

Analysis of Magneto-Poroelastic Wave Propagation Under Combined Static and Initial Stress

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ABSTRACT

This study investigates the two-dimensional vibrational response of a poroelastic medium subjected to both an initial static stress state and an external magnetic field. The theoretical formulation is based on Biot's theory of poroelasticity and incorporates electromagnetic effects through Maxwell's equations, enabling a fully coupled description of solid deformation, pore-fluid pressure, and magnetoelastic interactions.

The governing equations account for mechanical pre-stress, fluid solid coupling, magnetic body forces, and variations in pore pressure. From these equations, a generalized wave equation is derived, and closed-form solutions for displacement components, stress fields, and pore pressure are obtained for harmonic wave propagation. The influence of key material and field parameters, including porosity, permeability, magnetic field intensity, and initial stress, on wave number and frequency characteristics is systematically analyzed.

Numerical results demonstrate that both magnetic loading and pre-stress significantly modify the effective stiffness of the medium, leading to noticeable changes in wave dispersion behavior. The outcomes of this work are relevant to applications in geomechanics, seismo-electromagnetic phenomena, and the development of magneto-sensitive porous materials.

Keywords: Poroelasticity; Biot theory; Magnetoelasticity; Initial stress; Two-dimensional wave propagation; Coupled field modelling; Pore-fluid interaction; Magnetic body forces; Compressional and shear waves; Elastic porous media.

INTRODUCTION

Poroelastic solids consist of a deformable porous skeleton permeated by fluid, and their mechanical behaviour depends on the interaction between solid deformation and fluid movement. Biot's theory offers an established foundation for describing such media, predicting the existence of two compressional waves fast and slow and a shear wave, each influenced by factors such as porosity, permeability, and the strength of solid-fluid coupling.

Understanding the vibrational response of poroelastic materials is essential in various applications, including subsurface characterization, soil-structure interaction, seismic exploration, acoustic damping, and smart material engineering. In many practical engineering and geological environments, poroelastic materials are subject not only to dynamic disturbances but also to pre-existing static stresses.

These initial stresses may originate from in situ geological loads, residual manufacturing stresses, or deliberate pre-stressing in structural components. Such stresses significantly influence the elastic response of porous media, modifying wave velocities, deformation patterns, and dynamic stability.

Therefore, incorporating initial stress into vibration analysis yields a more realistic representation of natural and engineered systems. Another influential factor is the presence of a magnetic field, especially in porous media containing electrically conductive fluids or magnetic particles.

Magnetoelastic coupling arising from the interaction of mechanical deformation with electromagnetic fields governed by Maxwell's equations—introduces additional forces that can stiffen or soften the medium. Magnetic fields can alter wave speeds, control attenuation, and enable tunable properties, making them relevant in sensors, non-destructive assessment, and magneto-sensitive composite structures.

Although poroelasticity, initial stress, and magnetoelasticity have each been studied independently, the combined influence of these factors on two-dimensional vibrations has received limited attention. In reality, many systems experience mechanical loading, pore-fluid pressure effects, and magnetic influences simultaneously, leading to complex multiphysics behaviour. Addressing these interactions requires an integrated mathematical formulation that accounts for all coupling mechanisms.

Biot [1] first formulated the fundamental theory describing the propagation of elastic waves through a porous solid saturated with fluid. His subsequent work [2] further expanded the mechanics governing deformation and acoustic behavior in such media.

Willson [3] examined the behavior of magnetoelastic plane waves, while Narain [4] analyzed torsional magnetoelastic waves in an initially stressed bar. Mahmoud [5] explored how magnetic fields and initial stress influence wave propagation in bone.

Pandey and Sharma [6] studied shear wave behavior in a dissipative magneto-poroelastic isotropic medium, and Singh and S. Singh [7] investigated shear wave propagation in a magneto-poroelastic layer enclosed between two media. The combined influence of initial stress and rotation on magnetoelastic waves in an isotropic half-space was reported by Abd-Alla [8], who also examined magnetic field and initial stress effects in poroelastic materials [9].

Lopatnikov [10] developed a thermodynamically consistent formulation for magneto-poroelastic materials, while Dorfmann and Ogden [11] provided insight into nonlinear interactions in magnetoelastic systems. Ramagiri et al. [12] analyzed the role of initial stress on torsional vibrations in an anisotropic magnetoporoelastic hollow cylinder.

Magneto-poroelastic wave propagation under initial stress was considered by AlShujairi and Younis [13], and Sharma and Tomar [14] studied the combined effects of magnetic fields and initial stress on coupled waves in poroelastic solids.

Singh and Rai [15] examined the dynamic behavior of prestressed poroelastic media subjected to a magnetic field within a generalized thermoelastic framework. Magneto-thermo-poroelastic wave behavior in fluid-filled porous materials under initial stress was investigated by Sahu and Acharya [16].

Al-Bazaz and Othman [17] explored temperature-dependent magneto-poroelastic wave characteristics in media with pre-existing stress. Plane wave propagation under magnetic fields and initial stress, based on Biot's theory, and was discussed by Singh and Tripathi [18].

El-Sayed and Abd-Alla [19] studied wave propagation in a magneto-poroelastic half-space with uniform initial stress, while Pandey and Gupta [20] analyzed the impact of magnetic fields and prestress on wave dispersion in porous elastic structures. The effect of magnetic fields on transversely isotropic poroelastic materials was presented by Manjula and Sree Lakshmi [21], and Manjula Ramagiri [22] investigated wave motion in magneto-thermoelastic solids in the presence of static stress.

The present study introduces a unified framework for analyzing two-dimensional vibrations in a poroelastic medium subjected to both static stress and a transverse magnetic field. By integrating Biot's poroelastic theory with magnetoelastic and initial stress effects, a comprehensive set of governing equations is formulated.

Analytical solutions for harmonic wave propagation are derived, and the influences of poroelastic material parameters, magnetic field strength, and pre-stress levels on wave dispersion and modal behavior are systematically evaluated. The findings contribute to an improved understanding of coupled multi-field dynamics

in porous media and have direct relevance to seismic wave modelling, geomechanics, and the design of magneto-responsive smart materials.

