INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)
ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

Fermatean Fuzzy Maximal Subgroup and its Characterizations

A.Sheela Roselin[1], D.Jayalakshmi[2], G.Subbiah[3*]

[1]Research scholar, Reg.No.22123272092005, Department of Mathematics, Vivekananda college, Agasteeswaram, Kanyakumari-629 701, Tamilnadu, India.

[2]Associate Professor, Department of Mathematics, Vivekananda College, Agasteeswaram, Kanyakumari-629 701, Tamilnadu, India.

[3]Associate Professor, Department of Mathematics, Sri K.G.S Arts College, Srivaikuntam-628 619, Tamilnadu, India.

DOI: https://doi.org/10.51583/IJLTEMAS.2026.15020000037

Received: 18 February 2025; Accepted: 25 February 2026; Published: 05 March 2026

ABSTRACT

In this article, we investigate the concept of fermatean fuzzy characteristic subgroup of a group and fermatean fuzzy normal subgroup. The level subset of fermatean fuzzy subgroup and its properties also defined. Finally, the homomorphic image of fermatean fuzzy subgroup is also discussed.

Keywords: fuzzy set, fermatean fuzzy set, subgroup, characteristic, level subset, homomorphism.

AMS subject classification (2020): 03F55, 06D72, 08A72.

INTRODUCTION

The concept of fuzzy set is first introduced by L.A.Zadeh in 1965 [23]. After the tremendous development intuitionistic fuzzy set [4], pythagoren fuzzy set [22], (3,2) fuzzy set[15], picture fuzzy set [10], Neutrosophic fuzzy set [19] was studied. Recently fermatean fuzzy set [18] was studied by Yager and its application is briefly explained by [18]. Due to the inadequacy of FST in the sense that it admits only the membership grade, intuitionistic fuzzy sets (IFSs) was introduced [5, 6] by incorporating membership and non-membership grades with the chance for hesitation margin. Various properties of IFSs were presented [4,5] and IFSs have been applied in real-life problems [4, 5, 6]. Consequently, Biswas [8, 9] introduced intuitionistic fuzzy subgroups (IFSGs) based on IFSs, Ahn et.al. [2] deliberated on sublattice of lattice of IFSGs of a group, some properties of IFSGs were discoursed in [1]. In addition, Yuan et.al. [21] shared some light on the description of IFSGs, Bal et.al. [7] presented a brief note of kernel subgroups on IFGs and derived some properties of IFGs. Senapati.T [18] introduced fermatean fuzzy sets in 2020, an extension of intuitionistic fuzzy set (IFSs) and pythagorean fuzzy set (PFSs). AVs and NAVs of fermatean fuzzy sets reveal their dependence on greater powers with sum of cubes is less than 1. Ibrahim.H.Z introduced and applied n, m-rung orthopair fuzzy sets in MCDM.. The concepts of fuzzy modules and fuzzy sub-modules was introduced by Nagotia and Ralescu [16] in 1975. The concept of essential fuzzy modules was introduced by Hadi [13] in 2000. Using this idea. Abbas established the concept of essential fuzzy sub-modules and uniform fuzzy modules in 2012. In this article, we investigate the concept of fermatean fuzzy characteristic subgroup of a group and fermatean fuzzy normal subgroup. The level subset of fermatean fuzzy subgroup and its properties also defined. Finally, the homomorphic image of fermatean fuzzy subgroup is also discussed.

Preliminaries:

Throughout the article, the symbols S and G represent a non-empty set and a group respectively.

Definition-2.1(L.A.Zadeh): A fuzzy sub collection A of S is of the form

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

$A = \{(x, J_A(x))/x \in S\}$ where $J_A: X \to [0,1]$ is the membership grade of $x \in X$.

Definition-2.2(A.Rosenfeld): A fuzzy sub collection A of G is a fuzzy subgroup of G if

(i) $J_A(xy) \geq \min\{J_A(x), J_A(y)\}$
(ii) $J_A(x^{-1}) = J_A(x), \forall x, y \in G$.

where 'min' is a minimum operation.

Furthermore, $J_A(e) = J_A(xx^{-1}) \geq \min\{J_A(x), J_A(x)\} = J_A(x)$, $\forall x \in G$, where e is the unit element of G. Then $J_A(e)$ is the upper bound or tip of A.

Definition-2.3(K.T. Atanassov):

An intuitionistic fuzzy set A of S is of the form $A = \{(x, J_A(x), K_A(x))/x \in S\}$ where $J_A: X \to [0,1]$ and $K_A: X \to [0,1]$ are the membership and non-membership grades of $x \in X$ and $0 \leq J_A(x) + K_A(x) \leq 1$.

Definition-2.4(Yager's): A fermatean fuzzy set A of S is of the form $A = \{(x, J_A(x), K_A(x))/x \in S\}$ where $J_A: X \to [0,1]$ and $K_A: X \to [0,1]$ are the membership and non-membership grades of $x \in X$ and $0 \leq {J^3}_A(x) + {K^3}_A(x) \leq 1$.

