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# Mathematical Modelling and Analysis of Infiltration of Firearms and Its Insecurity Implications in the Northern Region of Nigeria

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## ABSTRACT

This research produces and analyzes an innovative nonlinear compartmental model to explore the infiltration of firearms into Nigeria and its resulting insecurity dynamics in Northern Nigeria. The model incorporates four interacting components: susceptible individuals, violent actors, arms and ammunition, and recovered individuals, capturing the bidirectional feedback between arms infiltration and violence recruitment. An effective reproduction number, $R_e$, was derived to characterize the threshold conditions governing the persistence or elimination of violence. Stability analyses, including Lyapunov-based proofs ($L(V,A) = w_1V + w_2A$), create the conditions for global stability of both the violence-free and endemic equilibria. Normalized sensitivity analysis using Python revealed that arms-related parameters, particularly arms inflow ($\sigma$ =+0.312), arms to violence activation ($\alpha$= -0.020), and arms-induced susceptibility ($\gamma$=+0.254), are the most influential drivers of violence propagation. Numerical simulations were used to validate the investigative results, which showed how reducing arms availability can substantially lower the long-term levels of violent actors. These findings showed that illicit arms proliferation is the primary catalyst sustaining endemic violence. The necessary interventions, such as addressing arms inflow, distribution networks, and de-escalation processes, are most effective for violence reduction. The simulation results establish that the long-term dynamics of violence in Northern Nigeria are strongly driven by the circulation and availability of illicit firearms. The trajectories for S(t), V(t), and A(t) showed that the reductions in arms-related parameters produce meaningful declines in violence, confirming the sensitivity analysis results. This study provides a quantitative framework and actionable insights to support evidence-based security policy, arms-control strategies, and conflict-mitigation efforts in Northern Nigeria.

**Keywords:** Violence, Simulation, Dynamics, Arms-Control

## INTRODUCTION

The infiltration of firearms into Nigeria, particularly through the Sahel region and Central African countries, has emerged as one of the most critical drivers of insecurity in northern region of Nigeria (Bassey *et al*., 2025). The proliferation of small arms and light weapons is adjudged as the most immediate security challenge to individuals, societies, and states worldwide, fuelling civil wars, organized criminal violence, insurgency, and terrorist activities, posing great obstacles to sustainable security and development of nations (Bashir, 2014). The infiltration of small arms and light weapons (SALW) in Nigeria has become a critical concern for national security and socio-economic stability, particularly in the northern part of Nigeria (Bassey *et al*., 2025). Unfortunately, too, the Northern region of Nigeria faces an array of violent threats, including banditry, killings by Boko Harm, farmer-herder clashes, terrorism, and communal conflicts that have grown in scale, frequency, and lethality over the past decade (Agaba and Upkabio, 2023; Isah *et al*., 2024). Nigeria's porous borders, spanning more than 4,000 km, serve as major entry points for illegal smuggling of small arms and light weapons

(SALWs), that have been estimated to exceed 6 million in circulation within the country (Agaba and Upkabio, 2023; Isah *et al*., 2024).

Worrisomely, empirical evidence has continued to link the availability of small arms to increased insecurity, banditry, kidnapping, and terrorism in across the West Africa countries (Abiodun *et al*., 2018; Small Arms Survey, 2019). In Nigeria, illicit firearms flow mainly from Libya, Chad, Niger, and Cameroon as a result of regional instability (Akinola, 2020) has continued to remain the simplest means through which all kinds of violence groups access their weapons including armed robbers, kidnappers, Boko Haram, killer herdsmen to foment, promoted and sustained the general insecurity in the northern region of Nigeria and worst still, the ECOWAS Convention on SALWs has achieved limited success due to weak enforcement mechanisms (Bah, 2017). Fundamentally, it is important to note that the infiltration of firearms creates a reinforcing cycle of violence: increased access to weapons encourages the formation and recruitment of armed groups, which in turn escalates conflict intensity and insecurity, thereby increasing demand for more weapons (Karp, 2018; Afuzie *et al*., 2021). Interestingly, mathematical modeling provides a rigorous framework for understanding how these interconnected processes evolve and how targeted interventions can disrupt the violence-arms cycle (Brauer & Dunne, 2011). This study develops a dynamic compartmental model that captures the interactions among susceptible individuals S(t), Violent groups V(t), the inflow of illegal firearms/ammunition A(t), and recovery individuals R(t). The model incorporates sociopolitical drivers unique to northern region of Nigeria and provides quantitative insights into how firearm inflow amplifies insecurity in the region.

Currently, research applying formal mathematical models to arms proliferation and the violence it enables or enhances in Nigeria is an emergent but interesting growing field of research. Nevertheless, although there are different modelling traditions apparently in the literature, such as agent-based and network modelling, game-theoretic and economic models of demand and diversion (Morgan, 2020; Alusala, 2023), compartmental or deterministic population modelling methodology was employed in the present investigation. Meanwhile, many researchers and authors have so far reported on the use of mathematical models and simulations to study the infiltration and spreading of firearms and weapons with respect to kidnapping, banditry, terrorists and Islamic extremists' activities and the implications for West African regional security. In that regard, Kambai (2023) reported on a mathematical model and analysis of the proliferation of arms and weapons in Nigeria and control, and a seven-compartmental model with well-defined classes was utilized as his designed model for analysis of the influence of proliferation of arms and weapons used and kidnapping activities on population dynamics. In addition, Kambai (2023) employed the Runge-Kutta method of order four to derive the numerical solutions of the model's hypotheses.

In another report, Akpienbi and Ibrahim (2024) have reported the mathematical modelling of security forces – insurgent dynamics in Nigeria. In the work, the authors developed a compartmental model where the populace was compartmentalized into protection forces (security forces), insurgents, and rehabilitated compartments. The model's equilibrium points for local approach stability of equilibrium were determined using the Routh-Hurwitz condition, and the model's local stability and numerical simulations were performed by employing parameter values with codes implemented using MATLAB 2012b software to generate a diagram of population trends. In conclusion, the study recommended a modified version of the prey-predator model of security forces and criminal dynamics, which was proposed as reported by Oduro *et al.* (2015). In other related reports, Bassey *et al*. (2025) reported small arms and light weapons proliferation in Nigeria: problems and prospects, and Bashir (2014) reported small arms and light weapons proliferation and its implications for West African regional security, respectively. In their submission, both authors maintained that the infiltration of small arms and light weapons (SALW) into Nigeria has become a critical concern for nationwide security and socio-economic stability by fuelling all kinds of violence activities including civil wars, organized criminal violence, insurgency, and terrorist activities, thereby constituting great hindrances to sustainable security, growth and developmental progress in Nigeria. Furthermore, the authors have adjudged the infiltration and spreading of small arms and light weapons (SALW) in Nigeria as the greatest direct security challenge not only to individuals and societies, but also to states and nations worldwide (Bashir, 2014; Bassey *et al*., 2025).

Meanwhile, despite the numerous reports and mathematical modellings and simulations of the effect of proliferation of arms and weapons on kidnapping activities and its general consequences on the West African regional security elsewhere in the literature, there appears to be very little or none of such information about the mathematical modelling and analysis of infiltration of firearms and its insecurity implications in the Northern part of Nigeria. Based on the foregoing, this work therefore is aimed at utilizing the mathematical tools of modelling and simulation to analyze the possible means of infiltration of firearms and their insecurity implications in Northern Nigeria, and possible means of reducing the inflow of firearms into Nigeria using normalized sensitivity analysis and simulation of a designed model to curtail the rate of insecurity in the Northern part of Nigeria. This, we believe, will go a long way to form a quantitative framework and actionable insights to support evidence-based security policy, arms-control strategies, and conflict-mitigation efforts by governments at various levels toward the Northern part of Nigeria.

This paper, in addition also argues that using a mathematical model and simulation, the possession of illegal small and light weapons through infiltration of firearms across the breadth and length of the northern part of Nigeria in particular and Nigeria at large, which fuels insecurity, insurgency, banditry, farmers-herders clashes, Boko haram, killers herd men and terrorism in the northern Nigeria must be addressed along the line of controlling the infiltration into Nigeria and the spreading of illegal firearms and light weapons cross the country and other west African countries must cooperate and collaborate to address this huge and complex insecurity challenge bedevilling the northern part of Nigeria in particular and Nigeria in general.

## MATERIALS AND METHODS

### Mathematical Models of Conflict Dynamics

This study utilizes a model by including northern Nigeria–specific parameters such as border permeability and armed group recruitment. Mathematical modeling and analysis of infiltration of firearms into Nigeria and its insecurity implications in the Northern region is developed and analyzed. The invariant region was determined, and the positivity of the solution of the model was analyzed, in order to ascertain that the model is well- posed mathematically and bounded.

### The Designed Model

To undertake the study, the proposed designed mathematical model is subdivided into four compartments at time $t$: Susceptible population $S(t)$ (populations not directly involved in violence), $V(t)$ (violence actors such as bandit, Boko Haram, kidnapping, Asaru etc.), $A(t)$ (arms/ammunitions that have infiltrated the region, treated as a resource that fuels $V(t)$), $R(t)$ (population who have escaped the violence zone or have been rehabilitated(recovered).

The birth and death rate for susceptible populations are $\pi$ and $\mu$ respectively. The susceptible population, S(t) becomes violence actors at the rate of $\beta SV$ and is been reintegrated back to S(t) at the rate of $\rho$ and dies out at the rate of $\mu$. $\delta$ and $\phi AV$ are the permanent removal from V and the removal from V via A's action respectively. The $\gamma$ and $\varepsilon SV$ are the de-mobilization/outflow from A and mobilization from S due to V's presence respectively. The arm/ammunitions are recruited at the rate $\sigma$ and fuels the violence actors at the rate $\alpha$ and decay or seized at the rate $\mu$.

### Model Hypotheses

i. Do firearms increase the recruitment rate of new violence actors?
ii. Does violent conflict decrease as arms are removed or actors are demobilized?
iii. Do arms enter the system through smuggling, theft, and black-market trade?
iv. Can Government interventions reduce firearm stock and violent groups?

The systematic diagram as well as the detailed descriptions of the model's variables and the parameters are given in the Figure 1, and Tables 1 and 2.