Governing equations

Consider an isotropic poroelastic solid in cartesian coordinate system (x, y, z) . Let (u, v, w) and (U, V, W) be the solid and fluid displacements. The governing equations are taken from [23], with initial stress and magnetic field, fluid pressures are given in [1, 3].

$$\begin{aligned}
 [1 + (1 - \varrho) \frac{\partial w_0}{\partial z}] [\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}] + [1 + 2\varrho \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{xz}}{\partial z} - P_0 [\frac{\partial w_3}{\partial y} + \frac{\partial w_2}{\partial z}] + F_1 &= \frac{\partial^2}{\partial t^2} (\rho_{11}u + \rho_{12}U), \\
 [1 + (1 - \varrho) \frac{\partial w_0}{\partial z}] [\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}] + [1 + 2\varrho \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{yz}}{\partial z} - P_0 \frac{\partial w_3}{\partial x} + F_2 &= \frac{\partial^2}{\partial t^2} (\rho_{11}v + \rho_{12}V), \\
 [1 + (1 - \varrho) \frac{\partial w_0}{\partial z}] [\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y}] + [1 + 2\varrho \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{zz}}{\partial z} + P_0 \frac{\partial w_2}{\partial z} + F_3 &= \frac{\partial^2}{\partial t^2} (\rho_{11}w + \rho_{12}W), \\
 \frac{\partial s}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{12}u + \rho_{22}U), \\
 \frac{\partial s}{\partial y} &= \frac{\partial^2}{\partial t^2} (\rho_{12}v + \rho_{22}V), \\
 \frac{\partial s}{\partial z} &= \frac{\partial^2}{\partial t^2} (\rho_{12}w + \rho_{22}W).
 \end{aligned}
 \tag{1}$$

In eq. (1) σ_{ij} are the stresses, s is the fluid pressure. $\rho_{11}, \rho_{12}, \rho_{22}$ are the mass coefficients. F_1, F_2, F_3 are the components of Lorentz force long the x, y, z directions. Taking into the account the absence of displacement current the linearized Maxwell equations governing the electro-magnetic fields for slowly moving solid medium having electrical conductivity are [24]

$$\text{Curl } \vec{h} = \vec{J}, \text{Curl } \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}, \text{div } \vec{h} = 0, \text{div } \vec{E} = 0, \vec{h} = \text{Curl}(\vec{\mu} \times H_0),
 \tag{2}$$

In eq. (2) $\vec{E}, \vec{J}, H_0, \mu_0, \vec{h}$ are the electric intensity, electric current density, primary magnetic field, magnetic permeability, perturbed magnetic field over the constant primary magnetic field. Solving \vec{J} of eq. (2) and then put the value of \vec{J} in the equation of Lorentz force $\vec{F} = \mu_0(\vec{J} \times H_0)$ we get the components of Lorentz force as

$$\begin{aligned}
 F_1 &= \mu_0 H_0^2 (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}), F_2 = \mu_0 H_0^2 (\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}), F_3 = \mu_0 H_0^2 (\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial z \partial y} + \frac{\partial^2 w}{\partial z^2}), \\
 w_2 &= \frac{1}{2} (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}), w_3 = \frac{1}{2} (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}).
 \end{aligned}
 \tag{3}$$

Now considering the problem in $x-z$ direction and substituting the eq. (3) in eq. (1) we get the following equations

$$\begin{aligned}
 & [1 + (1 - \mathcal{G}) \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{xx}}{\partial x} + [1 + 2\mathcal{G} \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{xz}}{\partial z} + [\mu_0 H_0^2 - P_0] \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right] = \frac{\partial^2}{\partial t^2} (\rho_{11} u + \rho_{12} U), \\
 & [1 + (1 - \mathcal{G}) \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{zz}}{\partial x} + [1 + 2\mathcal{G} \frac{\partial w_0}{\partial z}] \frac{\partial \sigma_{zz}}{\partial z} + [\mu_0 H_0^2 + P_0] \left[\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right] = \frac{\partial^2}{\partial t^2} (\rho_{11} w + \rho_{12} W), \\
 & \frac{\partial s}{\partial x} = \frac{\partial^2}{\partial t^2} (\rho_{12} u + \rho_{22} U), \\
 & \frac{\partial s}{\partial z} = \frac{\partial^2}{\partial t^2} (\rho_{12} w + \rho_{22} W).
 \end{aligned}
 \tag{4}$$

In eq. (4), P_0 is the initial stress, $\mathcal{G} = \frac{N}{2(A + N)}$ is the poisson ratio, σ_{ij} are stresses and fluid pressure s is given in [1]

$$\begin{aligned}
 \sigma_{ij} &= 2N e_{ij} + (Ae + Q\varepsilon) \delta_{ij}, \quad (i, j = x, y, z) \\
 s &= Qe + R\varepsilon
 \end{aligned}
 \tag{5}$$

In eq. (2) e_{ij} are strain displacements, A, N, Q, R are poroelastic constants, δ_{ij} is the kroneckar's delta function, e and ε are the solid and fluid dilatations. Substituting eq. (5) in eq. (4) we get the following equations

