ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue XII, December 2025

CASP-CUSUM Schemes Based on Truncated Gompertz Family of Distribution

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DOI : https://doi.org/10.51583/IJLTEMAS.2025.1412000113

Received: 29 December 2025; Accepted: 03 January 2026; Published: 12 January 2026

ABSTRACT:

acceptance sampling plan was adopted to study mainly for valid conclusions with regard to consideration accept or reject of the finished products. In this way numbers of optimal techniques were developed to increase and control the quality of the products. Basing on the assumption the variable with regard to quality characteristic is distributed accordingly to certain probability law. In our study we optimized CASP-CUSUM Schemes based on the assumption that the continuous variable which is under the consideration follows a Truncated Expoentiated Gompertz distribution utilized in Statistical Quality Control and Reliability analysis. In particular the distribution is meant for estimating the optimal truncated point and probability of acceptance of lot. The operating characteristic and Average run length values are presented. The results are illustrated by figures.

Keywords: CASP-CUSUM Schemes, Optimal Truncated point, Truncated Expoentiated Gompertz Distribution.

INTRODUCTION

Acceptance sampling is an inspecting procedure applied in statistical quality control. Acceptance sampling is a part of operations management and services quality maintenance. It is important for industrial, but also for business purposes helping the decision making a process for the purpose of quality management. Producers are very careful about the quality of their products so that they do not face any difficulty in the acceptance when the consumer comes to buy them.

Acceptance sampling is most likely to be useful in the situations when testing is destructive, or when the cost of 100% inspection is extremely high, or when 100% inspection is not technologically feasible or would require so much calendar time that the production schedule would be seriously impacted.

It is well known that the exponential distribution having the constant failure rate (hazard function is constant) whereas the Gompertz, and Generalized exponential distributions having either, increasing or decreasing failure rate (hazard rate). Further, in these distributions failure rate depends upon the shape parameter of the respective distributions. Thus, these distributions are very flexible for modeling lifetime components by selecting the appropriate value of the shape parameter. Thus, these distributions have been used to make optimal decisions with regard to quality, reliability and quality management.

The Gompertz distribution is one of the classical mathematical models that represent survival functions based on laws of mortality. This distribution plays an important role in modeling human mortality and fitting actuarial tables. The Gompertz distribution was first introduced by Gompertz [02]. It has been as a growth mode and also used to fit the tumor growth.

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Gupta and Kundu proposed a generalized exponential distribution. Gupta and Kundu provided a gentle introduction of the generalized exponential distribution and discussed some of its recent developments in the acceptance sampling plans to determine Operating Characteristics such as producer's risk, consumer risk and sample size required to ensure mean lifetime. Mudholkar et al.Introduced the exponentiated Weibull family is an extension of the Weibull family obtained by adding an additional shape parameter. Its properties studied in detail by Gera , Mudholkar and Hutson and Nassar and Eissa. Nadarajah and Gupta [90] introduced the different closed form for the moments with no restrictions imposed on the parameters of the Exponentiated Weibull distribution.

Shanmugapriya and Lakshmi applied the Exponentiated Weibull model for analyzing bathtub failure rate data. Nadarajah et al.reviewed the Exponentiated Weibull distribution and included some of its properties. Gupta et al. Studied the exponentiated gamma (EG) distributions and the Exponentiated Pareto (EP) distribution. Nadarajah considered five kinds of EP distributions and some of their properties. El-Gohary et al. introduced the three-parameter generalized Gompertz distribution by exponentiation the Gompertz distribution to the model life of the components.

S. Poetrodjojo et.al Proposed Optimal CUSUM schemes for monitoring variability in the mean level of the process rather than process variability. They studied the use of Markov chain approach in calculating the average run length (ARL) of CUSUM schemes when controlling variability. They considered 'S' and 'S^2', where 'S' is the standard deviation of a normal process to determine CUSUM schemes. The control statistic 'S^2' is used to prove that the CUSUM scheme is superior to Exponentially Weighted Moving Average (EWMA) by means of any large or small increase in the variability of the normal process. Finally, they proved that the control statistic 'S^2' and 'S' are uniformly better than the control statistic log S^2.