Definition-2.5: Let α_1 and α_2 be fermatean fuzzy sets of S. Then

(i) $\alpha_1 = \alpha_2 \Leftrightarrow {J^3}_{\alpha_1}(x) = {J^3}_{\alpha_2}(x)$ and ${K^3}_{\alpha_1}(x) = {K^3}_{\alpha_2}(x)$, $\forall x \in S$.
(ii) $\alpha_1 \subseteq \alpha_2 \Leftrightarrow {J^3}_{\alpha_1}(x) \leq {J^3}_{\alpha_2}(x)$ and ${K^3}_{\alpha_1}(x) \geq {K^3}_{\alpha_2}(x)$, $\forall x \in S$.
(iii) $\alpha_1 \cap \alpha_2 = \left\{\left(x, \min\left\{{J^3}_{\alpha_1}(x), {J^3}_{\alpha_2}(x)\right\}, \max\{{K^3}_{\alpha_1}(x), {K^3}_{\alpha_2}(x)\}\right)/x \in S\right\}$
(iv) $\alpha_1 \cup \alpha_2 = \left\{\left(x, \max\left\{{J^3}_{\alpha_1}(x), {J^3}_{\alpha_2}(x)\right\}, \min\{{K^3}_{\alpha_1}(x), {K^3}_{\alpha_2}(x)\}\right)/x \in S\right\}$.

Definition-2.6: Let $A = (J_A, K_A)$ be a fermatean fuzzy set on a group X. Then A is a fermatean fuzzy subgroup of X if

(i) ${J_A}^3(x * y) \geq \min\{{J_A}^3(x), {J_A}^3(y)\}$ and

${K_A}^3(x * y) \leq \max\{{K_A}^3(x), {K_A}^3(y)\}$ for all $x, y \in X$.

(ii) ${J_A}^3(x^{-1}) \geq {J_A}^3(x)$ and ${K_A}^3(x^{-1}) \leq {K_A}^3(x)$ for all $x \in X$.

Definition-2.7: If α_1 and α_2 are fermatean fuzzy subgroups of a group G, then α_1 is a fermatean fuzzy subgroup of α_2 if $\alpha_1 \subseteq \alpha_2$, and α_1 is a proper fermatean fuzzy subgroup of α_2 if $\alpha_1 \subset \alpha_2$, which gives α_1 is strictly contained in α_2.

Definition-2.8: Let α_1 be a fermatean fuzzy subgroup of G. Then, the level set of α_1 is denoted by ${\alpha_1}^*$ and defined by ${\alpha_1}^* = \{x \in G/{J_{\alpha_1}}^3(x) \geq 0 \text{ and } {K_{\alpha_1}}^3(x) \leq 0\}$. Also ${\alpha_1}^*$ is a subgroup of G.

Definition-2.9: Suppose α_1 be a fermatean fuzzy subgroup of G. Then the strong $[\lambda, \mu]$-cuts of α_1 is denoted by $\alpha_{1[\lambda,\mu]}$ and defined by $\alpha_{1[\lambda,\mu]} = \{x \in G/{J_{\alpha_1}}^3(x) \geq \lambda \text{and} {K_{\alpha_1}}^3(x) \leq \mu\}$. Also it is a subgroup of G, where $\lambda, \mu \in [0, 1]$.

Similarly, the weak (λ, μ)-cuts of α_1 is denoted by $\alpha_{1(\lambda,\mu)}$ and defined by

$\alpha_{1(\lambda,\mu)} = \left\{x \in \frac{G}{{J_{\alpha_1}}^3(x)} > \lambda \text{ and } {K_{\alpha_1}}^3(x) < \mu\right\}$. Also it is a subgroup of G.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

Definition-2.10: A fermatean fuzzy subgroup α_1 of G commutative if

$J_{\alpha_1}{}^3(xh) = J_{\alpha_1}{}^3(hx)$ and

$K_{\alpha_1}{}^3(xh) = K_{\alpha_1}{}^3(hx)$ for all $x, h \in G$.

However, a fermatean fuzzy subgroup α_1 is commutative if G is a commutative group.

Definition-2.11:

Let α_1 be a fermatean fuzzy subgroup of a fermatean fuzzy subgroup α_2 of G. Then α_1 is a normal in α_2, denoted as $\alpha_1 < \alpha_2$ if

$J_{\alpha_1}{}^3(xh) = J_{\alpha_1}{}^3(hx)$ and $K_{\alpha_1}{}^3(xh) = K_{\alpha_1}{}^3(hx) \Rightarrow J_{\alpha_1}{}^3(xhx^{-1}) = J_{\alpha_1}{}^3(h)$ and

$K_{\alpha_1}{}^3(xhx^{-1}) = K_{\alpha_1}{}^3(h)$for all $x, h \in G$.

Definition-2.12:

Assume G and H groups and $\delta: G \to H$ is a homomorphism. Suppose α_1 and α_2 are fermatean fuzzy subgroups of G andH respectively. Then δ induces a homomorphism from α_1 to α_2 that satisfies

(i) $J_{\alpha_1}{}^3\left(\delta^{-1}(h_1h_2)\right) \geq \min\left\{J_{\alpha_1}{}^3\left(\delta^{-1}(h_1)\right), J_{\alpha_1}{}^3\left(\delta^{-1}(h_2)\right)\right\}$ and

$K_{\alpha_1}{}^3\left(\delta^{-1}(h_1h_2)\right) \leq \max\left\{K_{\alpha_1}{}^3\left(\delta^{-1}(h_1)\right), K_{\alpha_1}{}^3\left(\delta^{-1}(h_2)\right)\right\}$for all $h_1, h_2 \in H$.