$$\begin{aligned}
 & N \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + (A + N) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + Q \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 W}{\partial x \partial z} \right) + \frac{1}{2(A + N)Y} \times (\sigma_{zz_0} - Q \left(\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} \right)) \\
 & (P(2N \frac{\partial^2 u}{\partial x^2} + A \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z} \right) + Q \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 W}{\partial x \partial z} \right) - 2AN \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right)) + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) \\
 & - P_0 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) = \frac{\partial^2}{\partial t^2} (\rho_{11} u + \rho_{12} U), \\
 & N \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + (A + N) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + Q \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 W}{\partial x \partial z} \right) + \frac{1}{2(A + N)Y} \times (\sigma_{zz_0} - Q \left(\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} \right)) \\
 & (PN \frac{\partial^2 w}{\partial x^2} + NP \frac{\partial^2 u}{\partial x \partial z}) - \sigma_{zz_0} \left(\frac{\partial^2 u}{\partial x \partial z} \right) - 2A(2N \frac{\partial^2 w}{\partial z^2} + A \frac{\partial^2 u}{\partial x \partial z} + Q \left(\frac{\partial^2 W}{\partial z^2} + \frac{\partial^2 U}{\partial x \partial z} \right) + \mu_e H_0^2 \frac{\partial^2 w}{\partial x \partial z} \\
 & + P_0 \frac{\partial^2 w}{\partial x \partial z} = \frac{\partial^2}{\partial t^2} (\rho_{11} w + \rho_{12} W), \\
 & Q \frac{\partial e}{\partial x} + R \frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2} (\rho_{12} u + \rho_{22} U), \\
 & Q \frac{\partial e}{\partial z} + R \frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2} (\rho_{12} w + \rho_{22} W).
 \end{aligned}
 \tag{6}$$

In eq. (6) $Y = \frac{N(3A + 2N)}{A + N}$ is the young's modulus, σ_{zz_0} is the static stress. The solution of eq. (6) can be decomposed in the following form.

$$(u, w, U, W)(x, z, t) = (C_1, C_2, C_3, C_4)e^{ik(x \cos \theta + z \sin \theta - \omega t)} \quad (7)$$

Substituting eq. (7) in eq. (6) we get the following equations

$$\begin{aligned} & [(P + \mu_e H_0^2 - P_0)k^2 \cos^2 \theta + \frac{2P\sigma_{z_0} Nk^2 \cos^2 \theta + P\sigma_{z_0} Ak^2 \cos^2 \theta - 2\sigma_{z_0} ANk^2 \sin^2 \theta}{2(A+N)Y} - \omega^2 \rho_{11} \\ & + Nk^2 \sin^2 \theta]C_1 + [(P + H_0^2 - P_0)k^2 \sin \theta \cos \theta + \frac{2P\sigma_{z_0} Ak^2 \cos \theta \sin \theta - 2\sigma_{z_0} ANk^2 \sin \theta \cos \theta}{2(A+N)Y}]C_2 + \\ & [Qk^2 \sin^2 \theta + \frac{P\sigma_{z_0} Qk^2 \cos^2 \theta}{2(A+N)Y} - \omega^2 \rho_{12}]C_3 + [Qk^2 \sin \theta \cos \theta + \frac{P\sigma_{z_0} Qk^2 \cos \theta \sin \theta}{2(A+N)Y}]C_4 = 0, \\ & [(A+N)k^2 \sin \theta \cos \theta + \frac{1}{2(A+N)Y} (PN\sigma_{z_0} k^2 \cos \theta \sin \theta - P\sigma_{z_0} k^2 \cos \theta \sin \theta - \\ & 2PA^2 k^2 \cos \theta \sin \theta)]C_1 + [Nk^2 \cos^2 \theta + (P + \mu_e H_0^2 + P_0)k^2 \sin^2 \theta \\ & + \frac{P\sigma_{z_0} Nk^2 \cos^2 \theta - 2\sigma_{z_0} PA^2 k^2 \sin^2 \theta}{2(A+N)Y} + (\mu_e H_0^2 k^2 - P_0 k^2) \sin \theta \cos \theta - \omega^2 \rho_{11}]C_2 \\ & - [2APQk^2 \cos \theta \sin \theta]C_3 + [Qk^2 \cos \theta \sin \theta + Qk^2 \sin^2 \theta - \frac{\sigma_{z_0} PAQk^2 \sin^2 \theta}{2(A+N)Y} - \omega^2 \rho_{12}]C_4 = 0, \\ & [Qk^2 \cos^2 \theta - \omega^2 \rho_{12}]C_1 + Qk^2 \cos \theta \sin \theta C_2 + [R^2 k^2 \cos^2 \theta - \omega^2 \rho_{22}]C_3 + Rk^2 \cos \theta \sin \theta C_4 = 0, \\ & Qk^2 \cos \theta \sin \theta C_1 + [Qk^2 \sin^2 \theta - \omega^2 \rho_{12}]C_2 + Rk^2 \cos \theta \sin \theta C_3 + [Rk^2 \sin^2 \theta - \omega^2 \rho_{22}]C_4 = 0. \end{aligned}$$

(8)

Numerical results

To demonstrate the combined influence of poroelastic properties, initial static stress, and magnetic field intensity on two-dimensional wave propagation, numerical computations were carried out based on the derived dispersion relation. The material is characterized using representative parameters consistent with Biot's theory and variations in magnetic field strength, porosity, permeability, and initial stress are considered to evaluate their impact on wave modes and velocities the eq. (8) reduces to the following form