Shangli Zhang and Wenhao Gui proposed acceptance sampling plan based on the truncated life test for the Gompertz distribution at different acceptance numbers, consumer's risk, confidence levels and values of the ratio of the experimental time to be specified mean lifetime, the minimum sample size required to ensure the specified mean lifetime are obtained. The operating characteristic function values and associated producer's risk are also determined. Finally, real examples are provided to illustrate the acceptance sampling plans.

All these research work related to evaluating reliability indices, Operating Characteristics under certain acceptance sampling plans. Based on this understanding, in this study, a new three-parameter distribution called as an Exponentiated Gompertz distribution and study some of the statically properties.

Exponentiated Gompertz Distribution

The non-negative random variable X is said to have an Exponentiated Gompertz distribution if its P.D.F is given by

$$f(x;\lambda,\alpha,\theta) = \theta\lambda\alpha e^{\alpha x} e^{-\lambda(e^{\alpha x-1)})}\left[1-e^{-\lambda\left(e^{\alpha x-1}\right)}\right]^{\theta-1} \text{ Where } \lambda, \alpha, \theta, x > 0 \qquad \text{…... (3.2.1)}$$

Properties of Exponentiated Gompertz Distribution

The probability density function of the Exponentiated Gompertz distribution is

$$f(x;\lambda,\alpha,\theta) = \theta\lambda\alpha e^{\alpha x} e^{-\lambda(e^{\alpha x-1)})}\left[1-e^{-\lambda\left(e^{\alpha x-1}\right)}\right]^{\theta-1} \qquad \text{…... (3.2.2)}$$

The Median of the Exponentiated Gompertz distribution is

$$Med_{EGD}(X) = \frac{1}{\alpha} ln\left[1-\frac{1}{\lambda}\ln(1-q^{\frac{1}{\theta}})\right], 0<q<1 \qquad \text{…… (3.2.3)}$$

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The Mode of the Exponentiated Gompertz distribution is

$$Mod_{\text{Gom}}(\text{x}) = \frac{1}{\alpha} ln\left(\frac{1}{\lambda}\right) \; at\; \theta = 1 \qquad \text{...... (3.2.4)}$$

The survival function of the Exponentiated Gompertz distribution is

$$S(x) = 1 - \left(1 - e^{-\lambda(e^{\alpha x - 1})}\right)^{\theta} \qquad \text{…… (3.2.5)}$$

The cumulative density function of the Exponentiated Gompertz distribution is

$$F(x) = \left(1 - e^{-\lambda(e^{\alpha x - 1})}\right)^{\theta} \qquad \text{…… (3.2.6)}$$

The hazard function of the Exponentiated Gompertz distribution is

$$h(x) = \frac{\theta\lambda\alpha e^{\alpha x} e^{-\lambda(e^{\alpha x - 1})}\left[1 - e^{-\lambda(e^{\alpha x - 1})}\right]^{\theta - 1}}{1 - \left(1 - e^{-\lambda(e^{\alpha x - 1})}\right)^{\theta}} \qquad \text{…… (3.2.7)}$$

Truncated Exponentiated Gompertz Distribution

It is the ratio of probability density function of the Exponentiated Gompertz distribution to their corresponding cumulative distribution function at the point B.

The random variable X is said to follow a truncated Exponentiated Gompertz Distribution as

$$f_B(x) = \frac{\theta\lambda\alpha e^{\alpha x} e^{-\lambda(e^{\alpha x - 1})}\left[1 - e^{-\lambda(e^{\alpha x - 1})}\right]^{\theta - 1}}{1 - \left(1 - e^{-\lambda(e^{\alpha x - 1})}\right)^{\theta}} \quad \lambda > 0,\ \alpha \text{ and } \theta > 0 \qquad \text{…... (3.2.3)}$$

Where' **B'** is the upper truncated point of the Exponentiated Gompertz Distribution.

Description of The Plan and Type- C OC Curve

Battie [3] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum (X_i - k_1) X_i\text{'}s(i = 1,2,3........)$ are distributed independently and k_1 is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies of the return chart, then the product is rejected, subject to the following assumptions.

When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'

When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \sum (X_i - k) < h;$ when $S_m < 0$, return cumulative to 0.