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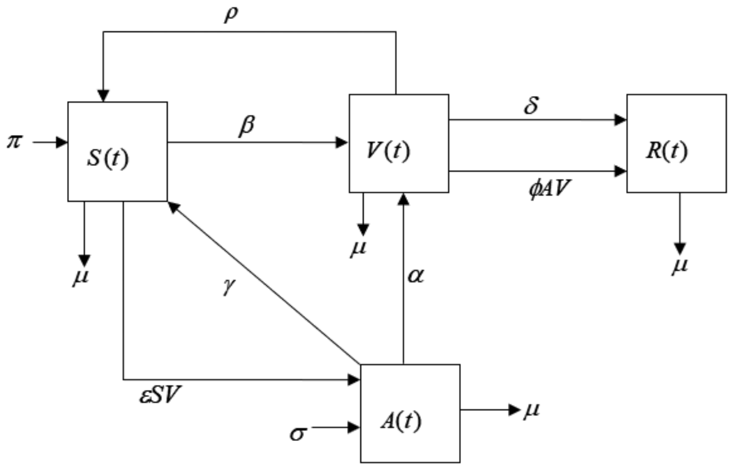


**Figure 1: Schematic Diagram of the Conflict Flow Model**

| Variables | Interpretations |
|---|---|
| S | Susceptible individuals. |
| V | Violence actors. |
| A | Arms/ammunitions. |
| R | Recovered individuals. |

**Table 1: State variables and their interpretations**

| Parameters | Interpretations |
|---|---|
| $\pi$ | Recruitment or birth rate into S |
| $\rho$ | Displacement rate (effect of violent class V reducing susceptible). |
| $\gamma$ | Rate at which arms get to susceptible individuals. |
| $\beta$ | baseline rate by which susceptible move into violence class |
| $\varepsilon$ | Rate at which the violent actor's 'V' produces additional armed individual's 'A' via contact with susceptible (term εSV). |
| $\mu$ | Natural exit rate (death, migration, ageing out, decay, seizures). |
| $\alpha$ | Rate at which arms/ammunitions fuels violence groups 'V'. |
| $\delta$ | Transition rate from violence groups 'V' to recovered class R(capture, death, reintegration). |
| $\phi$ | Coupling coefficient representing additional removal/transition intensity that depends on A interacting with V. |
| $\sigma$ | External inflow to the arms/ammunitions class 'A' (e.g., firearms importation or supply). |

**Table 2: Model Parameters and their Interpretations**

**The Model Equations**

Considering the systematic diagram, Figure 1, the possible differential equations derived are:

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$$\frac{dS}{dt} = \pi - \rho V + \gamma A - \beta S - \varepsilon SV - \mu S, \qquad (1)$$

$$\frac{dV}{dt} = \beta S + \alpha A - (\delta + \phi A + \mu)V, \qquad (2)$$

$$\frac{dA}{dt} = \sigma + \varepsilon SV - (\gamma + \alpha + \mu)A, \qquad (3)$$

$$\frac{dR}{dt} = \delta V + \phi AV - \mu R, \qquad (4)$$

**Basic Mathematical Properties of the Designed Model**

The fundamental properties of the equations (1) to (4) of our designed model such as the invariant region, positivity of solutions of the model, the equilibrium state (violence-free equilibrium states and its local and global stabilities) were considered, the effective reproduction number were obtained, and the sensitivity analysis of the model equations was also computed.

**The Invariant Region/Boundedness**

The invariant region that the model solutions are bounded was obtained, and the total population, $N(t)$ was given by; $N(t) = S + V + R$, ………………………………………(5a)

Since A(arms/ammunitions) is a stock or inhuman, with initial condition;

$N(0) = N_0$, Hence, the differentiation of $N(t)$ with regards to '$t$' leads to;

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dV}{dt} + \frac{dR}{dt}. \qquad \text{………………………………………………(5b)}$$

Substituting equations (8) to (15) into (16) and simplifying further, we have;

$$\frac{dN}{dt} = \pi - \mu(S + V + R) + A(\gamma + \alpha) + V(\rho - \varepsilon S); \qquad \text{………………………………(6)}$$

Since $N(t) = S + V + R$,

Then equation (6) reduces to,

$$\frac{dN}{dt} = \pi - \mu N + A(\gamma + \alpha) + V(\rho - \varepsilon S) \qquad \text{……………………………………(7)}$$

In the absence of violence and arms (since its inhuman), V = A = 0, and

Introducing inequality in (7), we have;

$$\frac{dN}{dt} \le \pi - \mu N, \qquad \text{……………………………………………………(8)}$$

Applying method of integrating factor and further simplification of (8), leads to;

$$N(t) \le \frac{\pi}{\mu}\left(1 - e^{-\mu t}\right) + N_0 e^{-\mu t}, \qquad \text{…………………………………………(9)}$$

This result confirms the boundness of the total population as; $t \to \infty$, the term

$e^{-\mu t} \to 0$

(assuming $\mu > 0$) , in equation (9);

$$N(t) \to \frac{\pi}{\mu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(10)$$

This means the feasible solution of the model of Equations (1) to (4) is in the region,

$$\Omega = \{(S, V, A, R) \in R_+^4; S, V, A, R \geq 0 : N(t) \leq \frac{\pi}{\mu}\}.; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(10b)$$

Hence, this implies that the proposed conflict model systems (1) to (4) are well modelled mathematically and with boundedness. Hence, they are sufficient enough to analyze and ascertain the dynamics of the model in this region, Ω

**Positivity of Solution**

The solution of the model is shown to be positive as it represents a human population that cannot be negative. To validate this, it has been assumed that the given initial condition of the model is non-negative, and the illustration has shown that the solutions of the model are positive. In that mathematical sense of consideration, the following theorems were proposed:

Let $\Omega = \{S, V, A, R\} \in R_+^4$ be solution set such that $S(0) = S_0$, $V(0) = V_0$, $A(0) = A_0$, $R(0) = R_0$, are positive, then the solution of the set $\Omega$ are all positive for $t \geq 0$.

From equation (1), we have: $\frac{dS}{dt} = \pi - \rho V + \gamma A - \beta S - \varepsilon SV - \mu S$; ………… (11)

Which means that; $\frac{dS}{dt} \geq -[\beta + \varepsilon V + \mu]S$, ……………………………………..(11b)

Using the exponential growth criterion and integrating (11) with initial conditions $S(0) = S_0$ gives;

$$S(t) \geq S_0 e^{\int -(\beta + \varepsilon V + \mu) dt} \geq 0,$$

$$S(t) \geq S_0 e^{-(\beta + \varepsilon V + \mu)t} \geq 0,$$

Also, from equation (2), we have: $\frac{dV}{dt} = \beta S + \alpha A - (\delta + \phi A + \mu)V$ ; which means that;

$$\frac{dV}{dt} \geq \beta S + \alpha A - (\delta + \phi A + \mu)V \; ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (12a)$$

Using the exponential growth criterion and integrating (12) with initial conditions $V(0) = V_0$ gives;

$$V(t) \geq V_0 e^{\int -(\delta + \phi A + \mu) dt} \geq 0$$

$$V(t) \geq V_0 e^{-(\delta + \phi A + \mu)t} \geq 0$$

From equation (3), we have;

$$\frac{dA}{dt} = \sigma + \varepsilon SV - (\gamma + \alpha + \mu)A \; ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12b)$$

which means that;

$$\frac{dA}{dt} \geq -(\gamma + \alpha + \mu)A \quad \text{.......................................................} (13)$$

Using the exponential growth criterion and integrating (13) with initial conditions $A(0) = A_0 e$ gives:

$$A(t) \geq A_0 e^{-(\gamma+\alpha+\mu)t} \geq 0$$

From equation (4), we have;

$$\frac{dR}{dt} = \delta V + \phi AV - \mu R$$

which means that;

$$\frac{dR}{dt} \geq -\mu R \quad \text{..........................................}(14)$$

Using the exponential growth criterion and integrating (14) with initial conditions $R(0) = R_0$ gives:

$$R(t) \geq R_0 e^{-\mu t}$$

Therefore, the set solution identified as $\{S, V, A, R\}$ for the systems of the Equations (1) to (4)) are positive only for all $t \geq 0$; since the exponential functions and their initial conditions have been proved to be positive.

**The Violence-Free Equilibrium State of the Model**

The state where there is no conflicts (violence) is referred to as the violence-free equilibrium. Therefore, the point was obtained by equating the right-hand side of all the equations (1) – (4) of the system to zero and then imposing $V_0 = 0$. Here, we let $E_0$ be the violence -free point.

Then, substituting, $V_0 = 0$ into (3), we have;

$$\sigma - A(\gamma + \alpha + \mu) = 0, \text{..............................................}(15a)$$

Therefore, $$A_0 = \frac{\sigma}{(\gamma + \alpha + \mu)} \quad \text{..........................}(15b)$$

Substituting $V_0$ and $A_0$ into (1), we have;

$$\pi + \gamma A_0 - (\beta + \mu)S_0 = 0,$$

$$S_0 = \frac{\pi + \gamma A_0}{(\beta + \mu)} \quad \text{.....................................................}(16)$$

Therefore, $$E_0 = (S_0, V_0, A_0, R_0) = \left[\frac{\pi + \gamma A_0}{(\beta + \mu)}, 0, \frac{\sigma}{(\gamma + \alpha + \mu)}, 0\right], \text{.........}(17)$$

**Effective Reproduction Number**

We treat the violent class "V" and the arms/ammunition class "A" as the infected compartment. Thus, the infected vector is $X = (V, A)^T$

We follow the standard decomposition (Driessche & Watmough, 2002). For each infected compartment i, write $x_i = F_i (x) - V_i (x)$ where $F_i$= is the rate of appearance of new voilence in compartment i and $V_i$ is the net transmission (other gains minus losses) in that compartment. Evaluate Jacobians at the CFE $E_0$ and compute the next-generation $K = FV^{-1}$. The Effective Reproduction Number $R_e$ is the spectral radius ρ(K).

From the model: New infections that are directly produced by infected classes:

New V from arms: $F_v = \alpha A$;

Where; 'A' receives new contributions from 'V' through εSV. So $F_A = \varepsilon SV$.

$$F = \begin{pmatrix} \alpha A \\ \varepsilon SV \end{pmatrix}, \quad \text{.......................................................(18)}$$

The remaining terms define V(x) (losses and transfers excluding F).