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = 0. \quad (9)$$

Where

$$\begin{aligned}
 B_{11} &= (P + \mu_e H_0^2 - P_0)k^2 \cos^2 \theta + \frac{2P\sigma_{z_0} Nk^2 \cos^2 \theta + P\sigma_{z_0} Ak^2 \cos^2 \theta - 2\sigma_{z_0} ANk^2 \sin^2 \theta}{2(A + N)Y} \\
 &\quad - \omega^2 \rho_{11} + Nk^2 \sin^2 \theta, \\
 B_{12} &= (P + \mu_e H_0^2 - P_0)k^2 \sin \theta \cos \theta + \frac{2P\sigma_{z_0} Ak^2 \cos \theta \sin \theta - 2\sigma_{z_0} ANk^2 \sin \theta \cos \theta}{2(A + N)Y}, \\
 B_{13} &= Qk^2 \sin^2 \theta + \frac{P\sigma_{z_0} Qk^2 \cos^2 \theta}{2(A + N)Y} - \omega^2 \rho_{12}, \\
 B_{14} &= Qk^2 \sin \theta \cos \theta + \frac{P\sigma_{z_0} Qk^2 \cos \theta \sin \theta}{2(A + N)Y}, \\
 B_{21} &= (A + N)k^2 \sin \theta \cos \theta + \frac{1}{2(A + N)Y} (PN\sigma_{z_0} k^2 \cos \theta \sin \theta - P\sigma_{z_0} k^2 \cos \theta \sin \theta - 2PA^2 k^2 \cos \theta \sin \theta), \\
 B_{22} &= Nk^2 \cos^2 \theta + (P + \mu_e H_0^2 + P_0)k^2 \sin^2 \theta + \frac{P\sigma_{z_0} Nk^2 \cos^2 \theta - 2\sigma_{z_0} PA^2 k^2 \sin^2 \theta}{2(A + N)Y} \\
 &\quad + (\mu_e H_0^2 k^2 - P_0 k^2) \sin \theta \cos \theta - \omega^2 \rho_{11}, \\
 B_{23} &= 2APQk^2 \cos \theta \sin \theta, \\
 B_{24} &= Qk^2 \cos \theta \sin \theta + Qk^2 \sin^2 \theta - \frac{\sigma_{z_0} PAQk^2 \sin^2 \theta}{2(A + N)Y} - \omega^2 \rho_{12}, \\
 B_{31} &= Qk^2 \cos^2 \theta - \omega^2 \rho_{12}; B_{32} = Qk^2 \cos \theta \sin \theta; B_{33} = R^2 k^2 \cos^2 \theta - \omega^2 \rho_{22}; B_{34} = Rk^2 \cos \theta \sin \theta, \\
 B_{41} &= Qk^2 \cos \theta \sin \theta; B_{42} = Qk^2 \sin^2 \theta - \omega^2 \rho_{12}; B_{43} = Rk^2 \cos \theta \sin \theta; B_{44} = Rk^2 \sin^2 \theta - \omega^2 \rho_{22}.
 \end{aligned}$$

To ensure a non-trivial solution to the system, the determinant of the coefficient matrix must vanish. This condition leads to the following frequency equation.

$$|B_{lm}| = 0, \quad l, m = 1, 2, 3, 4$$

(10)

Equation (10) provides an implicit relationship linking the frequency with the wavenumber. For the numerical evaluation, the material properties reported in [25] and [26] are used.

Material-I (Sand stone saturated with kerosene)

$$\begin{aligned}
 A &= 0.4436 \times 10^{10}, N = 0.2765 \times 10^{10}, Q = 0.7635 \times 10^{10}, R = 0.0326 \times 10^{10}, \\
 \rho_{11} &= 1.926137 \times 10^3, \rho_{12} = -0.002137 \times 10^3, \rho_{22} = 0.21537 \times 10^3.
 \end{aligned}$$

Material-II (Sand stone saturated with water)

$$\begin{aligned}
 A &= 0.306 \times 10^{10}, N = 0.922 \times 10^{10}, Q = 0.013 \times 10^{10}, R = 0.0637 \times 10^{10}, \\
 \rho_{11} &= 1.90302 \times 10^3, \rho_{12} = 0, \rho_{22} = 0.2268 \times 10^3.
 \end{aligned}$$

For given materials, the above obtained frequency equation, (10), constitute a relation between the frequency and the wavenumber for different angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ and magnetic field and initial stress $H_0 = P_0 = 0.1, 0.2, 0.3$. Figures 1–5 illustrate the variation of frequency with wavenumber for Material-I under a fixed static stress level (ST-5), considering different propagation angles, initial stress values, and magnetic field strengths. From these figures, it is evident that an increase in initial stress, magnetic field intensity, or

wavenumber leads to a corresponding rise in frequency. Similarly, Figures 6–10 present the frequency–wavenumber behavior for Material-II under the same static stress condition (ST-5). These plots show a trend comparable to Material-I, where higher initial stress, stronger magnetic fields, and larger wavenumbers result in higher frequencies. In the limiting case where both magnetic field and initial stress are absent, the results reduce to those reported in [27].

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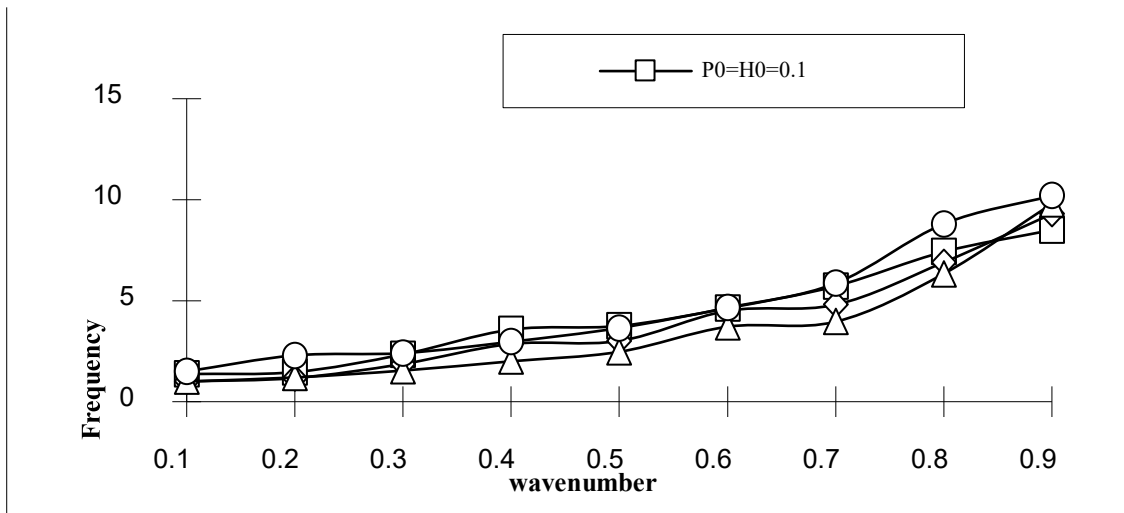


Fig:1 Variation of frequency with wavenumber for Material-I at ($\theta = 0^\circ$)