(ii) $J_{\alpha_2}{}^3\left(\delta(x_1x_2)\right) \geq \min\left\{J_{\alpha_2}{}^3\left(\delta(x_1)\right), J_{\alpha_2}{}^3\left(\delta(x_2)\right)\right\}$ and

$K_{\alpha_2}{}^3\left(\delta(x_1x_2)\right) \leq \max\left\{K_{\alpha_2}{}^3\left(\delta(x_1)\right), K_{\alpha_2}{}^3\left(\delta(x_2)\right)\right\}$for all $x_1, x_2 \in G$.

Main Results:

In the classical setting, we call a subgroup H of a group G is characteristic if $M^\delta = M$ for every automorphism, δ of G, where $M^\delta = \delta(M)$. Now, we establish the analogue of this idea as follows.

Definition-3.1: Let A be fermatean fuzzy subgroup of G, then A is characteristic (δ-invariant) if

${J^3}_{A^\delta}(x) = {J^3}_A(x)$,

${K^3}_{A^\delta}(x) = {K^3}_A(x)$, for all $x \in G$ for every automorphism δ of G.

Hence $\delta(A) \subseteq A$ for every δ of $\delta \in Aut(G)$.

Definition-3.2: Let A be fermatean fuzzy subgroup in G and δ is a mapping of G. Define a fermatean fuzzy set A^δ in G by

${J^3}_{A^\delta}(x) = {J^3}_A(x^\delta)$,

${K^3}_{A^\delta}(x) = {K^3}_A(x^\delta)$, where $x^\delta = \delta(x) = x$, for all x in G.

Proposition-3.3: Let A be fermatean fuzzy subgroup in G and $x \in G$. Suppose δ is an automorphism of G for $\delta(x) = gxg^{-1}$ for all $x \in G$, then $A^g = A^\delta$.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

Proof: For A be fermatean fuzzy subgroup in G and $x \in G$, assume that $\delta: G \to G$ is characterized by $\delta(x) = gxg^{-1}$ for all $x \in G$.

Then ${J^3}_{A^g}(x) = {J^3}_A(gxg^{-1}) = {J^3}_A(\delta(x)) = {J^3}_{A^\delta}(x)$,

${K^3}_{A^g}(x) = {K^3}_A(gxg^{-1}) = {K^3}_A(\delta(x)) = {K^3}_{A^\delta}(x)$.

Proposition-3.4: Let A be fermatean fuzzy subgroup in G and δ be an automorphism of G. Then A^δ is fermatean fuzzy subgroup in G.

Proof: For $x, g \in G$, we have

${J^3}_{A^\delta}(xg) = {J^3}_A((xg)^\delta) = {J^3}_A(x^\delta g^\delta)$ and

${K^3}_{A^\delta}(xg) = {K^3}_A((xg)^\delta) = {K^3}_A(x^\delta g^\delta)$ because δ is homomorphism.

Since A is a fermatean fuzzy subgroup of G, we have

${J^3}_A(x^\delta g^\delta) \geq \min\{{J^3}_A(x^\delta), {J^3}_A(g^\delta)\} = \min\{{J^3}_{A^\delta}(x), {J^3}_{A^\delta}(g)\}$ and

${K^3}_A(x^\delta g^\delta) \leq \max\{{K^3}_A(x^\delta), {K^3}_A(g^\delta)\} = \max\{{K^3}_{A^\delta}(x), {K^3}_{A^\delta}(g)\}$.

Thus, ${J^3}_{A^\delta}(xg) \geq \min\{{J^3}_{A^\delta}(x), {J^3}_{A^\delta}(g)\}$ and ${K^3}_{A^\delta}(xg) \leq \max\{{K^3}_{A^\delta}(x), {K^3}_{A^\delta}(g)\}$.

In addition, ${J^3}_{A^\delta}(x^{-1}) = {J^3}_A((x^{-1})^\delta)$

$= {J^3}_A\left((x^\delta)^{-1}\right)$

$= {J^3}_A(x^\delta) = {J^3}_{A^\delta}(x)$ and

${K^3}_{A^\delta}(x^{-1}) = {K^3}_A((x^{-1})^\delta) = {K^3}_A\left((x^\delta)^{-1}\right)$

$= {K^3}_A(x^\delta) = {K^3}_{A^\delta}(x)$.

Hence A^δ is fermatean fuzzy subgroup in G.

Theorem-3.5: Suppose $\delta: G \to G$ is an automorphism and A is a fuzzy subgroup in G. Then

A^δ is a fermatean fuzzy subgroup in G if and only if A is a fermatean fuzzy subgroup in G.

Proof: Suppose A is a fermatean fuzzy subgroup in G. Then using the procedure in the proposition-3.4, it is certain that A^δ is a fermatean fuzzy subgroup in G.