$V(x) = (\delta + \phi A + \mu)V - \alpha A$, and $V(x) = (\gamma + \alpha + \mu)A$,

Therefore, $V(x) = \begin{pmatrix} (\delta + \phi A + \mu)V - \alpha A \\ (\gamma + \alpha + \mu)A \end{pmatrix}$ ......................................(19 )

Taking the partial derivative of F and V with regards to the violence vector (V, A) at VFE, we have the Jacobian matrices

$$F(E_0) = \begin{pmatrix} 0 & \alpha \\ \varepsilon S_0 & 0 \end{pmatrix}, \quad \text{................................(20)}$$

and

$$V(E_0) = \begin{pmatrix} \delta + \mu + \phi A_0 & 0 \\ 0 & \gamma + \alpha + \mu \end{pmatrix} \quad \text{........(21a)}$$

.

Taking the inverse of V, we have;

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\delta + \mu + \phi A_0} & 0 \\ 0 & \dfrac{1}{\gamma + \alpha + \mu} \end{pmatrix}$$

The next generation matrix is $K = FV^{-1}$.

$$K = \begin{pmatrix} 0 & \dfrac{\alpha}{\gamma + \alpha + \mu} \\ \dfrac{\varepsilon S_0}{\delta + \mu + \phi A_0} & 0 \end{pmatrix}$$

Thus, by computing the eigen-values to evaluate the effective reproduction number $R_c$ by taking the spectral radius (dominant eigen-value) of the matrix $K$.

This is computed by $|K - \lambda I| = 0$, and after further simplification, we have:

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$$\lambda = \pm\sqrt{\frac{\varepsilon\alpha S_0}{(\gamma+\alpha+\mu)(\delta+\mu+\phi A_0)}} \quad \text{………………(21b)}$$

Therefore, the effective reproduction number for violence is the spectral radius (largest absolute eigen-value) of the matrix. Substituting the expression for $S_0$ and $A_0$ we have;

$$R_e = \sqrt{\frac{\varepsilon\alpha(\pi(\gamma+\alpha+\mu)+\gamma\sigma)}{(\gamma+\alpha+\mu)(\beta+\mu)((\gamma+\alpha+\mu)+\phi\sigma)}} \quad \text{………………..(22a)}$$

**The Stability Threshold.**

The effective reproduction number, $R_c$, was determined by the decomposition technique in van den Driessche and Watmough, (2002). Thus, an effective reproduction number attained by this method defines the local stability of the Conflict (violence)-free equilibrium point which is locally asymptotically stable for $R_c < 1$ and unstable for $R_c > 1$.

Thus, the condition for long-term control of firearm-driven violence is to ensure that

$$\left(\sqrt{\frac{\varepsilon\alpha(\pi(\gamma+\alpha+\mu)+\gamma\sigma)}{(\gamma+\alpha+\mu)(\beta+\mu)((\gamma+\alpha+\mu)+\phi\sigma)}}\right) < 1 \quad \text{……….(22b)}$$

**The Existence of Endemic Equilibrium States of the Model**

The endemic equilibrium state of the model is a steady-state solution where violence exists in the susceptible population. For the determination of the endemic equilibrium point $E^*$, the steady state of the system from Equations (1) to (4) of all state variables was considered. Then;

$$\pi - \rho V^* + \gamma A^* - \beta S^* - \varepsilon S^* V^* - \mu S^* = 0 \quad \text{, ……………………………….(23)}$$

$$\beta S^* + \alpha A^* - (\delta + \phi A^* + \mu)V^* = 0 \quad \text{,……………………………………….(24)}$$

$$\sigma + \varepsilon S^* V^* - (\gamma + \alpha + \mu)A^* = 0 \quad \text{,…………………………………………(25)}$$

$$(\delta + \phi A^*)V^* - \mu R^* = 0 \quad \text{,…………………………………………………(26)}$$

Since we are at endemic equilibrium, $V^* \neq 0$

From (26), solving for $R^*$

$$R^* = \frac{(\delta+\phi A^*)}{\mu}V^* \quad \text{………………………..(27)}$$

From (24), solve for $S^*$

$$\beta S^* = (\delta + \phi A^* + \mu)V^* - \alpha A^*$$

$$S^* = \frac{(\delta + \phi A^* + \mu)V^* - \alpha A^*}{\beta} \quad \text{………………………(28)}$$

From (25), solving for $A^*$

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$$(\gamma+\alpha+\mu)A^* = \sigma + \varepsilon S^* V^*$$

$$A^* = \frac{\sigma + \varepsilon S^* V^*}{(\gamma+\alpha+\mu)}, \quad \text{……………………………………}(29)$$

Substitute (28) into (23), we have;

$$(\gamma+\alpha+\mu)A^* = \sigma + \varepsilon V^* \left(\frac{(\delta+\phi A^* + \mu)V^* - \alpha A^*}{\beta}\right) \text{……….}(30)$$

$$(\gamma+\alpha+\mu)A^* = \sigma + \frac{\varepsilon}{\beta}\left[(\delta+\mu)(V^*)^2 + \phi A^*(V^*)^2 - \alpha A^* V^*\right], \text{………………}(31)$$

Substituting (29) into (30) and simplifying further will yield a high complex equation and can be solved numerically for specific parameter values. Therefore, the actual value of $V^*$ is the positive root of a high-degree polynomial function $F(V^*) = 0$

The equation (31) is a polynomial of high degree in $V^*$, which cannot be solved analytically. The state of stability of endemic equilibrium is often related to the trans-critical bifurcation that occurs at $R_0 = 1$. If $R_0 > 1$; the endemic equilibrium $E^*$ generally occurs and is locally asymptotically stable and if $R_0 < 1$; the endemic equilibrium $E^*$ is either non-existent or unstable and the system moves towards the violence free equilibrium $E_0$.

**Global Stability of Violence Free Equilibrium**

At this point, our target is to investigate the global asymptomatic stability of the violence free equilibrium state. In that regard, we proposed the **theorem 1:** if $R_c < 1$, then $E_0$ is globally asymptomatically stable in the feasible region $(S(t), V(t), A(t), R(t)) \to E_0$ as $t \to \infty$ an unstable if $R_c > 1$.

**Proof:** Since the violence variables are V(t) and A(t) and these form the next-generation matrix from which $R_e$ is derived, a natural Lyapunov function is the weighted linear combination of V and A using the left eigenvector of the next-generation matrix.

Let; $K = FV^{-1}$; where F and V are the new violence and transition matrices evaluated at $E_0$.

However, let $w = (w_1 + w_2) > 0$ be the left perron eigenvector satisfying; $wF = R_e w$.

Then, define the Lyapunov function;

$L(V, A) = w_1 V + w_2 A$. Where; $w_1 > 0$, $w_2 > 0$.

Clearly, $L \geq 0, L = 0 \Leftrightarrow (V, A) = (0,0)$.

This isolates the violence subsystem and is the standard structure for global stability proofs in epidemic systems.

Let $$\frac{dL}{dt} = w_1 \frac{dV}{dt} + w_2 \frac{dA}{dt} \text{ ………………………………………………}(32)$$

Substitute the derivative of V and A in (32), we have;

$$\frac{dL}{dt} = w_1(\beta S + \alpha A - (\delta + \phi A + \mu)V) + w_2(\sigma + \varepsilon SV - DA) \text{ …………………..}(33)$$

Where $D = \gamma + \alpha + \mu$. And from (33), we rearranging the terms in the form (34);

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$$\frac{dL}{dt} = w_1\beta(S - S_0) + w_2\sigma - w_2 D(A - A_0) + (w_2\varepsilon S - w_1(\delta + \mu))V + (w_1\alpha - w_2 D) \qquad (34)$$

At the VFE: $\beta S_0 + \alpha A_0 = (\delta + \mu)V_0 = 0,\ \sigma - DA = 0.$

Therefore, the first bracket vanishes when (S, A) = ($S_0$, $A_0$).

(for stability analysis, we use the fact that (S(t), A(t) remain bounded and positive)

Using the next generation matrix relation, that is eigenvalue relation, we have; $wF = R_e wV$.

This yields the inequality (standard in the van den Driessche-Watmough framework).

$$w_1(\beta S_0) + w_2(\varepsilon\beta S_0 A_0) = R_e(w_1(\delta + \mu) + w_2 D). \quad \ldots\ldots(35)$$

Because $S(t) \le S_0$ and $A(t) \le A_0$ for all t (due to boundness of solution), we obtain;

$$w_1\beta S + w_2\varepsilon\beta SA \le R_e(w_1(\delta + \mu) + w_2 D). \quad \ldots\ldots(36)$$

This inequality is the central device allowing us to control $\dot{L}$. Therefore, using the inequality (36), the derivative satisfies,

$$\dot{L} \le (R_e - 1)(w_1(\delta + \mu)V + w_2 DA) - w_1\phi AV. \quad \ldots\ldots(37)$$

From (25) the last term $-w_1\phi AV \le 0.$; and the factor multiplying the first bracket is $(R_e - 1)$. Thus, if; $R_e < 1$, then; $\dot{L} \le (R_e - 1)(w_1(\delta + \mu)V + w_2 DA) - w_1\phi AV \le 0.$

Equality $\dot{L} = 0$ occurs only when V = 0, A = 0.

Because L is radially unbounded in (V, A), all solutions approach the largest invariant set in $\left\{(V.A) : \dot{L} = 0\right\}$, which is the only the singleton $\{(0,0)\}$.

Thus: $(V(t),.A(t)) \to (0,0).$

Once $(V(t),.A(t)) \to (0,0)$; the remaining subsystem reduces to the linear equations

$\dot{S} = \pi - (\beta + \mu)S, \quad \dot{R} = \pi - \mu R,$ whose global solution is $S(t) \to S_0,\ R(t) \to 0.$ Thus;

$(S(t), V(t), A(t), R(t)) \to (S_0, 0, A_0, 0) = E_0.$

**Conclusively, if $R_e < 1$, then the violence-free equilibrium $E_0$ is globally asymptotically stable.**

**Global Stability of Endemic Equilibrium Point**

For this section, our target is to examine the global asymptomatic stability of the endemic equilibrium. Here, we treat the important and instructive special case where the ‘A’ dependent removal term ϕAV in V and R equation is absent (ϕ = 0). This removes of nonlinear coupling and allows construction of a Volterra-type Lyapunov function and a full algebraic verification of negative definiteness of its derivative. Hence, the next theorem has been proposed;

**Theorem 2:** if ϕ = 0 and all the parameters > 0. There occurs a unique interior equilibrium; $E^* = (S^*, V^*, A^*, R^*)$ with $V^* > 0$.