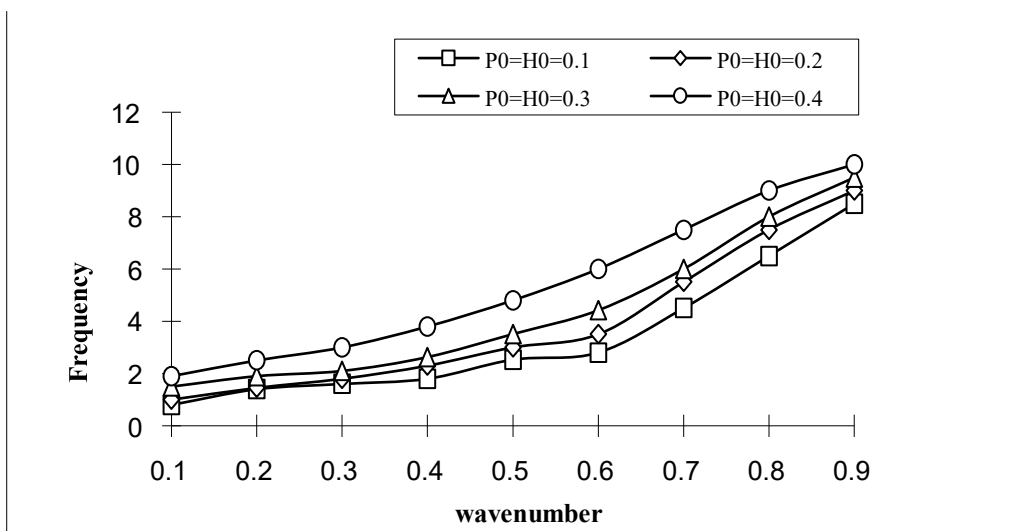


Fig:2 Variation of frequency with wavenumber for Material-I at ($\theta = 30^\circ$)

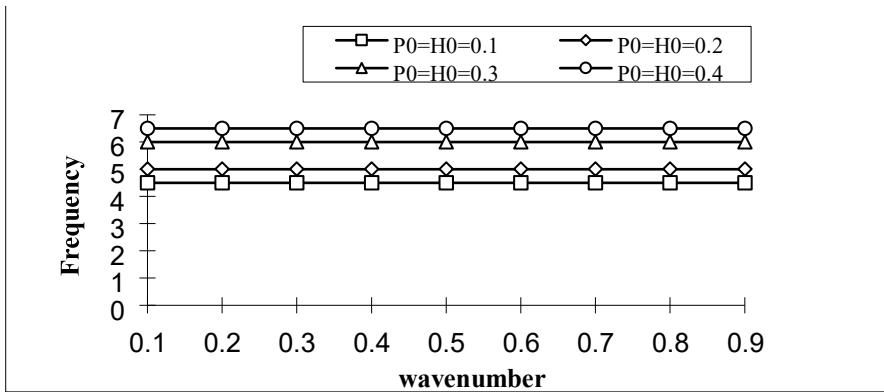


Fig:3 Variation of frequency with wavenumber for Material-I at ($\theta = 45^\circ$)

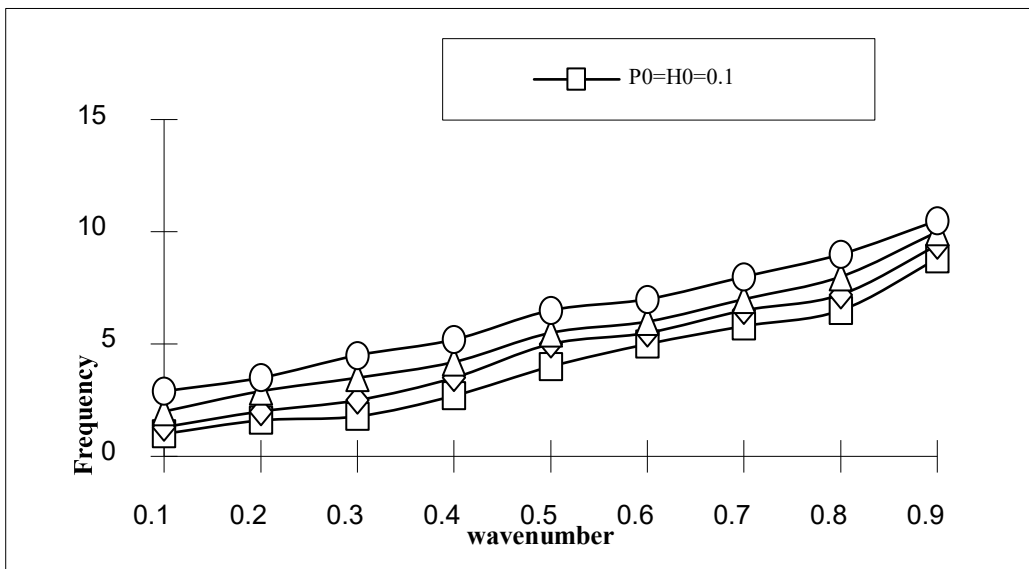


Fig:4 Variation of frequency with wavenumber for Material-I at ($\theta = 60^\circ$)