Conversely, if A^δ is a fermatean fuzzy subgroup in G. Then

${J^3}_{A^\delta}(xg) \geq \min\{{J^3}_{A^\delta}(x), {J^3}_{A^\delta}(g)\}$ and ${J^3}_{A^\delta}(x^{-1}) = {J^3}_{A^\delta}(x)$ and

${K^3}_{A^\delta}(xg) \leq \max\{{K^3}_{A^\delta}(x), {K^3}_{A^\delta}(g)\}$ and ${K^3}_{A^\delta}(x^{-1}) = {K^3}_{A^\delta}(x)$, $\forall x, g \in G$.

Thus, ${J^3}_{A^\delta}(xg) = {J^3}_A((xg)^\delta) = {J^3}_A(\delta(xg)) = {J^3}_A(xg)$

$\Rightarrow {J^3}_A(xg) \geq \min\{{J^3}_A(x), {J^3}_A(g)\}$, $\forall x, g \in G$.

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

In addition, $J^3{}_{A^\delta}(x^{-1}) = J^3{}_A((x^{-1})^\delta)$

$$= J^3{}_A\left(\left(x^\delta\right)^{-1}\right)$$

$$= J^3{}_A\left(\left(\delta(x)\right)^{-1}\right)$$

$$= J^3{}_A(x^{-1})$$

$\Rightarrow J^3{}_A(x^{-1}) = J^3{}_A(x)$ and

$K^3{}_{A^\delta}(x^{-1}) = K^3{}_A((x^{-1})^\delta)$

$$= K^3{}_A\left(\left(x^\delta\right)^{-1}\right)$$

$$= K^3{}_A\left(\left(\delta(x)\right)^{-1}\right)$$

$$= K^3{}_A(x^{-1})$$

$\Rightarrow K^3{}_A(x^{-1}) = K^3{}_A(x)$, $\forall x \in G$.

Hence A is a fermatean fuzzy subgroup of G.

Theorem-3.6: Every fermatean fuzzy characteristic subgroup if a fermatean fuzzy group is a fermatean fuzzy normal subgroup.

Proof: Let $x, g \in G$ and let A be fermatean fuzzy characteristic subgroup.

We need to show that $J^3{}_A(xg) = J^3{}_A(gx)$ and $K^3{}_A(xg) = K^3{}_A(gx), \forall x, g \in G$.

Let us assume that δ is the automorphism of G defined by $\delta(h) = x^{-1}g'x, \forall\, g' \in G$.

Now, since A is fermatean fuzzy characteristic subgroup we have $A^\delta = A$.Then

$J^3{}_A(xg) = J^3{}_A(\delta(xg)) = J^3{}_A((xg)^\delta) = J^3{}_A(x^{-1}(xg)x) = J^3{}_A(gx)$ and

$K^3{}_A(xg) = K^3{}_A(\delta(xg)) = K^3{}_A((xg)^\delta) = K^3{}_A(x^{-1}(xg)x) = K^3{}_A(gx)$

Hence A^δ is a normal subgroup.

Remark-3.7: Let A, B, C be fermatean fuzzy subgroups in G in which $C \subseteq B \subseteq A$

(i) If C is a fermatean fuzzy characteristic subgroup of B and B is fermatean fuzzy characteristic subgroup of A, then C is a fermatean fuzzy characteristic subgroup of A.

(ii) If C is a fermatean fuzzy normal subgroup of B and B is fermatean fuzzy normal subgroup of A, then C is a fermatean fuzzy normal subgroup of A.

Proposition-3.8: If B is fermatean fuzzy subgroup in G and A is fermatean fuzzy characteristic subgroup of B, then A is a characteristic subgroup of G. In addition, A is a

characteristic subgroup of G.

Proof: It is certain that A^* is a subgroup of G.

To explain that A^* is characteristic in G, it is sufficient to show that $\delta(A^*) \subseteq A, \forall\, \delta \in Aut(G)$.

Let $\delta \in \mathrm{Aut}(G)$, then $J^3{}_{A^\delta}(x) = J^3{}_A(x)$ and $K^3{}_{A^\delta}(x) = K^3{}_A(x)$, since A is fermatean fuzzy characteristic subgroup of B.

Let $x \in A^*$, then $J^3{}_A(x) \geq 0$ and $K^3{}_A(x) \leq 0$, which gives

$J^3{}_{A^\delta}(x) = J^3{}_A(\delta(x)) = J^3{}_A(x) \geq 0$ and $K^3{}_{A^\delta}(x) = K^3{}_A(\delta(x)) = K^3{}_A(x) \leq 0$.

So $\delta(x) \in A^*$. Hence $\delta(A^*) \subset A^*$ which concludes the result.

In addition, because A is a fermatean fuzzy characteristic subgroup of B and A^* is a

characteristic subgroup of G, it shows A^* is a characteristic subgroup of B^*.

Theorem-3.9: Suppose G is finite and A is a fermatean fuzzy characteristic subgroup of B in G. Then $A_{[r,s]}$ and $A_{(r,s)}$ are characteristic subgroups of G.

