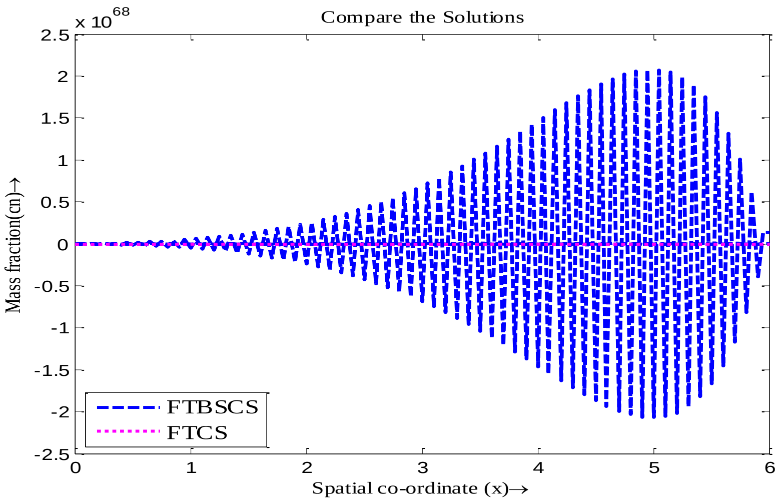

Figure 4.5: Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.122 $pe \approx 1$

Concentration distribution profiles of CDE with Δx = 0.05, Δt = 0.122, $pe \approx 1$ is presented. Here, the stability conditions for both the schemes FTBSCS and FTCS of CDE are not satisfied at Δt = 0.122 and $pe \approx 1$ the transport is spreading out, like blob.

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Table 1 displays the numerical solution of our considered problem for different values of diffusion coefficient, D = 0.002 m2/s and viscosity coefficient, ν = 0.002 m2/s for distance, x = 42 meters and time, t = 3000 seconds and compare the result with another numerical solution obtained by the spline function and finite elements in [6] by using the same values of parameters in the same domain.

Distance x(m)	D = 0.002 m2/s, ν = 0.002 m2/s		
	Proposed methods for CDE		Reference method [6]
	FTBSCS	FTCS	SF-FE [74]
0.0	0.0200	0.0200	1.000
⋮	⋮	⋮	⋮
18.0	0.0200	0.0200	1.000
19.0	0.0199	0.0200	0.999
20.0	0.0199	0.0200	0.998
21.0	0.0198	0.0200	0.996
22.0	0.0196	0.0199	0.990
23.0	0.0193	0.0198	0.978
24.0	0.0188	0.0196	0.957
25.0	0.0181	0.0191	0.922
26.0	0.0172	0.0183	0.870
27.0	0.0160	0.0172	0.799
28.0	0.0146	0.0155	0.708
29.0	0.0129	0.0134	0.602

Distance x(m)	D = 0.002 m2/s, ν = 0.002 m2/s		
	Proposed methods for CDE		Reference method [6]
	FTBSCS	FTCS	SF-FE [74]
30.0	0.0110	0.0110	0.488
31.0	0.0091	0.0085	0.375
32.0	0.0073	0.0062	0.272
33.0	0.0055	0.0041	0.185

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34.0	0.0041	0.0026	0.118
35.0	0.0028	0.0015	0.070
36.0	0.0019	0.0008	0.038
37.0	0.0012	0.0004	0.020
38.0	0.0007	0.0002	0.009
39.0	0.0004	0.0001	0.004
40.0	0.0002	0.0000	0.002
41.0	0.0001	0.0000	0.001
42.0	0.0000	0.0000	0.000

Table 1: Comparison of results for D = 0.002 m2/s, ν = 0.002 m2/s with Δt = 6 s and Δx = 0.25 m.

Selection of Numerical values of parameters for the solution of CDE

For temporal variable t, we consider the domain $t\in(0,6)$ in minute and for spatial variable x, we consider the domain $x\in(0,6)$ in meter. The initial concentration is considered as $c_0(x) = maxc_0 \times e^{-10x}$, where $maxc_0$ is the maximum of the concentration c(t, x).

For u = 0.01 m/s and D = 0.001m2/s at time from 1 minute to 6 minutes, the solution of CDE for the numerical scheme FTBSCS is shown in Figure 5.1, which shows that the concentration distribution within the described domain.