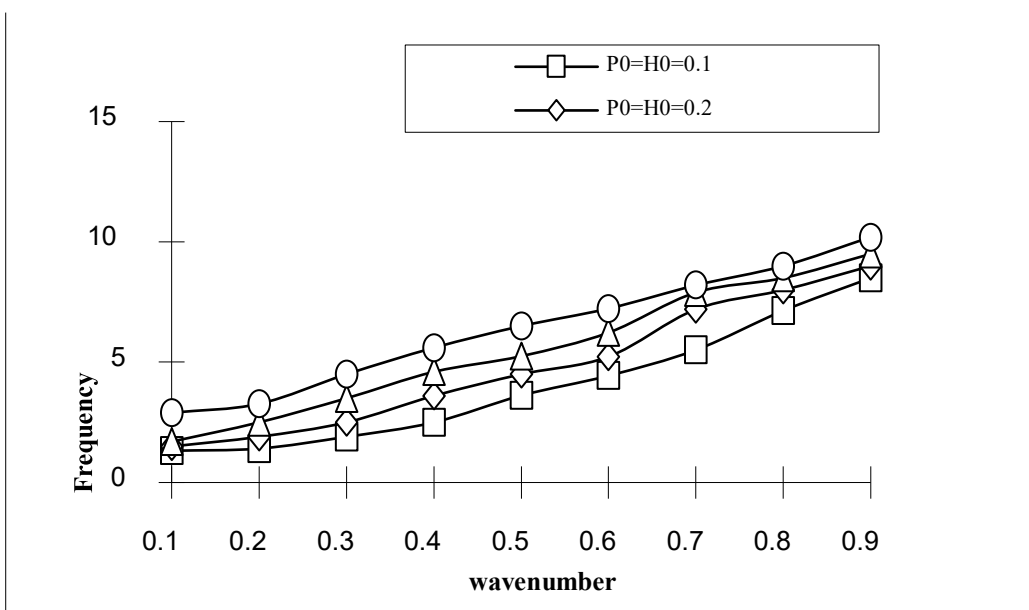


Fig:5 Variation of frequency with wavenumber for Material-I at ($\theta = 90^\circ$)

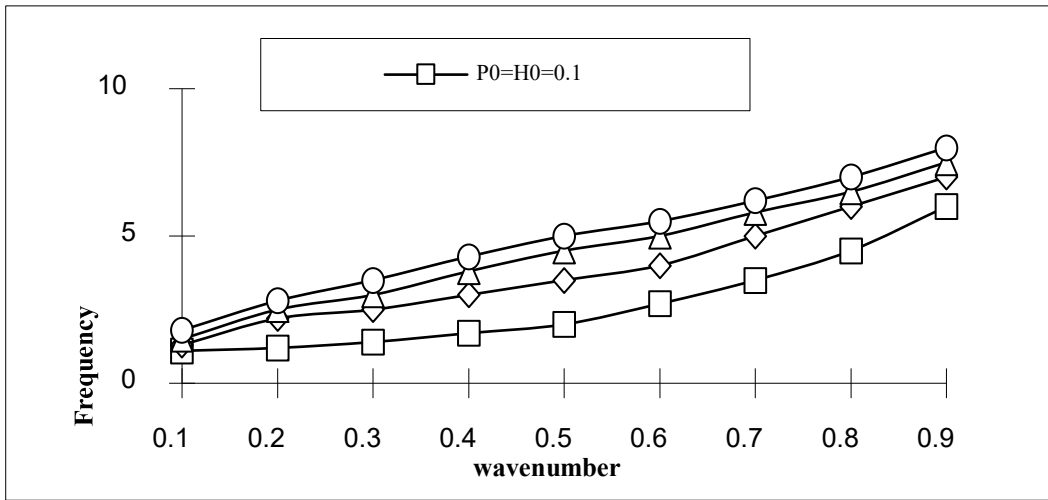


Fig:6 Variation of frequency with wavenumber for Material-II at ($\theta = 0^\circ$)

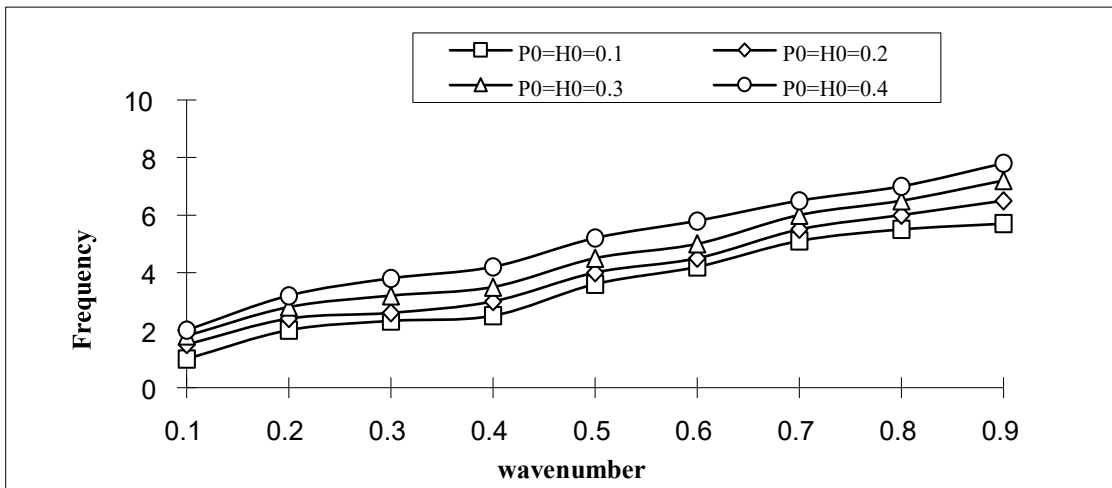


Fig:7 Variation of frequency with wavenumber for Material-II at ($\theta = 30^\circ$)