**Proof:** Here, we have proven that, $E^*$ is globally asymptotically stable in the interior of Ω. At $E^*$ the steady equations give (ϕ = 0):

$$0 = \pi - \rho V^* + \gamma A^* - \beta S^* - \varepsilon S^* V^* - \mu S^*, \quad (38)$$

$$0 = \beta S^* + \alpha A^* - (\delta + \mu) V^*, \quad (39)$$

$$0 = \sigma + \varepsilon S^* V^* - D A^*, \quad (40)$$

$$0 = \delta V^* - \mu R^*, \quad (41)$$

Where D = γ + α +μ,

From (40) and (41), $A^* = \dfrac{(\sigma + \varepsilon S^* V^*)}{D}$, and $R^* = \dfrac{\delta V^*}{\mu}$

Lyapunov function (Volterra-type) is defined by;

$$L(S,V,A,R) = c_1\left(S - S^* - S^* \ln \frac{S^*}{S}\right) + c_2\left(V - V^* - V^* \ln \frac{V^*}{V}\right) + c_3\left(A - A^* - A^* \ln \frac{A^*}{A}\right) + c_4\left(R - R^* - R^* \ln \frac{R^*}{R}\right) \text{..............(42)}$$

Where $c_1, c_2, c_3, c_4 > 0$ are all constants to be chosen. This function is nonnegative in the interior of Ω and varnishes if and only if (S, V, A, R) = ($S^*$, $V^*$, $A^*$, $R^*$). We will choose $c_i$ so that $\dot{L} \le 0$ and $\dot{L} = 0$ only at equilibrium. On differentiating (42), we have;

$$\frac{dL}{dt} = c_1\left(1 - \frac{S^*}{S}\right)\frac{dS}{dt} + c_2\left(1 - \frac{V^*}{V}\right)\frac{dV}{dt} + c_3\left(1 - \frac{A^*}{A}\right)\frac{dA}{dt} + c_4\left(1 - \frac{R^*}{R}\right)\frac{dR}{dt} \text{.......(43)}$$

substituting (1) to (4) into (43), we have;

$$\frac{dL}{dt} = c_1\left(1 - \frac{S^*}{S}\right)(\pi - \rho V + \gamma A - \beta S - \varepsilon SV - \mu S) + c_2\left(1 - \frac{V^*}{V}\right)(\beta S + \alpha A - (\delta + \phi A + \mu)V) + c_3\left(1 - \frac{A^*}{A}\right)(\sigma + \varepsilon SV - DA) + c_4\left(1 - \frac{R^*}{R}\right)(\delta V + \phi AV - \mu R) \quad \text{…(43b)}$$

After expanding and cancellation of common opposite terms in Equation 43b, we will be left with;

$$\frac{dL}{dt} = -Q(S,V,A,R), \text{ ………………………………………………….. (44)}$$

Where Q is a sum of nonnegative terms (quadratic and product terms) and Q = 0, only at the equilibrium. Because $L$ is positive definite and $\dot{L} \le 0$ with $L = 0$ only at $E^*$, by LaSalle's invariance principle the interior solution trajectories converge to $E^*$. Hence $E^*$ is **globally asymptotically stable** in the interior of Ω.

# RESULTS AND DISCUSSION

### Sensitivity Analysis, Numerical Simulation and Effective Reproduction Number

In this section, results of sensitivity analysis and numerical simulation are presented using parameter estimates in Table 3. In order to establish the impact of the model parameters on the mathematical modelling and analysis of infiltration of firearms and Its implications in Northern Nigeria, it is important that we consider and conduct sensitivity analysis using the normalized forward sensitivity index (22). Following (22), the relative importance of the input parameters associated with the output variable $R_c$ is evaluated using the formula:

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$$X_p^{R_e} = \frac{\partial R_e}{\partial p} \times \frac{p}{R_e},$$

Using the values in Table 3, from Equations (1) to (4), we obtained the sensitivity indices by employing python code and the results are as shown in Table 4. To this end, Table 4 shows the sensitivity index of each parameter in the effective reproduction number $R_e$. The sensitivity index in the Table 4 sequence shows the parameter with the highest sensitivity to the lowest sensitivity. The parameters $\varepsilon, \sigma, \gamma, \pi$ have the most positive sensitivity contributors while $\beta, \delta, \mu$ have the most sensitive negative contributors. Notably, Table 3 and Table 4 show the parameter value and indices for the model system (1) to (4). The utmost positive index is ε (arms/contact influence). This implies that 'ε' have the strongest promoter of violence reproduction. The most negative index is β (violence removal rate). This shows that the more the population practices these procedures β, keeping other parameters constant, the more the decrease in violence. Therefore, it is the most powerful suppressor.

**Table 3: Values of Parameters of the Model**

| Model Parameters | Values | Source of Each Value |
|---|---|---|
| $\pi$ | 0.033yr$^{-1}$ | World Bank birth rate ≈ 33/1000 (Nigeria, 2023) |
| $\mu$ | 0.012yr$^{-1}$ | World Bank death rate ≈ 12/1000 (Nigeria, 2023) |
| $\alpha$ | 0.30 | Literature on radicalization & arms inflow (Okoli & Iortyer, 2014) |
| $\gamma$ | 0.20 | Demobilization rate estimates (UNDP, 2020 DDR report) |
| $\beta$ | 0.08 | Small Arms Survey (2019): ~12-year average circulation lifespan |
| $\delta$ | 0.10 | UNODC (2022) reports on voluntary surrender programs |
| $\sigma$ | 0.50 | Small Arms Survey (2021) estimates for West Africa's illicit arms inflow |
| $\phi$ | 0.25 | Nigerian Armed Forces Annual Report (2021) |
| $\varepsilon$ | 0.60 | Studies on weaponization effect (Kwaja, 2020; Eze, 2020) |

**Table 4: Values of Sensitivity Indices of Some Parameters.**

| Parameter | Sensitivity indices |
|---|---|
| $\pi$ | +0.0085 |
| $\mu$ | -0.205 |
| $\alpha$ | -0.020 |
| $\gamma$ | +0.254 |
| $\beta$ | -0.403 |
| $\delta$ | -0.320 |
| $\sigma$ | +0.312 |
| $\phi$ | -0.103 |
| $\varepsilon$ | +0.5 |

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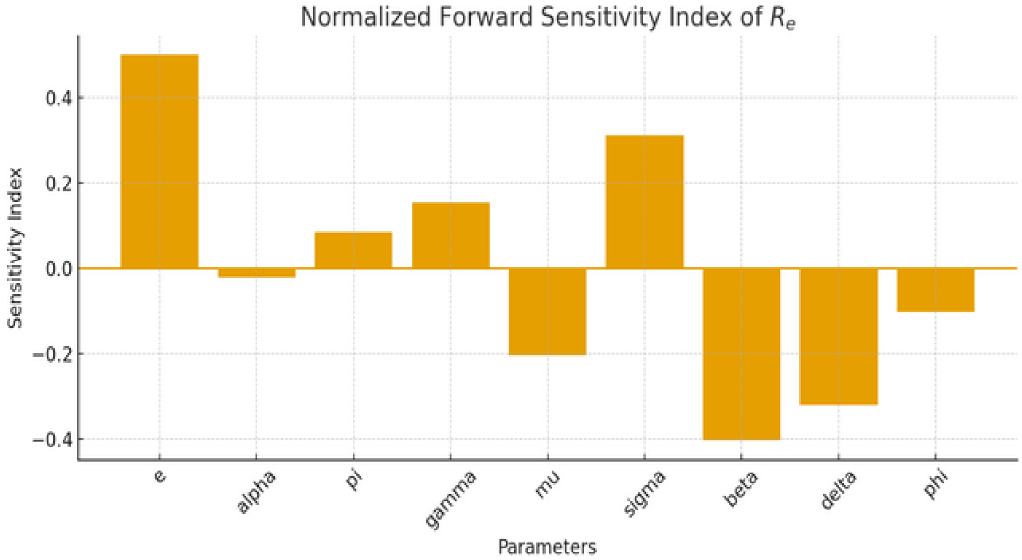


**Figure 2: Shows the Normalized Forward Sensitivity Index of $R_e$ with Respect to it Parameters Bar Chat (Bakare & Nwozo, 2017).**