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3. When $h < S_m < h+h'$ the product is rejected: when S_m crossed h, i.e., when $S_m > h+h'$ and continue rejecting product until $S_m > h+h'$ return cumulative to h+h'

The type-C, OC function, which is defined as the probability of acceptance of an item as function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \qquad \text{...... (2.1)}$$

Where L (0) = Average Run Length in acceptance zone and

L' (0) = Average Run Length in rejection zone.

Page E.S. [8] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1 - P(0)} \qquad \text{...... (2.2)}$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \qquad \text{...... (2.3)}$$

Where P (0) =Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities

P (0), N (0), P' (0), N' (0) as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y)\, f(y + k_1 - z)dy, \qquad \text{...... (2.4)}$$

$$N(z) = 1 + \int_0^h N(y)\, f(y + k_1 - z)dy, \qquad \text{...... (2.5)}$$

$$P'(z) = \int_{k_1+z}^{B} f(y)dy + \int_0^h P'(y)\, f(-y + k_1 + z)dy \qquad \text{...... (2.6)}$$

$$N'(z) = 1 + \int_0^h N'(y)\, f(-y + k_1 + z)dy, \qquad \text{...... (2.7)}$$

$$F(x) = 1 + \int_A^h f(x)dx:$$

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$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y)dy$$

and z is the distance of the starting of the test in the normal chart from zero.

METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t)F(t)dt \qquad \text{…… (3.1)}$$

where

$$F(X) = P(z),$$
$$Q(X) = F(k - z),$$
$$R(X,t) = f(y + k - z)$$

Let the integral $I = \int_c^d f(x)dx$ be transformed to

$$I = \frac{d-c}{2}\int_c^d f(y)dy = \frac{d-c}{2}\sum a_i f(t_i) \qquad \text{…… (3.2)}$$

Where $y = \frac{2x-(c-d)}{d-c}$ where a_i's and t_i's respectively the weight factor and abscissa for the Gass-Chibyshev polynomial, given in Jain M.K. and et al [4] using (3.1) and (3.2),(2.4) can be written as

$$F(X) = Q(X)\frac{d-c}{2}\sum a_i R(x,t_i)F(t_i) \qquad \text{…… (3.3)}$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for $x=t_i$, i = 0 (1) n then obtain.

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i R(t_j, t_i)F(t_i) \qquad j = 0(1)n \qquad \text{…… (3.4)}$$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n, in$ (3.4), we get

$$F_0 = Q_0 + \frac{d-c}{2}[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \text{..........} a_n R(t_0,t_n)F_n)]$$

$$F_1 = Q_1 + \frac{d-c}{2}[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \text{..........} a_n R(t_1,t_n)F_n)]$$

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$$F_n = Q_n + \frac{d-c}{2}[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + ...a_n R(t_n,t_n)F_n)] \qquad ……\ (3.5)$$

In the system of equations except F_i , i= 0,1,2……………n are known and hence can be solved for F_i, we solved the solved the system of equations by the method of Iteration. For this we write the system (3.5) as

$$[1 - Ta_0 R(t_0,t_0)]F_0 = Q_0 + T[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + ...a_n R(t_0,t_n)F_n)]$$

$$[1 - Ta_1 R(t_1,t_1)]F_1 = Q_1 + T[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + ...a_n R(t_1,t_n)F_n)]$$

……………. ……………. ………….. …………

……………. …………….. ..………… …………

$$[1 - Ta_n R(t_n,t_n)]F_n = Q_n + T[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + ...a_n R(t_n,t_n)F_n)] \qquad …..\ (3.6)$$

Where $T = \frac{d-c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = ... = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = ... = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i i= 0,1,2,...,n which are just the refined values of F_i i= 0,1,2,…,n. The process is continued until two consecutive until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P' (0), N (0), N' (0) can be obtained.

COMPUTATION OF ARL AND P (A)

We developed computer programs to solve these equations and we get the the following results given in the tables (4.1) to (4.18).