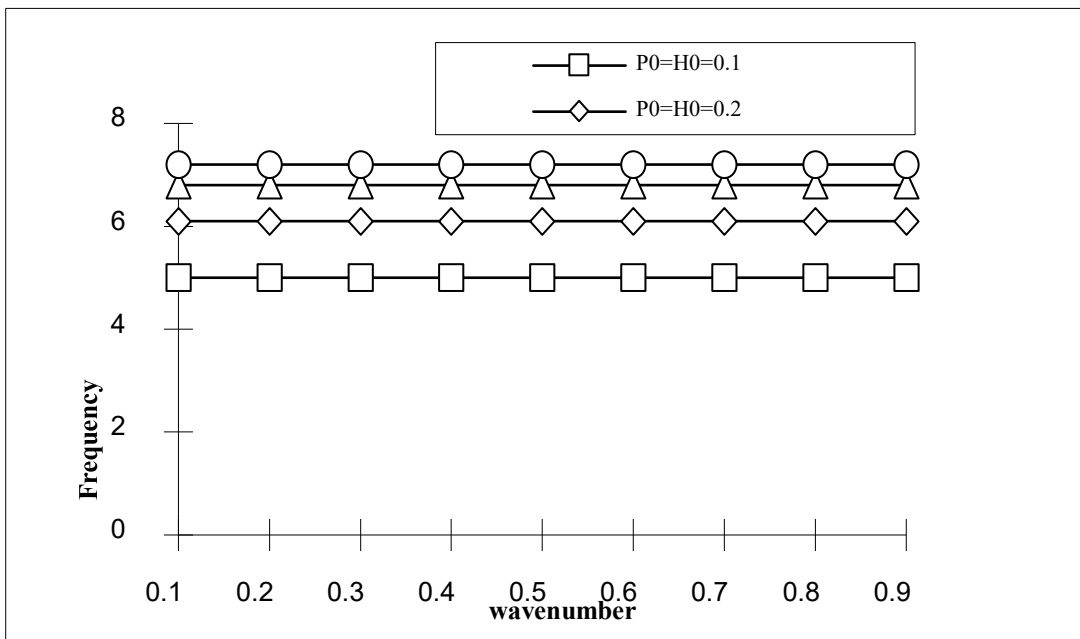


Fig:8 Variation of frequency with wavenumber for Material-II at ($\theta = 45^\circ$)

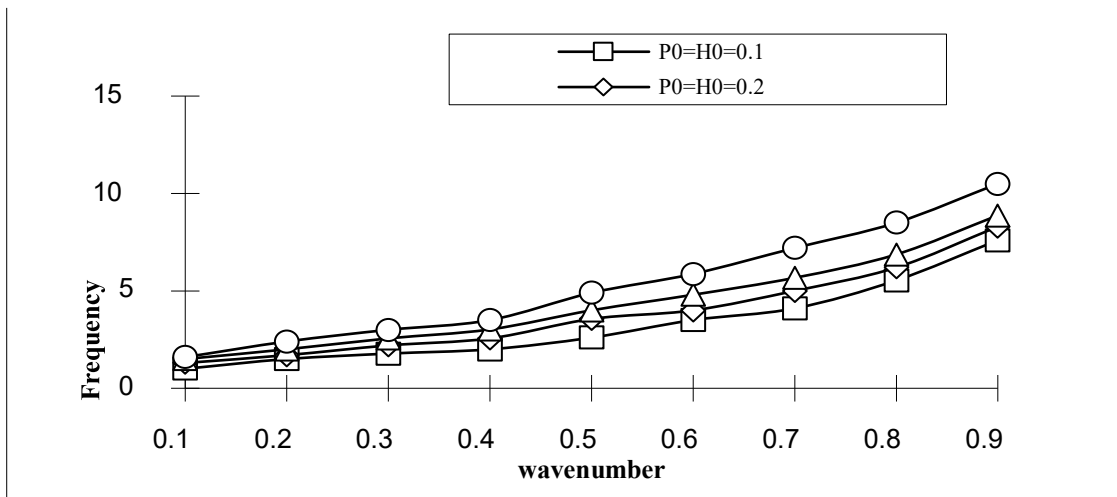


Fig:9 Variation of frequency with wavenumber for Material-II at ($\theta = 60^\circ$)

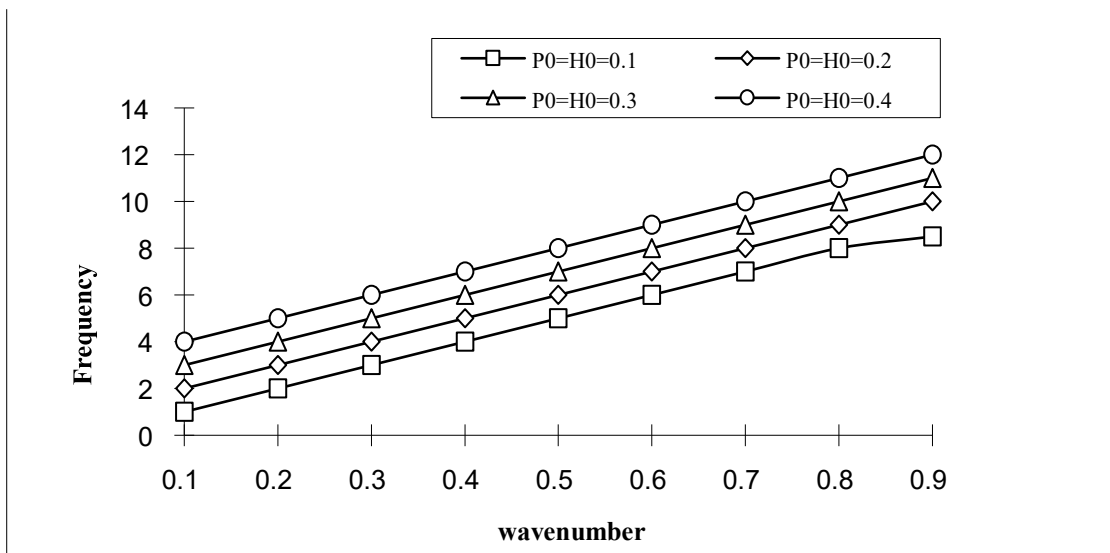


Fig: 10 Variation of frequency with wavenumber for Material-II at ($\theta = 90^\circ$)

CONCLUSION

This study presents a detailed investigation of two-dimensional wave propagation in a poroelastic solid subjected to an initial static stress and an external magnetic field. By integrating Biot's poroelastic framework with magnetoelastic coupling and pre-stress effects, a comprehensive set of governing equations was established. The resulting analytical formulation yields a generalized dispersion relation describing waves, along with shear-type modes affected by the coupled poroelastic–magnetoelastic environment.

The numerical results reveal that initial static stress significantly alters the dynamic behaviour of the medium, modifying wave velocities, increasing anisotropy, and influencing dispersion for waves at various propagation angles. The magnetic field enhances stiffness through Lorentz-type forces, leading to increased frequencies. Interactions among magnetic intensity, porosity, and permeability produce distinct trends not observed in purely elastic or conventional poroelastic systems.