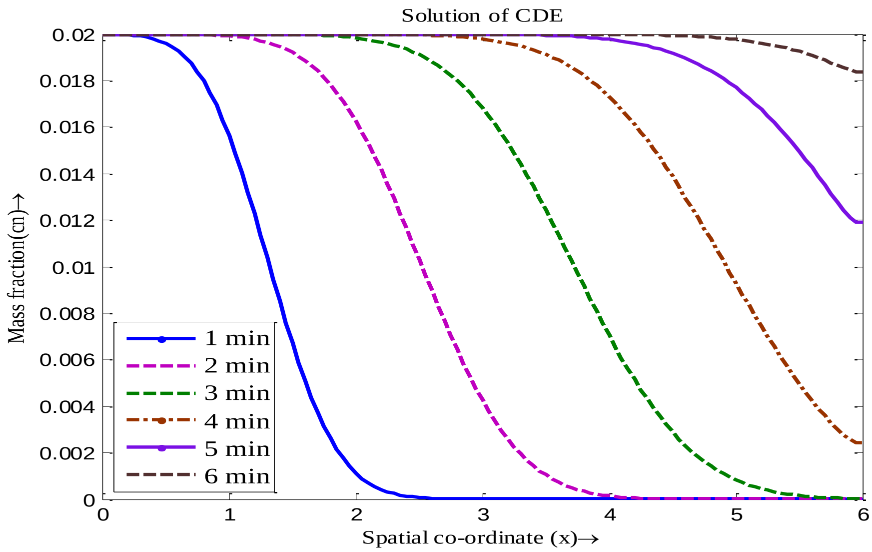

Figure 5.1: Solution of CDE by FTBSCS at different time with ν=0.01 m2/s and D=0.001 m2/s.

For u = 0.01 m/s = 36 m/h and D = 0.01m2/s = 36 m2/h at time from 1 minute to 6 minutes, the solution of CDE for the numerical scheme FTCS is shown in Figure 5.2, which shows that the pollutant distribution within the described domain.

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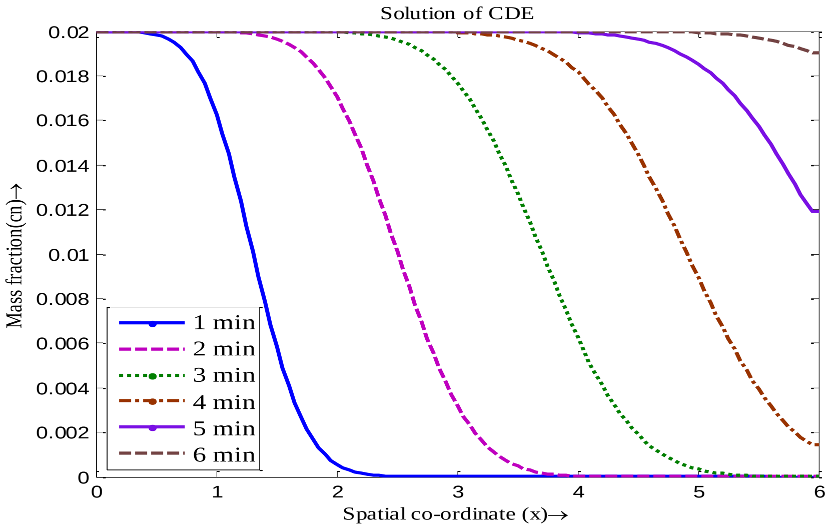

Figure 5.2: Solution of CDE by FTCS at different time with ν=0.01 m2/s and D=0.01 m2/s.

CONCLUSION

In this study, we investigated two finite difference schemes—Forward Time Backward Space Centered Space (FTBSCS) and Forward Time Centered Space (FTCS)—for solving the convection-diffusion equation (CDE) associated with viscous Burgers' equation. We derived the stability conditions for both schemes and conducted a stability analysis to establish constraints on the Peclet number in terms of the time step, spatial step, advection coefficient (u), and diffusion coefficient (D).

Figures 4.1–4.3 demonstrate stable numerical solutions, as the chosen parameters satisfy the derived stability conditions. In contrast, Figures 4.4–4.5 exhibit instability due to violations of these conditions within the prescribed domain. A comparative analysis with an existing numerical solution from [6], presented in Table 1, confirms the accuracy and reliability of our results.

Finally, in Figures 5.1 and 5.2, we illustrate the numerical solutions of the CDE using artificially selected parameter values to further validate the behavior of the schemes. The findings highlight the critical role of stability conditions in ensuring accurate and convergent solutions for convection-diffusion problems.

REFERENCES

1. Roache, P. J. (1972). Computational Fluid Dynamics. Hermosa Publishers.
2. Fletcher, C. A. J. (1991). Computational Techniques for Fluid Dynamics. Springer-Verlag.
3. Berger, M. J. (1984). "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations." Journal of Computational Physics, 53, 484-512.
4. Changjun Zhu and Shuwen Li, "Numerical Simulation of River Water Pollution Using Grey Differential Model," Journal of computers, Vol. No.9, 2010.
5. D.J. Evans, A.R. Abdullah, The group explicit method for the solution of Burger's equation, Computing 32 (1984) :239-253.
6. A. Kumar, D. K. Jaiswal and N. Kumar, Analytical solution of one-dimensional Advection diffusion equation with variable coefficients in a finite domain, J. Earth Syst. Sci. 118, No.5, pp. 539-549, October 2009.