Proof: It is obvious that, $A_{[r,s]}$ is a subgroup of G. Further we will clear that $A_{[r,s]}$ is characteristic in G by showing that $\delta(A_{[r,s]}) \subset A_{[r,s]}$, $\forall \delta \in \mathrm{Aut}(G)$.

For $\delta \in \mathrm{Aut}(G)$, we get $J^3{}_{A^\delta}(x) = J^3{}_A(x)$ and $K^3{}_{A^\delta}(x) = K^3{}_A(x)$, because A is a fermatean fuzzy characteristic subgroup.

Let $x \in A_{[r,s]}$. Then $J^3{}_A(x) \geq r$ and $K^3{}_A(x) \leq s$, so $\delta(x) \in A_{[r,s]}$. Thus $\delta(A_{[r,s]}) \subset A_{[r,s]}$. Hence, $A_{[r,s]}$ is a characteristic subgroup of G.

Similarly, $A_{(r,s)}$ is a characteristic subgroup of G.

Remark-3.10: Because A is a fermatean fuzzy characteristic subgroup of B, and $A_{[r,s]}$ and $A_{(r,s)}$ are characteristic subgroups of G, it follows that $A_{[r,s]}$ is a characteristic subgroup of $B_{[r,s]}$ and $A_{(r,s)}$ is a characteristic subgroup of $B_{(r,s)}$.

Theorem-3.11: Let A be a fermatean fuzzy subgroup of B in G, where G is finite. If $A_{[r,s]}$ and $A_{(r,s)}$ are characteristic subgroups of G, then A is fermatean fuzzy characteristic subgroup of B.

Proof: Since G is finite, $|A| < \infty$. Let $\mathrm{Im}(A) = \{(r_0, s_0), (r_1, s_1), \dots\dots, (r_n, s_n)\}$ with

$r_0 > r_1 > \cdots > r_n$ and $s_0 < s_1 < \cdots < s_n$.

By assumption, $A_{[r,s]} = \{x \in G / J^3{}_A(x) \geq r_j; K^3{}_A(x) \leq s_j\}$ is a characteristic subgroup of G, $\forall\, j = 0,1,2, \dots, n$.

Let $\delta \in \mathrm{Aut}(G)$. Because $J^3{}_{A^\delta}(x) = J^3{}_A(\delta(x)) = J^3{}_{\delta^{-1}(A)}(x) = J^3{}_A(x)$ and

$K^3{}_{A^\delta}(x) = K^3{}_A(\delta(x)) = K^3{}_{\delta^{-1}(A)}(x) = K^3{}_A(x)$, then $\mathrm{Im}(A^\delta) = \mathrm{Im}(A)$.

Moreover, $\forall\, p = 0,1,2, \dots, n$, we have $(A^\delta)_{[r_p, s_p]}$.

Since $x \in (A^\delta)_{[r_p, s_p]} \Leftrightarrow J^3{}_{A^\delta}(x) \geq r_p, K^3{}_{A^\delta}(x) \leq s_p \Leftrightarrow J^3{}_A(\delta(x)) \geq r_p, K^3{}_A(\delta(x)) \leq s_p$

$\Leftrightarrow \delta(x) \in A_{[r_p, s_p]} \Leftrightarrow x \in \delta^{-1}\left(A_{[r_p, s_p]}\right) \Leftrightarrow x \in A_{[r_p, s_p]}$.

Thus $A^\delta = A$ and hence A is fermatean fuzzy characteristic subgroup of B.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

In what follows we validate that theorem-3.9 and theorem-3.11 are still true by discarding the finite order of G.

Theorem-3.12: Suppose A is fermatean fuzzy subgroup of B in G. Then the following statements are the same;

(i) A is fermatean fuzzy characteristic subgroup of B.
(ii) $A_{[r,s_p]}$ and $A_{(r,s_p)}$ are fermatean fuzzy characteristic subgroups in G.

Proof: In this proof (i)⇒(ii)

Let $r, s \in \mathrm{Im}(A)$, $\delta \in \mathrm{Aut}(G)$ and $x \in A_{[r,s]}$.

Assume that A is fermatean fuzzy characteristic subgroup of B. Then

$J^3{}_A(\delta(x)) = J^3{}_A(x) \geq r$ and $K^3{}_A(\delta(x)) = K^3{}_A(x) \leq s$.

Thus $\delta(x) \in A_{[r,s]}$ and so, $\delta(A_{[r,s]}) \subseteq A_{[r,s]}$.

But $A_{[r,s]} \subseteq \delta(A_{[r,s]})$ by symmetry.

Let $x \in A_{[r,s]}$ and $g \in G$ where in $\delta(g) = x$.

Then, $J^3{}_A(g) = J^3{}_A(\delta(g)) = J^3{}_A(x) \geq r$ and $K^3{}_A(g) = K^3{}_A(\delta(g)) = K^3{}_A(x) \leq s$.

$\Rightarrow g \in A_{[r,s]}$, thereforex $\in \delta(A_{[r,s]})$.

Hence $A_{[r,s]} \subseteq \delta(A_{[r,s]})$.

Thus, $A_{[r,s]}$ is a fermatean fuzzy characteristic subgroup in G.

Similarly, $A_{(r,s)}$ is a fermatean fuzzy characteristic subgroup in G.