Overall, the combined action of initial stress, poroelastic coupling, and magnetic fields results in complex and enriched wave behaviour. The developed formulation offers a strong theoretical basis for analysing magnetoporoelastic media and is applicable to geomechanical studies, earthquake engineering, reservoir characterization, biomedical elastography, and magnetically tunable porous materials. Future extensions may

incorporate nonlinearity, anisotropy, and numerical schemes such as finite-element models or physics-informed neural networks (PINNs) for enhanced predictive capabilities.

REFERENCES:

1. M. A. Biot "Theory of propagation of elastic waves in a fluid-saturated porous solid, *J. Acoust. Soc. Am.*, vol. 28, no. 2, pp. 168–178, 1956.
2. M.A. Biot "Mechanics of deformation and acoustic propagation in porous media," *J. Appl. Phys.*, vol. 33, no. 4, pp. 1482–1498, 1962.
3. A. J. Willson "Propagation of magneto-elastic plane waves," *Math. Proc. Cambridge Philos. Soc.*, vol. 62, pp. 275–289, 1966.
4. S. Narain, "Magnetoelastic torsional waves in a bar under initial stress," *Proc. Indian Acad. Sci.*, vol. 75, pp. 81–90, 1972.
5. S. R. Mahmoud, "Effect of initial stress and magnetic field on wave propagation in bone," *Boundary Value Problems*, vol. 2014, pp. 1–14, 2014.
6. S. K. Pandey and S. P. Sharma, "Shear wave propagation in magneto-poroelastic dissipative isotropic medium," *Int. J. Pure Appl. Math.*, vol. 118, no. 2, pp. 1–12, 2018.
7. Y. Singh and S. Singh, "Shear wave propagation in magneto-poroelastic medium sandwiched between layers," *Engineering Reports*, vol. 2, pp. 1–10, 2020.
8. A. M. Abd-Alla, "Effect of initial stress and rotation on magnetoelastic wave propagation in isotropic halfspace," *Acta Mech.*, vol. 235, pp. 3105–3117, 2024.
9. A. M. Abd-Alla, "Effect of magnetic field and initial stress on waves in poroelastic media," *J. Theor. Appl. Mech.*, vol. 63, pp. 455–468, 2025.
10. S. L. Lopatnikov, "A thermodynamically consistent formulation of magneto-poroelastic materials," *Int. J. Solids Struct.*, vol. 35, pp. 3891–3906, 1998.
11. L. Dorfmann and R. W. Ogden, "Nonlinear magnetoelastic effects," in *Magnetoelasticity: Materials and Models*. Singapore: Springer, 2014, pp. 231–265.
12. M. Ramagiri, T. S. Lakshmi, and A. Chandulal, "Investigation of initial stress on torsional vibrations in an anisotropic magneto-poroelastic hollow cylinder, *PONTE J.*, vol. 79, no. 7, pp. 50–65, 2023.
13. M. J. Al-Shujairi and H. K. Younis, "Magneto-poroelastic wave propagation in saturated porous media under initial stress," *International Journal of Solids and Structures*, vol. 206, pp. 160–175, 2020.
14. R. K. Sharma and S. K. Tomar, "Effect of magnetic field and initial stress on coupled waves in poroelastic solids," *Journal of Applied Geophysics*, vol. 192, 104376, 2021.
15. B. Singh and B. K. Rai, "Dynamic response of pre-stressed poroelastic media in a magnetic field: A generalized thermoelastic approach," *Acta Mechanica*, vol. 232, no. 9, pp. 3201–3220, 2021.
16. A. K. Sahu and D. P. Acharya, "Magneto-thermo-poroelastic wave analysis in fluid-saturated media with initial stress," *Mechanics Research Communications*, vol. 119, 103848, 2022.
17. T. M. Al-Bazaz and M. I. Othman, "Wave characteristics in a magneto-poroelastic continuum with temperature dependence and pre-existing stress," *Journal of Thermal Stresses*, vol. 45, no. 11, pp. 1328–1349, 2022.
18. S. P. Singh and N. B. Tripathi, "Plane wave propagation in a poroelastic solid under initial stress and magnetic field using Biot's theory," *Wave Motion*, vol. 114, 102939, 2023.
19. H. A. El-Sayed and A. M. Abd-Alla, "Propagation of waves in a magneto-poroelastic half-space under uniform initial stress," *Applied Mathematical Modelling*, vol. 119, pp. 694–712, 2023.
20. M. K. Pandey and R. C. Gupta, "Impact of pre-stress and magnetic field on wave dispersion in porous elastic structures," *Journal of Mechanics of Materials and Structures*, vol. 18, no. 4, pp. 455–472, 2023.
21. Manjula Ramagiri and Sree Lakshmi, Influence of magnetic field on transversely isotropic poroelastic solids, *International Journal in Engineering and Science*, vol. 1. no.1, pp.24-29, 2024.
22. Manjula Ramagiri, Wave propagation in magnetothermoelastic solids in the presence of static stress, *International Journal of Engineering Research and Modern Education* vol.6. no.2, pp. 1-9, 2021.
23. Mott G, Equations of elastic motion of an isotropic medium in the presence of body forces and static stresses, *Journal of the Acoustical Society of America*, USA 50(3), pp. 856-868, 1971.

24. Manik Chandra Singh, Nilratan Chakraborty, Effect of magnetic field on reflection of thermo elastic waves from the boundary of a half space using G-N model of type-II for different nature of the boundary, *International Journal of Applied Computational Mathematics*, 2, pp. 625-640, 2016.
25. Yew, C.H, and Jogi, P.N, Study of wave motions in fluid-saturated porous rocks, *Journal of the Acoustical Society of America*, 60, pp.2-8, 1976.
26. Fatt I, The Biot-Willis elastic coefficient for a sand stone, *Journal Applied Mechanics*, pp.296-297, 1957.
27. Rajitha, G., Srisailam, A., Malla Reddy, P. (2015). Flexural vibrations of poroelastic solids in the presence of static stresses. *Journal of Vibration and Control*, 21(11), pp. 2266–2272, 2015.