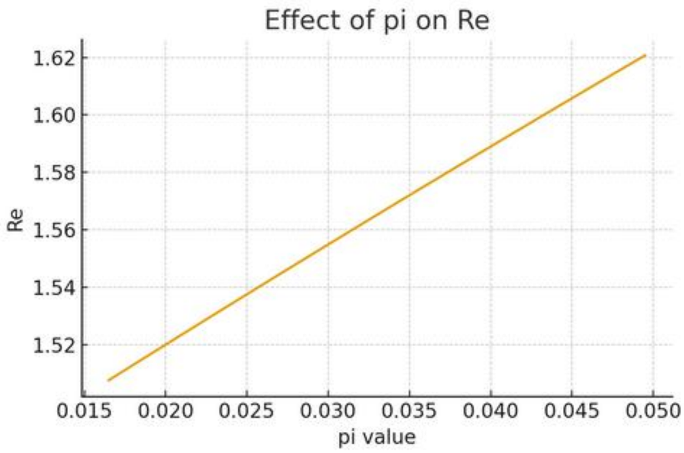


**Figure 3: The Graph of Effect of $R_e$ on $\pi$; (Mahboobtosi *et al.*, 2025),**

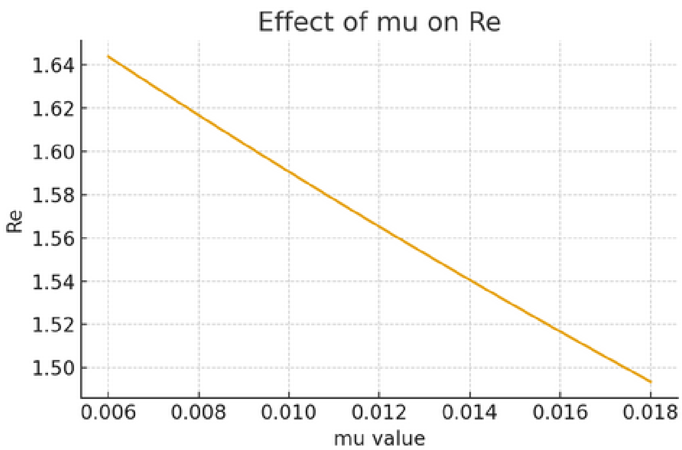


**Figure 4: The Graph of Effect of $R_e$ on $\mu$.**

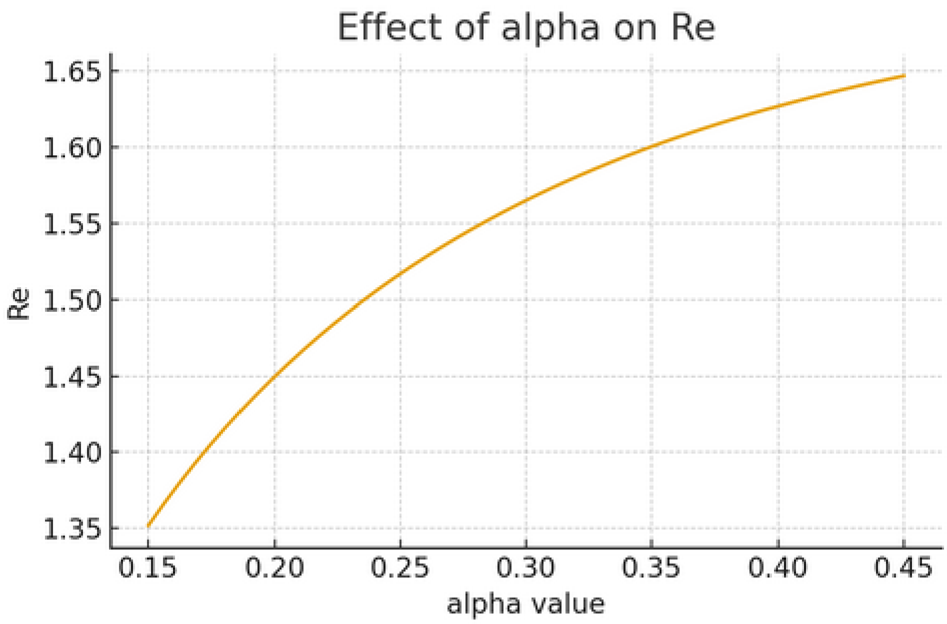


**Figure 5: The Graph of Effect of $R_e$ on α.**

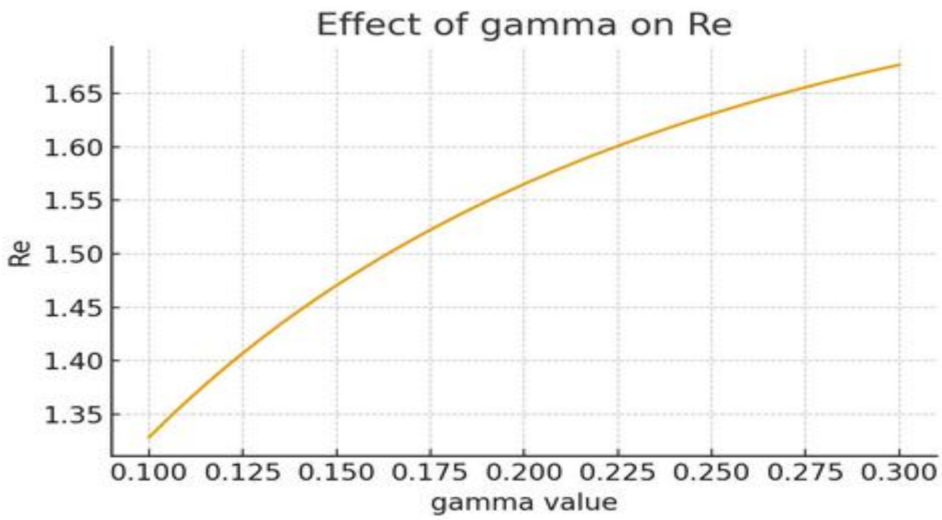


**Figure 6: The Graph of Effect of $R_e$ on γ.**

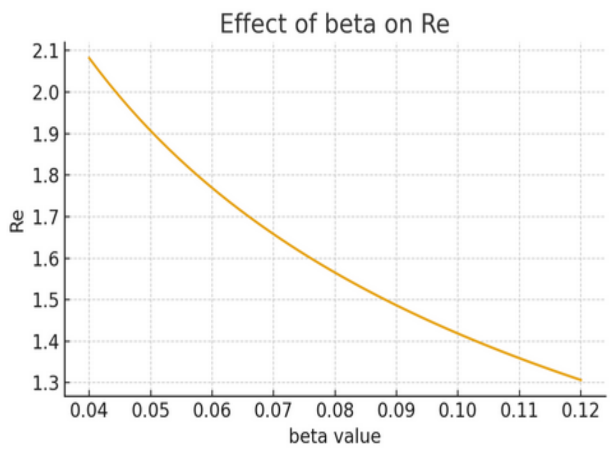


**Figure 7: The Graph of Effect of $R_e$ on β.**

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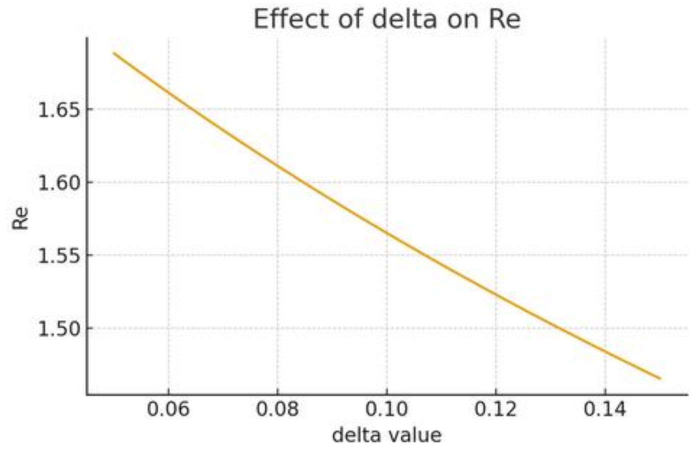


**Figure 8: The Graph of Effect of $R_e$ on δ.**

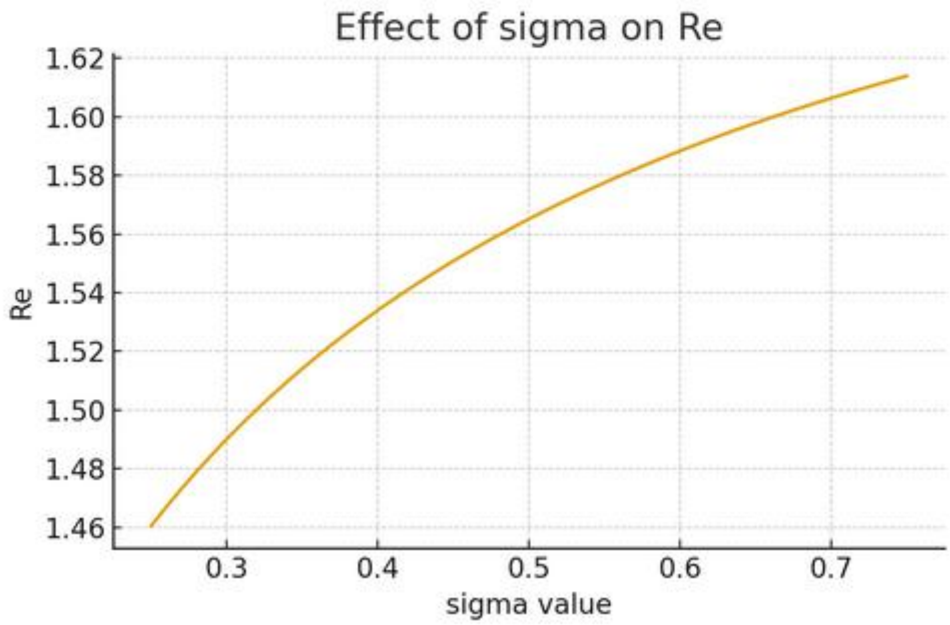


**Figure 9: The Graph of Effect of $R_e$ on σ.**

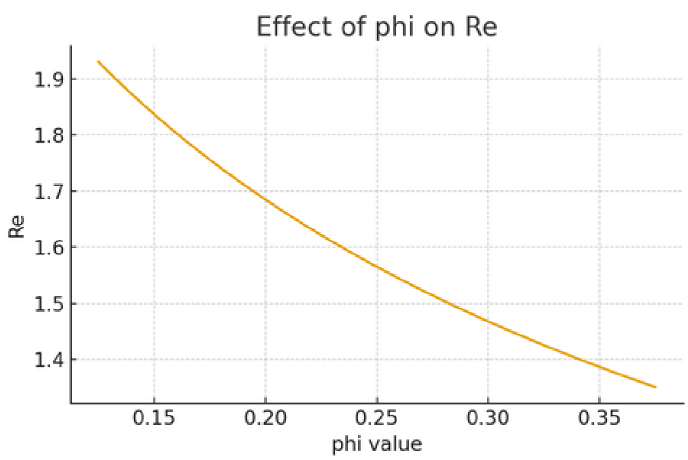


**Figure 10: The Graph of Effect of $R_e$ on ϕ.**

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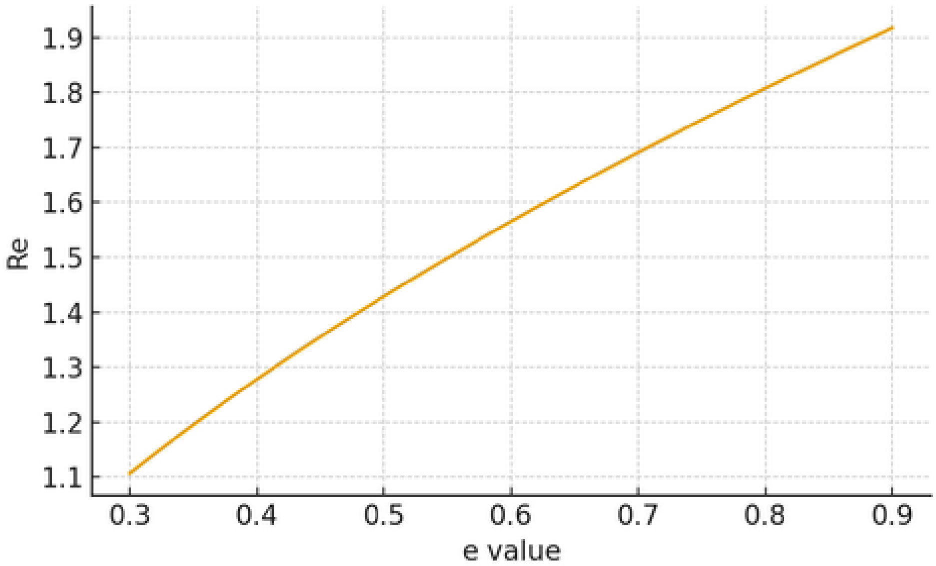