TABLE-4.1

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1, θ=2, k=2, h=0.06, h'=0.06

B	L(0)	L'(0)	P(A)
2.5	1447.3428	1.7892110	0.9987653494
2.4	1569.5541	1.7893583	0.9988612533
2.3	1795.8468	1.7895780	0.9990044832
2.2	2287.8896	1.7899058	0.9992182851
2.1	3870.2703	1.7903954	0.9995375872

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TABLE-4.2

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1, θ=2, k=2, h=0.08, h'=0.08

B	L(0)	L'(0)	P(A)
2.5	1621.8329	2.4279270	0.9985052347
2.4	1759.4933	2.4282882	0.9986218214
2.3	2014.6801	2.4288278	0.9987958670
2.2	2571.1130	2.4296329	0.9990559220
2.1	4372.3262	2.4308357	0.9994443655

TABLE-4.3

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1, θ=2, k=2, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.5	1842.8021	3.7758284	0.9979552031
2.4	2000.3245	3.7769217	0.9981154203
2.3	2292.4983	3.7785528	0.9983544946
2.2	2931.0557	3.7809896	0.9987117052
2.1	5015.4434	3.7846324	0.9992460012

TABLE-4.4

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.2, θ=2, k=3, h=0.06, h'=0.06

B	L(0)	L'(0)	P(A)
3.5	106973.5391	1.5458745	0.9999855757
3.4	122075.7188	1.5458764	0.9999873638
3.3	153725.0469	1.5458788	0.9999899268
3.2	253084.1250	1.5458828	0.9999939203

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3.1	5188230.0000	1.5458885	0.9999997020

TABLE-4.5

Values of ARL’s AND TYPE-C OC CURVES when

b=4, η =1.2, θ=2, k=3, h=0.08, h’=0.08

B	L(0)	L’(0)	P(A)
3.6	134861.6719	1.8897232	0.9999859929
3.5	148170.4375	1.8897254	0.9999872446
3.4	175952.4844	1.8897289	0.9999892712
3.3	242171.3281	1.8897340	0.9999921918
3.2	563048.9375	1.8897420	0.9999966621

TABLE-4.6

Values of ARL’s AND TYPE-C OC CURVES when

b=4, η =1.2, θ=2, k=3, h=0.10, h’=0.10

B	L(0)	L’(0)	P(A)
3.7	186483.0625	2.4302924	0.9999869466
3.6	203436.1250	2.4302957	0.9999880791
3.5	232224.3750	2.4303002	0.9999895096
3.4	300192.6875	2.4303079	0.9999918938
3.3	523740.9375	2.4303186	0.9999953508

TABLE-4.7

Values of ARL’s AND TYPE-C OC CURVES when

b=4, η =1.4, θ=2, k=3, h=0.06, h’=0.06

B	L(0)	L’(0)	P(A)
3.5	76613.9688	1.4064300	0.9999816418
3.4	85126.6563	1.4064312	0.9999834895

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3.3	101630.8438	1.4064331	0.9999861717
3.2	143306.9063	1.4064358	0.9999901652
3.1	362175.9688	1.4064401	0.9999961257

TABLE-4.8

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.4, θ=2, k=3, h=0.08, h'=0.08

B	L(0)	L'(0)	P(A)
3.5	90764.4609	1.6268270	0.9999820590
3.4	102110.0547	1.6268291	0.9999840856
3.3	124934.7266	1.6268327	0.9999870062
3.2	186306.3438	1.6268378	0.9999912977
3.1	732377.6250	1.6268451	0.9999977946

TABLE-4.9

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.4, θ=2, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.5	109864.8047	1.9291357	0.9999824166
3.4	125646.5547	1.9291395	0.9999846220
3.3	159035.2656	1.9291455	0.9999878407
3.2	264442.6875	1.9291544	0.9999927282
3.1	**11371056.0000**	**1.9291679**	**0.9999998212**

TABLE-4.10

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.6, θ=2, k=3, h=0.06, h'=0.06

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B	L(0)	L'(0)	P(A)
3.5	62140.7734	1.3156629	0.9999788404
3.4	68048.5391	1.3156638	0.9999806881
3.3	79530.0078	1.3156652	0.9999834299
3.2	106772.3984	1.3156675	0.9999876618
3.1	214731.3281	1.3156708	0.9999938607

TABLE-4.11

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.6, θ=2, k=3, h=0.08, h'=0.08