Next, we prove (ii)⇒(i).

Let $x \in G$, $\delta \in \mathrm{Aut}(G)$ and $J^3{}_A(x) = r$, $K^3{}_A(x) = s$.

Then $x \in A_{[r,s]}$ and $x \notin A_{[r^*,s^*]}$ for $r^*, s^* > r, s$.

By hypothesis $\delta(A_{[r,s]}) = A_{[r,s]}$, so $\delta(x) \in A_{[r,s]}$ and hence

$J^3{}_A(x) = J^3{}_A(\delta(x)) \geq r$ and $K^3{}_A(x) = K^3{}_A(\delta(x)) \leq s$.

Let $r^* = J^3{}_A(\delta(x))$ and $s^* = K^3{}_A(\delta(x))$.

Assume $r^*, s^* > r, s$, then $\delta(x) \in A_{[r^*,s^*]} = \delta(A_{[r^*,s^*]})$.

Because δ is injective, we get $x \in A_{[r^*,s^*]}$, which is a contradiction.

Thus, $J^3{}_A(\delta(x)) = r = J^3{}_A(x)$ and $K^3{}_A(\delta(x)) = s = K^3{}_A(x)$.

⇒A is fermatean fuzzy characteristic subgroup of B.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

Theorem-3.13: Let A be a fermatean fuzzy subgroup that is normal in a fermatean fuzzy subgroup B in G and assume δ is an automorphism of G which leaves an invariant subgroup A^*. Then δ induces an automorphism δ^* of B/A defined by $\delta^*(xA) = \delta(x)A$, $\forall x \in G$.

Proof: To start with, we confirm that δ^* is well defined.

Suppose $x, g \in G$ such that $xA = gA$. Then, it is sufficient to prove that $\delta(x)A = \delta(g)A$.

Since $xA = gA$, we get ${J^3}_{xA}(x) = {J^3}_{gA}(x)$ and ${J^3}_{xA}(g) = {J^3}_{gA}(g)$

$\Rightarrow {J^3}_A(e) = {J^3}_A(g^{-1}x)$ and ${J^3}_A(x^{-1}g) = {J^3}_A(e)$

$\Rightarrow {J^3}_A(g^{-1}x) = {J^3}_A(x^{-1}g) = {J^3}_A(e) \Rightarrow g^{-1}x, x^{-1}g \in A^*$.

Similarly, ${K^3}_{xA}(x) = {K^3}_{gA}(x)$ and ${K^3}_{xA}(g) = {K^3}_{gA}(g)$.

$\Rightarrow {K^3}_A(e) = {K^3}_A(g^{-1}x)$ and $K^3(x^{-1}g) = {K^3}_A(e)$.

$\Rightarrow {K^3}_A(g^{-1}x) = K^3(x^{-1}g) = {K^3}_A(e)$.

$\Rightarrow g^{-1}x$, $x^{-1}g \in A^*$.

By assumption, $\delta(A^*) = A^*$, then $\delta(g^{-1}x)$, $\delta(x^{-1}g) \in A^*$.

Thus, ${J^3}_A\big(\delta(g^{-1}x)\big) = {J^3}_A\big(\delta(x^{-1}g)\big)$ and ${K^3}_A\big(\delta(g^{-1}x)\big) = {K^3}_A\big(\delta(x^{-1}g)\big)$.

Now, let $x \in G$, then

$$
\begin{aligned}
{J^3}_{\delta(x)A}(x^*) &= {J^3}_A(\delta(x^{-1})x^*) \\
&= {J^3}_A(\delta(x^{-1})\delta(g)\delta(g^{-1})x^*) \\
&\geq \min\{{J^3}_A\big(\delta(x^{-1})\delta(g)\big), {J^3}_A(\delta(g^{-1})x^*)\} \\
&= \min\{{J^3}_A\big(\delta(x^{-1}g)\big), {J^3}_A(\delta(g^{-1})x^*)\} \\
&= {J^3}_{\delta(g)A}(x^*) \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
{K^3}_{\delta(x)A}(x^*) &= {K^3}_A(\delta(x^{-1})x^*) \\
&= {K^3}_A(\delta(x^{-1})\delta(g)\delta(g^{-1})x^*) \\
&\leq \max\{{K^3}_A\big(\delta(x^{-1})\delta(g)\big), {K^3}_A(\delta(g^{-1})x^*)\} \\
&= \max\{{K^3}_A\big(\delta(x^{-1}g)\big), {K^3}_A(\delta(g^{-1})x^*)\} \\
&= {K^3}_{\delta(g)A}(x^*) .
\end{aligned}
$$

Thus, ${J^3}_{\delta(x)A}(x^*) \geq {J^3}_{\delta(g)A}(x^*)$ and ${K^3}_{\delta(x)A}(x^*) \leq {K^3}_{\delta(g)A}(x^*)$.

Similarly, ${J^3}_{\delta(g)A}(x^*) \geq {J^3}_{\delta(x)A}(x^*)$ and ${K^3}_{\delta(g)A}(x^*) \leq {K^3}_{\delta(x)A}(x^*)$.