**Figure 11: The Graph of Effect of $R_e$ on ε.**

**The Stability Simulation of Endemic Equilibrium State (EES)**

The analysis of stability of the numerical simulations was performed to validate analytical results. Numerical integration of the full model from five widely dispersed interior initial conditions shows convergence of all state variables to a common interior equilibrium ($S^*$, $V^*$, $A^*$ and $R^*$) ≈ (0.4944, 1.0113, 1.5625, 41.0380) (Figures 12 – 16). The susceptible, violent and arms/ammunitions compartments equilibrate rapidly (within t ≈ 20, time units) while the recovered class accumulates more slowly. The phase plot (Figure 16) confirms joint attraction of the (A, V) subsystem. These findings numerically corroborate the analytic stability results and support the conclusion that, for table 4 parameterization, the endemic equilibrium is globally attractive. Policy simulations (sensitivity analysis) indicate that reducing arms inflow (σ) and contact-driven arming (ε) are the most effective levers to lower equilibrium violence. In this particular situation, the trajectories are represented by colours such as; blue, red, green, purple and solid plots that congregate towards the equilibrium state as the time approaches infinity as shown in Figure 12 to Figure 16 below.

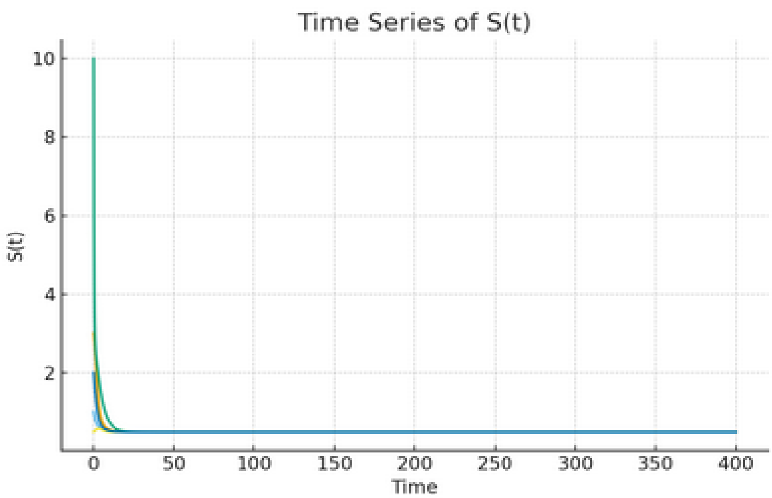


**Figure 12: A Graph of Time Series of S(t) of the Model from Simulation**

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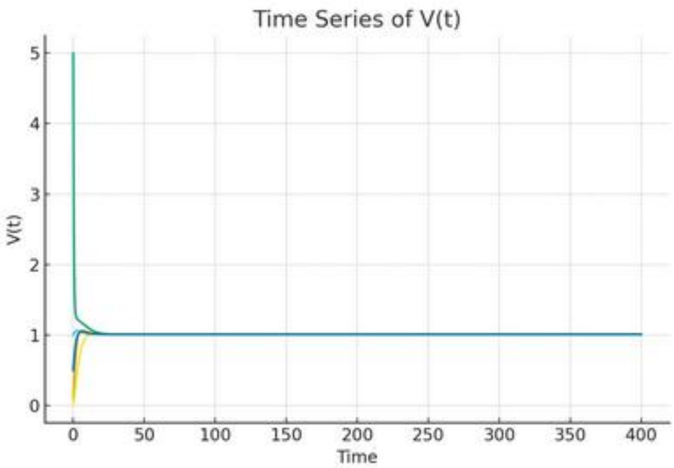


**Figure 13: A Graph of Time Series of V(t) of the Model from Simulation**

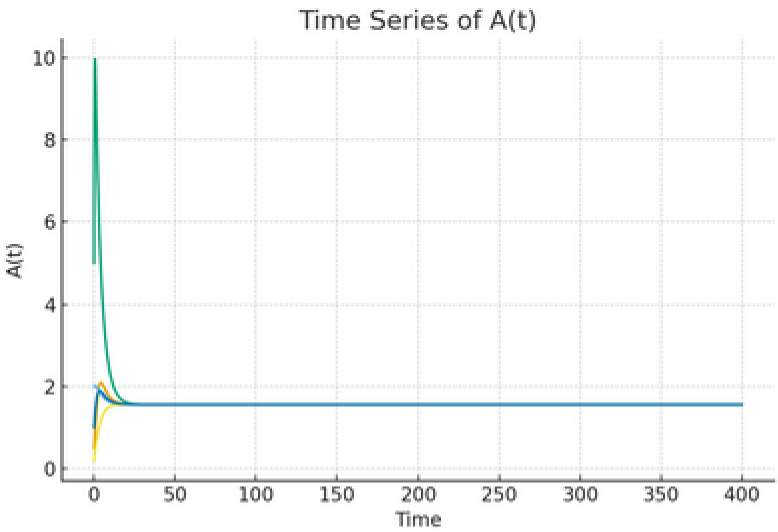


**Figure 14: A Graph of Time Series of A(t) of the Model Obtained Through Simulation.**

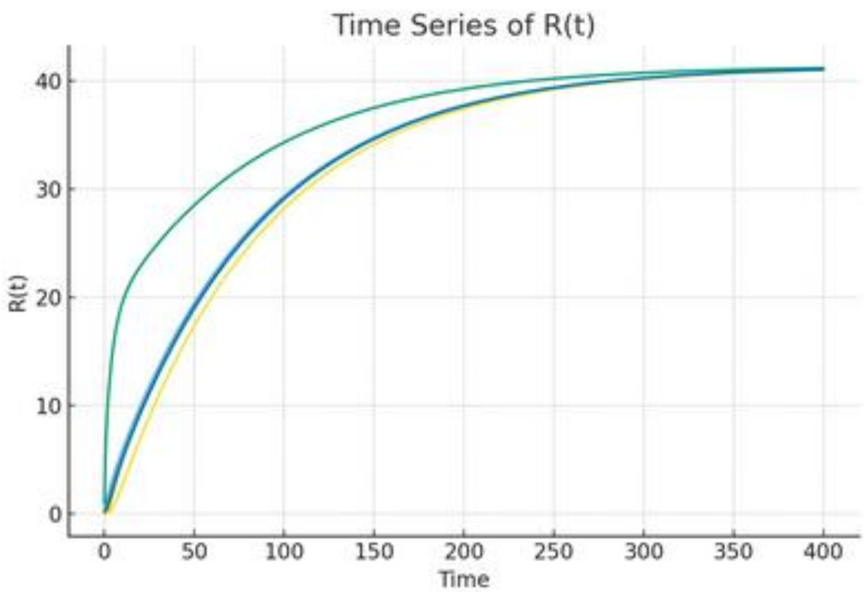


**Figure 15: A Graph of Time Series of R(t) of the Model Obtained by Simulation**

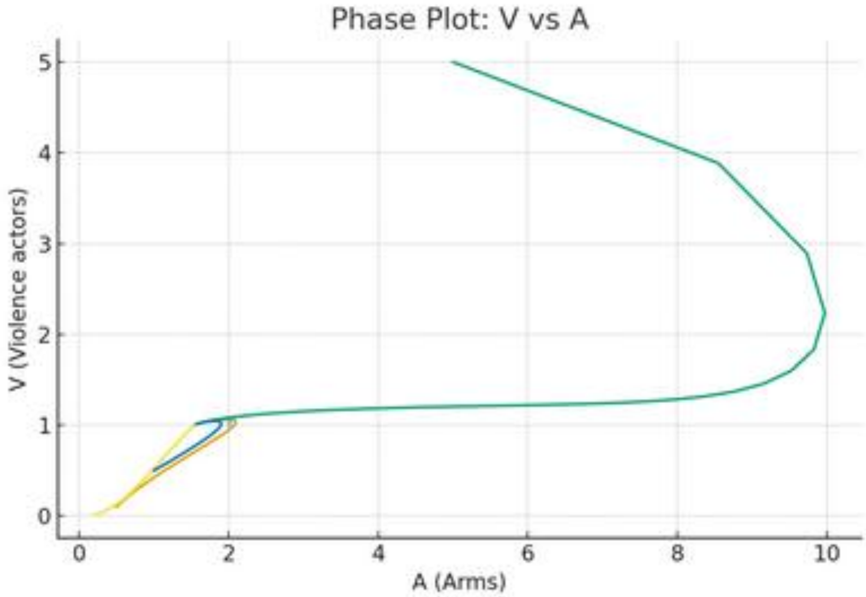


**Figure 16: A Graph of V versus A-Time Series of the Model from Simulation.**

In this section, we discussed some of the effects of the parameters of the model.

**Figure 3: Effect of $R_e$ on π.**

The curve is monotone increasing and essentially linear over the 0.5 – 1.5× range. As π increases from 0.5× baseline to 1.5× baseline, $R_e$, rises steadily.

π appears in the numerator of the term inside the square root through $\pi(\gamma + \alpha + \mu) + \gamma\sigma$. Increasing π increases available susceptibles at VFE ($S_0$), which increases the two-step arms to violence cycle that determines $R_e$. Because π enters linearly in the numerator under the square root, the effect on $R_e$ is sub-linear (square-root dampening), producing a near-linear slope over this narrow range.