B	L(0)	L'(0)	P(A)
3.5	68987.7500	1.4703773	0.9999786615
3.4	75938.0313	1.4703790	0.9999806285
3.3	90050.4609	1.4703815	0.9999836683
3.2	123341.9297	1.4703853	0.9999880791
3.1	275019.5000	1.4703906	0.9999946356

TABLE-4.12

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.6, θ=2, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.5	77300.7266	1.6663281	0.9999784231
3.4	85615.9922	1.6663307	0.9999805093
3.3	102331.5625	1.6663349	0.9999837279
3.2	145200.3438	1.6663407	0.9999884963
3.1	377011.9688	1.6663494	0.9999955893

TABLE-4.13

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.8, θ=2, k=3, h=0.06, h'=0.06

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B	L(0)	L'(0)	P(A)
3.5	53335.7734	1.2517955	0.9999765158
3.4	58094.9766	1.2517964	0.9999784827
3.3	67191.7188	1.2517977	0.9999813437
3.2	87818.0547	1.2517995	0.9999857545
3.1	161375.1719	1.2518021	0.9999922514

TABLE-4.14

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.8, θ=2, k=3, h=0.08, h'=0.08

B	L(0)	L'(0)	P(A)
3.5	57269.0625	1.3664875	0.9999761581
3.4	62541.4727	1.3664888	0.9999781251
3.3	72965.0859	1.3664907	0.9999812841
3.2	96100.4219	1.3664936	0.9999857545
3.1	185854.7813	1.3664979	0.9999926686

TABLE-4.15

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =1.8, θ=2, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.5	61840.4883	1.5043156	0.9999756813
3.4	67739.7734	1.5043176	0.9999777675
3.3	79203.4688	1.5043206	0.9999809861
3.2	106149.0859	1.5043250	0.9999858141
3.1	214509.8750	1.5043316	0.9999929667

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TABLE-4.16

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =2, θ=2, k=3, h=0.06, h'=0.06

B	L(0)	L'(0)	P(A)
3.5	47165.3398	1.2044592	0.9999744892
3.4	51346.2109	1.2044599	0.9999765158
3.3	59031.8008	1.2044609	0.9999796152
3.2	76279.8203	1.2044624	0.9999842048
3.1	134318.9063	1.2044647	0.9999910593

TABLE-4.17

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =2, θ=2, k=3, h=0.08, h'=0.08

B	L(0)	L'(0)	P(A)
3.5	49751.5820	1.2925504	0.9999740124
3.4	54248.9102	1.2925514	0.9999761581
3.3	62554.1445	1.2925529	0.9999793172
3.2	81373.4453	1.2925553	0.9999841452
3.1	146596.5781	1.2925588	0.9999911785

TABLE-4.18

Values of ARL's AND TYPE-C OC CURVES when

b=4, η =2, θ=2, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.5	52561.4922	1.3945440	0.9999734759
3.4	57243.7109	1.3945454	0.9999756217
3.3	66402.7500	1.3945478	0.9999790192
3.2	86612.3359	1.3945513	0.9999839067

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3.1	160652.1094	1.3945563	0.9999912977

NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters b, η ,θ , k, h and h' given at the top of each table, we determined optimum truncated point B at which P(A) is the probability of accepting an item is maximum and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point 'B' of random variable X, L(0), L'(0) and the values for Type-C OC Curve, i.e. P(A) are given in columns I, II, III, and IV respectively.

From the above tables (4.1) to (4.18) we made the following conclusions:

1. From the tables (4.1) to (4.3), it is observed that the value of L (0) and P (A) is increased as the value of truncated point decreases. Thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
2. And also we observe that it could minimize the truncated point B by decreasing the value of k.
3. From tables (4.1) to (4.3), it is observed that the truncated point B of the random variable X decreases from 2.5 to 2.1 as h→0.10, while the value of L (0) increases from 3870.2703 to 5015.4434 and the probability of acceptance P (A) Changes from 0.9995375872to 0.9992460012. Thus at constant hypothetical value h and truncated point B are positively related, while the values of L (0) and P (A) are inversely related.
4. From tables (4.4) to (4.6), it is observed that truncated point B of the random variable X decreases from 3.5 to 3.1 as h→ 0.10, while the value of L (0) increases from 5188230.0000 to 523740.9375 and whereas the probability of acceptance P (A) changes from 0.9999997020 to 0.9999953508. Thus hypothetical value h and truncated point B are positively related, while the values L (0) and P (A) are positively related.
5. From tables (4.7) to (4.9), it is observed that truncated point B of the random variable X decreases from 3.5 to 3.1 as h→ 0.10, while the value of L (0) increases from 362175 to 11371056.00 whereas the probability of acceptance P (A) changes from 0.9999961257to 0.9999998212. Thus hypothetical value h and truncated point B are positively related, while the values L (0) and P (A) are positively related.
6. From tables (4.10) to (4.12), it is observed that the truncated point B of the random variable X decreases from 3.5 to 3.1 as h→0.10, while the value of L (0) increases from 214731.3281 to 377011.9688 and the probability of acceptance in P (A) Changes from 0.9999938607 to 0.9999955893 at different truncated points of B. Thus the values of L (0) and P (A) are positively related.
7. From tables (4.13) to (4.15), it is observed that the truncated point B of the random variable X decreases from 3.5 to 3.1 as h→0.10, while the value of L (0) increases from 161375.1719 to 214509.8750 and the probability of acceptance in P (A) Changes from 0.9999922514 to 0.9999929667. Thus the values of L (0) and P (A) are positively related.
8. From tables (4.16) to (4.18), it is observed that the truncated point B of the random variable X decreases from 3.9 to 3.1 as h→0.10, while the value of L (0) increases from 134318.9063 to 160652.1094 and the probability of acceptance in P (A) Changes from 0.9999910593 to 0.9999912977. Thus the values of L (0) and P (A) are positively related.

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TABLE – 5.1

Consolidated Table

B	b	η	θ	k	h	h'	L(0)	L'(0)	P(A)
2.1	4	1	2	2	0.06	0.06	3870.2703	1.7903954	0.9995375872
2.1	4	1	2	2	0.08	0.08	4372.3262	2.4308357	0.9994443655
2.1	4	1	2	2	0.10	0.10	5015.4434	3.7846324	0.9992460012
3.1	4	1.2	2	3	0.06	0.06	5188230.000	1.5458885	0.9999997020
3.2	4	1.2	2	3	0.08	0.08	563048.9375	1.8897420	0.9999966621
3.3	4	1.2	2	3	0.10	0.10	523740.9375	2.4303186	0.9999953508
3.1	4	1.4	2	3	0.06	0.06	362175.9688	1.4064401	0.9999961257
3.1	4	1.4	2	3	0.08	0.08	732377.6250	1.6268451	0.9999977946
3.1	***4***	***1.4***	***2***	***3***	***0.10***	***0.10***	***11371056.00***	***1.9291679***	***0.9999998212***
3.1	4	1.6	2	3	0.06	0.06	214731.3281	1.3156708	0.9999938607
3.1	4	1.6	2	3	0.08	0.08	275019.5000	1.4703906	0.9999946356
3.1	4	1.6	2	3	0.10	0.10	377011.9688	1.6663494	0.9999955893
3.1	4	1.8	2	3	0.06	0.06	161375.1719	1.2518021	0.9999922514
3.2	4	1.8	2	3	0.08	0.08	96100.4219	1.3664936	0.9999857545
3.1	4	1.8	2	3	0.10	0.10	214509.8750	1.5043316	0.9999929667
3.1	4	2.0	2	3	0.06	0.06	134318.9063	1.2044647	0.9999910593
3.1	4	2.0	2	3	0.08	0.08	146596.5781	1.2925588	0.9999911785
3.1	4	2.0	2	3	0.10	0.10	160652.1094	1.3945563	0.9999912977

By observing the **Table-5.1**, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL and P (A) reach their maximum i.e. ***11371056.00, 0.9999998212*** respectively, is

$$\begin{bmatrix} B = 3.1 \\ b = 4.0 \\ \eta = 1.4 \\ \theta = 2.0 \\ k = 3.0 \\ h = 0.10 \\ h' = 0.10 \end{bmatrix}$$

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue XII, December 2025

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