Since $x^* \in G$ is arbitrary, then ${J^3}_{\delta(x)A}(x^*) = {J^3}_{\delta(g)A}(x^*)$ and ${K^3}_{\delta(x)A}(x^*) = {K^3}_{\delta(g)A}(x^*)$.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

$\Rightarrow \delta(x)A = \delta(g)A$.

Thus, δ is well defined.

In addition, let $x, g \in G$. Because δ is homomorphism,

$\delta(xg) = \delta(x)\delta(g)$ and so, $\delta(xg)A = \delta(x)A\,\delta(g)A$.

Hence $\delta^*(xgA) = \delta^*(x)A\,\delta^*(g)A$ and $\delta^*(xAgA) = \delta^*(xA)\delta^*(gA)$.

Therefore, $\delta^*$is homomorphism.

Corollary-3.14: Using the theorem-3.13 is an automorphism if G is finite and δ is an automorphism.

Proof: Let $\delta(x) = \delta(g)$, for $x, g \in G$.

Then $\delta(x)A = \delta(g)A$, which gives $\delta^*(xA) = \delta^*(gA)$.

Hence $xA = gA$, since δ^* is injective.

Thus, ${J^3}_{xA}(g) = {J^3}_{gA}(g)$.

$\Rightarrow {J^3}_A(x^{-1}g) = {J^3}_A(e)$ and ${K^3}_A(g) = {K^3}_{gA}(e)$.

$\Rightarrow {J^3}_A(x^{-1}g) = {J^3}_A(e)$.

So, $x^{-1}g \in A^*$ and $x^{-1}g = e$, since $A^* = \{e\}$.

Thus, ${J^3}_A(x) = {J^3}_A(g)$ and ${K^3}_A(x) = {K^3}_A(g) \Rightarrow x = g$.

Hence δ is injective, which completes the proof.

Corollary-3.15: Let a fermatean fuzzy subgroup A be normal in a fermatean fuzzy subgroup B in G. If δ is an automorphism of G and $A^\delta = A$, then δ induces an automorphism of δ^* of B defined by $\delta^*(xA) = \delta(x)A$, for all $x \in G$.

Proof: Let $x, g \in G$. Then $xA = gA \Rightarrow xA^\delta = gA^\delta$.

$\Leftrightarrow {J^3}_{xA^\delta}(x^*) = {J^3}_{gA^\delta}(x^*), {K^3}_{xA^\delta}(x^*) = {K^3}_{gA^\delta}(x^*), \forall x^* \in G$.

$\Rightarrow {J^3}_{A^\delta}(x^{-1}x^*) = {J^3}_{A^\delta}(g^{-1}x^*), {K^3}_{A^\delta}(x^{-1}x^*) = {K^3}_{A^\delta}(g^{-1}x^*), \forall x^* \in G$.

$\Rightarrow {J^3}_A(\delta(x^{-1}x^*)) = {J^3}_A(\delta(g^{-1}x^*)), {K^3}_A(\delta(x^{-1}x^*)) = {K^3}_A(\delta(g^{-1}x^*)), \forall x^* \in G$.

$\Rightarrow {J^3}_A(\delta(x^{-1})\delta(x^*)) = {J^3}_A(\delta(g^{-1})\delta(x^*)), {K^3}_A(\delta(x^{-1})\delta(x^*)) = {K^3}_A(\delta(g^{-1})\delta(x^*))$,

$\forall x^* \in G$.

$\Rightarrow {J^3}_{\delta(x)A}(\delta(x^*)) = {J^3}_{\delta(g)A}(\delta(x^*)), {K^3}_{\delta(x)A}(\delta(x^*)) = {K^3}_{\delta(g)A}(\delta(x^*))$.

Thus , $\delta^*$is well defined and injective. Certainly, δ^*maps B/A onto itself. Because δ^* is homomorphism, then the proof is completed by the similar wave length of the proof

theorem-3.13.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

Theorem-3.16: Suppose $\delta: G \rightarrow G'$ is a homomorphism, A and B are fermatean fuzzy subgroups of G and G′ respectively. Then $\delta(A) \subseteq B$ implies $J^3{}_B(\delta(x)) \geq J^3{}_A(x)$ and $K^3{}_B(\delta(x)) \leq K^3{}_A(x)$, for all $x \in G$.

Proof: Let $\delta(A) \subseteq B$ and $x \in G$.

Then, $J^3{}_A(\delta(x)) \geq J^3{}_{\delta(A)}(\delta(x))$

$= \min\{J^3{}_A(g^*)/\delta(g^*) = \delta(x)\}$

$= J^3{}_A(x)$ and

$K^3{}_A(\delta(x)) \leq K^3{}_{\delta(A)}(\delta(x))$

$= \max\{K^3{}_A(g^*)/\delta(g^*) = \delta(x)\}$

$= K^3{}_A(x)$.

By inversely, let $J^3{}_B(\delta(x)) \geq J^3{}_A(x)$ and $K^3{}_B(\delta(x)) \geq K^3{}_A(x)$, for all $x \in G$.