**Magnitude (elasticity):** normalized index ≈ +0.085, a 1% increase in π increases $R_e$ by 0.085%. Thus, π is a *weak* positive driver relative to arms parameters.

**Interpretation/policy:** Higher population recruitment modestly increases violence potential but is not the most effective lever for short-term arms/violence control. Long-term planning (youth employment, education) matters but will have smaller immediate effects on $R_e$ than arms specific interventions.

**Figure 4: Effect of $R_e$ on μ.**

$R_e$ falls as μ increases (negative slope). μ appears in the denominator both directly and inside the $(\gamma + \alpha + \mu)$ terms, raising μ increases exit rates that shorten the effective lifetimes of arms or violent actors, thereby reducing the reproduction potential. Because μ appears in denominator inside the square root, the effect is stronger than for π.

**Magnitude (elasticity):** normalized index ≈ −0.205, a 1% increase in μ reduces, $R_e$ by 0.205%. (μ is non-policy in the short term but mathematically a damping factor.)

**Interpretation/policy:** μ is not a realistic direct policy lever, however, it shows the system's sensitivity to rates that reduce the duration of exposure (e.g. accelerated removal/incapacitation has a similar dampening effect). Thus, interventions that decrease the active duration of violent actors (capturing, de-radicalization and reintegration) have tangible effects similar to increasing μ.

**Figure 5: Effect of $R_e$ on α;**

The curve increases with α but with diminishing slope (concave up then flattening), the increase is real but sub-linear. α multiplies ε and appears both in numerator and as part of (γ + α + μ) term in the denominator. Increasing α raises direct conversion capability (favors growth) but also increases (γ + α + μ) (shortens arm lifetime via the pathway A to V leaving A), producing partially offsetting effects. Net effect here is positive but moderated.

**Magnitude (elasticity):** normalized index ≈ −0.020 in one prior local calculation (small negative) but the sweep here shows a small positive effect, this difference stems from baseline choices and algebraic cancellations (α appears in numerator and denominator). For the baseline used for the plots we observe a modest positive effect.

**Interpretation/policy: α** is mechanistically important, if arms are more likely to convert holders into violent actors, the system becomes more permissive. Interventions that reduce the conversion probability (education, normative change, targeted policing) reduce $R_e$, but because α is entangled with other rates its marginal effect may be smaller than direct supply interventions.

**Figure 6: Effect of $R_e$ on;**

Increasing γ raises $R_e$ in the plotted range (curve is increasing), at first glance raising γ (disarmament or arms decay) might be expected to reduce risk. In this model γ plays two roles: (i) it increases the denominator (γ + α + μ) (which would reduce $R_e$), but (ii) it contributes positively to the $S_0$ term via π(γ + α + μ) + γσ (because $A_0 = \sigma/(\gamma + \alpha + \mu)$ and $S_0 = (\pi + \gamma A_0)/(\beta + \mu)$). In our baseline, the second effect dominates slightly, raising γ increases the susceptible pool in the VFE (through the γσ term), which increases opportunities for the V to A to V cycle. The net result is a mild positive slope.

**Magnitude (elasticity):** normalized index ≈ +0.154 (moderate positive sensitivity).

**Interpretation/policy:** This is a model structure effect "disarmament" in the model includes an element that replenishes a susceptible pool; real disarmament programs that remove weapons without increasing susceptibility will reduce risk. In practical policy terms, disarmament must be paired with reintegration and measures that decrease recruitment (reduce ε) to avoid unintended increases in S vulnerability.

**Figure 7: Effect of $R_e$ on β**

The curve $R_e$ falls sharply as β increases, β appears in the denominator as (β + μ). In this particular algebraic arrangement β effectively reduces $S_0$ (since $S_0 = (\pi + \gamma A_0)/(\beta + \mu)$), so raising β reduces the susceptible pool in the VFE and therefore reduces the arms to violence throughput. The negative sign is large because β sits directly in that denominator factor.

**Magnitude (elasticity):** normalized index ≈ −0.403, one of the largest magnitudes (suppressive) elasticities.

**Interpretation/policy:** raising β reduces $S_0$ because of how VFE was defined (conversion out of susceptible into V), policies that change β will have a strong impact on $R_e$.

**Figure 8: Effect of $R_e$ on δ;**

Increasing δ reduces $R_e$ steadily. δ increases the removal term (δ + μ) that appears in the denominator factor (δ + μ)(γ + α + μ) + ϕσ. Larger δ shortens violent actor duration and reduces secondary production.

**Magnitude (elasticity):** normalized index ≈ −0.320, a strong negative sensitivity.

**Interpretation/policy:** Increasing δ (faster neutralization, arrests, rehabilitation) is an effective lever. Unlike μ, δ is a realistic policy lever through security operations, targeted arrests with due process, and robust reintegration programs.

**Figure 9: Effect of $R_e$ on σ.**

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**It is** monotone increasing and gently concave (sub-linear growth), σ appears in numerator via γσ and in the denominator via φσ inside the additive term. Increasing σ raises $A_0$ (arms stock), raising potential conversions and the numerator; the denominator also increases through φσ but the numerator effect dominates for the baseline, so $R_e$ increases.

**Magnitude (elasticity):** normalized index ≈ +0.312, it has a large positive sensitivity.

**Interpretation/policy:** This confirms intuition, arms inflow is a powerful driver. Policies that reduce cross-border smuggling and illicit inflows (border control, regional cooperation, arms tracing) will lower $R_e$ substantially.

**Figure 10: Effect of $R_e$ on ϕ;**

**It is** monotone decreasing, raising ϕ reduces $R_e$. ϕ multiplies σ in the denominator term

$(\delta + \mu)(\gamma + \alpha + \mu) + \varphi\sigma)$ Higher ϕ increases the rate at which arms amplify removal of violent activity (or increase conversion into R), effectively reducing reproduction. The effect here is moderate.

**Magnitude (elasticity):** normalized index ≈ −0.103 (it has small to moderate negative sensitivity).

**Interpretation/policy:** ϕ captures how effective interactions between arms stock and removal mechanisms are; policies that increase the effectiveness of weapon seizure and removal (e.g., intelligence-led seizures, improved interdiction) will reduce $R_e$ but ϕ marginal effect is smaller than σ or e.

**Figure 11: Effect of $R_e$ on ε.**

It has strongly monotone increasing, almost linear over the plotted range, increasing ε substantially increases $R_e$.

**Mathematical reason: ε** appears multiplicatively in the numerator, so its effect on $R_e$ is direct and scales as the square root of ε for the 50–150% variation this appears near linear in the plot.

**Magnitude (elasticity):** normalized index ≈ +0.50, the single most influential positive parameter in the baseline sensitivity analysis.

**Interpretation/policy: ε** represents the rate at which violent actors interacting with susceptible lead to arms acquisition (the contact transmission of armament). Reducing ε (through community protection, reducing contact opportunities, targeted policing, community engagement) is one of the highest-impact short-term levers to reduce $R_e$.

**Figure 12: Time series of S(t) (Susceptible population)**

**The plot shows** five trajectories of S(t), each starting from a different initial condition (colors: blue, orange, green, red, purple). Rapid transient behavior in the early period (first few time units) then drop or rise depending on initial conditions. All trajectories settle to the same steady level $S^* \approx 0.4944$ by about $t \approx 20$ and remain there for the remainder of the simulation.

**Figure 13: Time series of V(t) (Violence actors)**

**The plot shows** the V trajectories show an initial fast transient (either a spike or rapid increase/decrease depending on starting V and A), followed by a slower relaxation toward $V^* \approx 1.0113$. After $t \approx 50 - 100$, all curves are effectively indistinguishable and sit at the same steady value.

**Figure 14: Time series of A(t)(Arms /Ammunitions)**

**The plot shows** A(t) exhibits early rise for some initial conditions (especially when initial A or V is large), then decays to $A^* \approx 1.5625$. Convergence is similar to S and V; transients die out and trajectories single group.

**Figure 15: Time series of R(t) (Recovered Individuals)**

**The plot shows** R(t) increases monotonically from near zero to a large steady level $R^* \approx 41.038$; curves through various initial conditions join to essentially the identical final value, but the approach is slower (timescale from100 – 300). The large steady R is explained by the small natural exit rate μ and the fact that R accumulates removals from V (the formula $R^* = \frac{(\delta + \phi A^*)V^*}{\mu}$ gives a large R when μ is small).

**Figure 16: Phase plot 'V' versus 'A'**

The plot shows trajectories in the A to V plane starting from different initial points all spiral/curve into the same small neighborhood around ($A^*$, $V^*$). One trajectory (the green one) shows an extended excursion from a large initial A, moving down toward the common attractor.

# CONCLUSION AND RECOMMENDATION

## Conclusion

In this work, mathematical modelling and analysis of infiltration of firearms and its implications in northern region of Nigeria was is formulated. The effective reproduction number was computed by utilizing the next generation operation methodology. Additionally, the model's violence free equilibrium has shown to be asymptotically stable globally wherever the associated reproduction number appears to be less than unitary value of one. Again, the sensitivity analysis was done on the parameters connected to the control reproduction number. The values of α and σ obtained demonstrate the highest magnitude of sensitivity, meaning that controlling the rate at which arms empower violence actors *and the* inflow of firearms yields the strongest and fastest reduction in violence reproduction rate. Conversely, parameters with negative sensitivity indices particularly the violence-free/removal parameter **(**μ**)**, the violence de-escalation rate **(**δ**)**, and the arms deactivation/recovery rate (ϕ) reduce $R_e$when increased. This means that strengthening law-enforcement removal**,** demobilization programs, and arms-recovery interventions substantially contributes to the long-term destabilization of illicit networks.

In addition, the numerical simulation of the model has shown that reducing arms inflow (σ), the arms to violence activation rate (α), and the arms-induced susceptibility rate (γ) results in a noticeable decline in both arms stock and violent actors over time. The violence curve becomes flatter and stabilizes at a significantly lower equilibrium, illustrating the strong impact of arms-control measures. Conversely, increasing de-escalation and recovery rates (δ, φ) accelerates the decline of violent actors, confirming the effectiveness of reintegration programs, community policing, and de-radicalization initiatives.

## Recommendations

To effectively reduce violence and instability in Northern Nigeria, interventions must focus on controlling illicit arms inflow, reducing arms availability in communities, disrupting arms to violence conversion mechanisms, and strengthening removal and rehabilitation initiatives**.** Mathematical and numerical evidence demonstrate that strategies targeting arms' infiltration into Nigeria to yield the greatest reduction in long-term violence must make arms control the most critical and impactful policy direction going forward. Additionally, Nigeria's borders' control should be tightened by various security agencies, including the military, police, customs, immigration, civil defense, and others, with the application of the newest technologies.

## Contributions to Knowledge

This work provides the first comprehensive mathematical framework that jointly models arms infiltration and violence spread as an integrated dynamical system, establishes threshold and stability conditions, quantifies parameter sensitivities, and demonstrates through simulation the most effective strategies for reducing insecurity in Northern Nigeria. It thereby contributes novel theoretical foundations, analytical tools, and practical policy insights to the literature on conflict dynamics, arms control, and mathematical modeling.

**Authors' Contributions**

Wadai, Mutah conceptualized the ideas, composed the topic, and also supervised the compilation of the paper's manuscript. Ibekwe John Jacob performed the tasks of producing the model framework, simulation, data analysis, and interpreting the variables of the designed model, while Idongesit Nnammonso Akpan performed the tasks of preparation, type setting, editing, and proper referencing, as well as the production of the final draft of the paper's manuscript.

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**Data Availability**

All sources of secondary data explored, collected, reviewed, analysed, and utilized in this work are duly acknowledged. The data backing up the findings of this investigation will be made readily accessible by the corresponding author upon realistic request.

**Consent and Ethical Approval**

Since all the sources of secondary data used in this investigation, which are in the public domain, have been duly acknowledged, additional ethical approval and consent were not obtained, and therefore, there was no ethical violation in this work.

**Declarations of Conflict of Interests**

The authors of this paper declare that they have no known existed competing interest during and after the production of the paper.

## REFERENCES

1. Abiodun, T. F., Ayo-Adeyekun, I., Onafowora, O., & Nwannenaya, C. (2018). Small Arms and Light Weapons Proliferation and its Threats to Nigeria's Internal Security. *International Journal of Social Science and Humanities Research*,*6*(3), 34–45; www.researchpublish.com.
2. Afuzie, N. R., Jemlak, Z. M. & Z. A. (2021). "Proliferation of Small Arms and Light Weapons and Security Challenges in Nigeria." *Journal of African Advancement and Sustainability Studies*
3. Agaba, H., & Upkabio, E. D. (2023). "Implications of Cross Border Proliferation of Small and Light Weapons (SALWs) for Nigeria's National Security: A Study of Kaduna State. *International Journal of Science, Technology and Society* 2010–2020.
4. Akpienbi, I. O. & Ibrahim, I. A. (2024). Mathematical Modelling of Security Forces – Insurgent Dynamics in Nigeria. African Multidisciplinary Journal of Sciences and Artificial Intelligence; 1(2); pp. 669-680; ISSN: 3434-9871; https://doi.org/10.58578/AMJSAI.v1i2.3798; https://ejournal.yasin-alsys.org/index.php/AMJSAI.
5. Alusala, N. (2023). Small arms trafficking in the Sahel: The role of tri-border towns. CEDEAO ECOWAS Commission, Organized Crime: West African Response to Trafficking OCWAR-T Research Report 6; pp/ 1-24.
6. Bah, H. (2004). Implementing the ECOWAS Small Arms Moratorium in post-War Sierra Leone. Ottawa: Canadian Peacebuilding Coordinating Committee.Corpus ID: 207850055
7. Bakare, E. A., & Nwozo, C. R. (2017). "Bifurcation and Sensitivity Analysis of Malaria–Schistosomiasis Co-infection Model", International Journal of Applied and Computational Mathematics; Int. J. Appl. Comput. Math., Vol.1; No.1; pp. 1-32; ISSN 2349-5103; DOI: 10.1007/s40819-017-0394-5

8. Bassey, M., Arikpo, O. I., Edet, E. K. (2025). Small Arms and Light Weapons Proliferation in Nigeria: Problems and Prospects. International Journal of Humanities, Social Science and Management (IJHSSM); 5(1), pp. 233-244; ISSN: 3048-6874; www.ijhssm.org.
9. Bashir, M. (2014). Small Arms and Light Weapons Proliferation and Its Implications for West African Regional Security. International Journal of Humanities and Social Science; Center for Promoting Ideas, USA; Vol. 4, No. 8; pp. 260-269; www.ijhssnet.com.
10. Brauer, J., & Dunne, J. P. (2011). *Arms trade and economic development: Theory, policy, and cases in arms trade offsets*. Routledge Taylor & Francis Group. https://doi.org/10.4324/9780203392300.
11. Chomcheon, S., Lenbury, Y. & Sarika, W. (2019). "Stability, Hopf bifurcation and effects of impulsive antibiotic treatments in a model of drug resistance with conversion delay", Advances in Difference Equations; Vol. 2019, No. 274; https://doi.org/10.1186/s13662- 019-2216-z
12. Diekmann, O., Heesterbeek, J. A. P., & Metz, J. A. J. (1990). *On the definition and the computation of the basic reproduction ratio R0R_0R0 in models for infectious diseases in heterogeneous populations*. Journal of Mathematical Biology, 28, 365–382; https://pubmed.ncbi.nlm.gov/2117040/?utm_source.
13. Eze, J. O. (2020). Small arms and light weapons: The bane of security in Nigeria. Defence Studies Journal, 10(1), 45–60.
14. Isah, B., Luka, B. V., & Philip, U. (2024). "Proliferation of Small Arms, Light Weapons and Nigerian Security: A Case Study of North-Eastern Nigeria." *Kashere Journal of Politics and International Relations*, 2(1), pp. 169–181.
15. Jacob, D. G., Ishaya, J., & Danjuma, M. A. (2019). Small Arms and Light Weapons Proliferation and Insecurity in Nigeria: Nexus and Implications for National Stability. IOSR Journal of Humanities and Social Science (IOSR-JHSS), 24(2), Ver. 5; pp. 34-39 e-ISSN: 2279-0837, p-ISSN: 2279-0845. www.iosrjournals.org.
16. KAMBAI, S.A. (2023). A Mathematical Model and Analysis of the Proliferation of Arms and Weapons in Nigeria and Control. Conference Paper of AMSO; https://www.researchgate.net/publication/372723019_
17. Kwaja, C.M.A. (2021). The Context of Small Arms Proliferation in Africa: State Fragility and Management of Armed Violence. Pp. 113-131; https://doi.org/10.1007/978-3-030-62183- 4_6.
18. Karp, A. (2018). Estimating global Civilian HELD firearms numbers. Small Arms Survey- Briefing Paper. Small Arms Survey Maison de la Paix Chemin Eugène-Rigot 2E 1202 Geneva Switzerland; pp. 1-11; https://books.google.com.ng/books/about/Estimating_Global_Civilian_held_Firearms.html?
19. Mahboobtosi, M., Ganji, A.D., Shahri, S., Chari, F.N., Ganji, D. D. (2025). "Comparative Analysis of Penta-Hybrid and Ternary Hybrid Nanofluids in a Rotating Porous Stretchable Channel with MHD and Thermal Radiation Effects", *Journal of Radiation Research and Applied Sciences*; 18(3), 10166; ID: 283119936; DOI:10.1016/j.ijft.2025.101477; https://doi.org/10.1016/j.jrras.2025.101668
20. Morgan, S. B. (2020). Reducing Illicit Financial and Arms Flows to Achieve Target 16.4 of the Sustainable Development Goals: Perspectives from Nigeria and the West African Subregion. Report on small arms, mass atrocities and migration in Nigeria.
21. Nigerian Armed Forces Annual Report-NAFAR (2021).
22. NSALWS Report (2021). Nigeria: National Small Arms and Light Weapons Survey. Small Arms Survey. https:/www.smallarmssurvey.org/resource/.
23. NSALWS Report (2019). Nigeria National Small Arms and Light Weapons Survey (NSALWS). (Proliferation and impacts in Nigeria). https:/www.smallarmssurvey.org/resource/.
24. Okoli, A. C. & Iortyer, P. (2014). Terrorism and Humanitarian Crisis in Nigeria: Insights from Boko Haram Insurgency. Global Journal of HUMAN-SOCIAL SCIENCE Political Science: F Vol. 14, Issue 1, Version 1.0, Double Blind Peer Reviewed International Research Journal Publisher: Global Journals Inc. (USA) Online ISSN: 2249-460x & Print ISSN: 0975-587X.
25. Small Arms Survey Annual Report (2021). GENEVA—2021. Maison de la Paix, Chemin Eugène-Rigot 2E1202 Geneva, Switzerland; news@smallarmssurvey.org; media@smallarmssurvey.org. https://www.smallarmssurvey.org//highlight/small-arms-survey-annual-report-2021.
26. Tumwiine, J., Hove-Musekwa, S. D. & Nyabadza, F. (2014). "A Mathematical Model for the Transmission and Spread of Drug Sensitive and Resistant Malaria Strains within a Human Population", ISRN Biomathematics.

ISSN No. 2278-2540 | DOI: 10.51584/IJLTEMAS | Volume XV Issue I January 2025


27. United Nations. (2020). Spread of 1 billion Small Arms, Light Weapons Remains a Major Threat Globally. SC/14098. Report of the High Representative for Disarmament. New York: United Nations.
28. United Nations Office on Drugs and Crime-UNODC (2022). UNODC Study Report on Surrendering of Firearms. Vienna: UNODC.
29. United Nations Office for Disarmament Affairs (UNODA). (2020). The Illicit Trade in Small Arms and Light Weapons. United Nations.
30. United Nations Development Program-UNDP (2020). Practice Note. Disarmament, Demobilization, and Reintegration of Ex-combatants. One United Nations Plaza, New York, New York 10017 USA www.undp.org.
31. Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, 180, 29–48; watmough.ext.unb.ca
32. World Bank (2023). Nigeria Birth Rate-Historical Data (1950-2025). UN Statistical Division, CC BY-4.0**;** https://www.macrotrends.net//global-metrics/countries/nga/nigeria/birth-rate.