Then, $J^3{}_{\delta(A)}(g) = \max\{J^3{}_A(x)/\delta(x) = g\}$

$\leq \max\{J^3{}_B(\delta(x))/\delta(x) = g\}$

$= J^3{}_B(g)$ and

$K^3{}_{\delta(A)}(g) = \min\{K^3{}_A(x)/\delta(x) = g\}$

$\geq \min\{K^3{}_B(\delta(x))/\delta(x) = g\}$

$= K^3{}_B(g)$ for all $g \in G'$.

Hence, $\delta(A) \subseteq B$.

CONCLUSION

In this article, the concepts of fermatean fuzzy characteristic maximal subgroups were shown. Various results concerning fermatean fuzzy characteristic subgroups were obtained and explained in detail.

ACKNOWLEDGEMENT

The authors are highly greatful to the refees for their valuable comments and suggestions to improve the paper.

REFERENCES

1. Ahn. T. C, Hur. K, Jang. K. W & Roh. S. B (2006), Intuitionistic fuzzy subgroups,
2. Honam Mathematical Journal, 28(1), 31–44.
3. Ahn. T. C, Jang. K. W, Roh. S. B & Hur. K (2005), A note on intuitionistic fuzzy
4. subgroups, Proceedings of KFIS Autumn Conference 2005, 15(2), 496–499.
5. Anthony. J. M & Sherwood. H (1982), A characterization of fuzzy subgroups, Fuzzy Sets and Systems, 7(3), 297–305.
6. Atanassov. K. T (1983), Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, June 1983
7. (Deposed in Central Science-Technical Library of Bulgaria Academic of Science, 1697/84)
8. (in Bulgarian). Reprinted: International Journal Bioautomation, 2016, 20(S1), S1–S6.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue II, February 2026

9. Atanassov. K. T (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1), 87–96.
10. Atanassov. K. T (1994), New operations defined over the intuitionistic fuzzy sets, Fuzzy
11. Sets and Systems, 61(2), 137–142.
12. Bal. M, Ahmad. K.D, Hajjari. A.A & Ali. R (2022), A short note on the kernel subgroup of intuitionistic fuzzy groups, Journal of Neutrosophic and Fuzzy Systems, 2(1), 14–20.
13. Biswas. R (1989), Intuitionistic fuzzy subgroups, Mathematical Forum, 10, 37–46.
14. Biswas. R (1997), Intuitionistic fuzzy subgroups, Notes on Intuitionistic Fuzzy Sets, 3(2),
15. 53–60.
16. B. C. Cuong, Picture fuzzy sets-first results part 2, Seminar Neuro-Fuzzy Systems with Applications, Preprint 04/2013, Institute of Mathematics, Journal of Computer Science and Cybernetics, 30(4)(2014), 409–420.
17. Ejegwa. P. A, Ajogwu. C. F & Sarkar. A (2023), A hybridized correlation coefficient
18. technique and its application in classification process under intuitionistic fuzzy setting,
19. Iranian Journal of Fuzzy Systems, 20(4), 103–120.
20. Fathi. M & Salleh. A. R (2009), Intuitionistic fuzzy groups, Asian Journal of Algebra,
21. 2(1), 1–10.
22. Hadi. I.M, On some special fuzzy ideal of fuzzy ring, Accepted in J. Soc. of
23. Phy-Math(2000).
24. Hamil. M.A, Semi prime fuzzy modules, Ibn. Al-Haitham J. pure and applied science, 25(1) (2012), 1-10.
25. H. Z. Ibrahim, T. M. Al-shami and O. G. Elbarbary, (3, 2)-fuzzy sets and their applications to topology and optimal choice, Computational Intelligence and Neuroscience, **2021** (2021), 14 pages.
26. Nagotia. X.V and Ralescu.D, Application of fuzzy sets and system analysis, Birkhauser, Basel, 1975.
27. Rosenfeld. A (1971), Fuzzy groups, Journal of Mathematical Analysis and Applications,
28. 35(3), 512–517.
29. T. Senapati and R.R. Yager, Fermatean fuzzy sets, Journal of Ambient Intelligence and Humanized computing, 11, (2) (2020), 663-674.
30. F. Smarandache (2003), Definition of Neutrosophic Logic – A Generalization of the Intuitionistic Fuzzy Logic, Proceedings of the Third Conference of the European Society for Fuzzy Logic and Technology, EUSFLAT 2003, September 10-12, 2003, Zittau, Germany; University of Applied Sciences at Zittau/Goerlitz, 141-146.
31. Xu. C (2008), New structures of intuitionistic fuzzy groups, In: Huang. D. S, Wunsch.
32. D. C, Levine D. S, & Jo. K. H (eds), Advanced Intelligent Computing Theories and Applications, With Aspects of Contemporary Intelligent Computing Techniques. Communications in Computer and Information Science, vol 15. Springer, Berlin,Heidelberg, 145–152.
33. Yuan. X. H, Li. H. X and Lee. E. S (2010), On the definition of the intuitionistic fuzzy
34. Subgroups, Computers and Mathematics with Applications, 59(9), 3117–3129.
35. R.R.Yager, Pythagorean fuzzy subsets, In:2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 2013, 36286152.
36. Zadeh. L. A (1965), Fuzzy sets, Information and Control, 8(3), 338–353.