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An Improved Exponential Ratio-Type Estimator with Two Auxiliary Variables in Double Sampling with Nonresponse

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ABSTRACT

This research introduces a new exponential ratio-type estimator for estimating the population mean when survey data are affected by nonresponse. The proposed estimator utilizes two auxiliary variables within a double sampling framework in order to enhance estimation accuracy. Expressions for the mean squared error (MSE), coefficient of variation (CV), and relative efficiency (RE) of the estimator were derived. The performance of the estimator was assessed using numerical illustrations and compared with some existing estimators. The empirical results reveal that the proposed estimator yields smaller MSE and CV values while achieving higher relative efficiency across different subsampling rates at a 5% nonresponse level. The findings indicate that the integration of auxiliary variables through exponential adjustments leads to substantial improvement in estimator performance. Consequently, the proposed estimator provides a more reliable and efficient alternative for estimating population means in the presence of nonresponse.

Keywords*:* Exponential ratio-type estimator, Nonresponse, Mean square error, Relative efficiency, Auxiliary information.

INTRODUCTION

In survey sampling, one of the primary objectives is to estimate unknown population parameters such as the population mean. To improve the accuracy of these estimates, researchers often make use of auxiliary information related to the study variable. When auxiliary variables exhibit correlation with the variable of interest, incorporating them into estimation procedures can significantly reduce sampling variability and increase the precision of the resulting estimates. Traditional estimation methods such as the ratio, product, and regression estimators are well-known techniques that exploit auxiliary information to produce more reliable population estimates.

Despite these advantages, many real-world surveys encounter the problem of nonresponse. Nonresponse arises when certain selected units in the sample do not provide the required information completely (unit nonresponse) or in part (item nonresponse) for the study variable. If this issue is not properly addressed, it can introduce bias and reduce the efficiency of estimators. One of the earliest approaches for handling nonresponse in survey sampling was developed by Hansen and Hurwitz (1946), who proposed a subsampling strategy for nonrespondents. Their method involves conducting follow-up surveys on a subset of the nonresponding units to obtain the missing information. This technique has become an important foundation for many later developments in survey sampling theory.

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Another important approach that improves estimation efficiency is double sampling or two phase sampling. This design is particularly useful in situations where the study variable is difficult or costly to collect, whereas auxiliary variables are easier to obtain. In a double sampling design, a large first-phase sample is used to collect information on auxiliary variables. Subsequently, a smaller second-phase sample is selected from the first sample to collect information on the study variable along with the auxiliary variables. The information obtained in the first phase can then be used to construct improved estimators for the second phase, leading to greater estimation precision and reduced survey costs.

To further enhance estimator performance, many researchers have proposed exponential ratio-type estimators that incorporate auxiliary information through nonlinear transformations. For instance, Bahl and Tuteja (1991) introduced an exponential ratio-type estimator for estimating the population mean under simple random sampling. Their work stimulated numerous extensions that adapted exponential-type estimators to more complex sampling situations. Singh et al. (2009) developed an exponential estimator that accounts for nonresponse in both the study and auxiliary variables. In a related contribution, Ozel Kadilar (2016) proposed another exponential estimator that effectively utilizes auxiliary information, while Ünal and Kadilar (2020) extended this approach to situations involving nonresponse in survey data. Other studies have developed exponential ratio-type estimators for population mean in the presence of nonresponse incorporating the use of auxiliary variables to improve accuracy in double sampling (Hazra, 2015; Khan & Khan, 2022; Oguagbaka et al., 2024).

Although considerable progress has been made in developing estimators for survey data with nonresponse, there is still a need for improved methods that effectively combine multiple auxiliary variables within a double sampling framework. In some practical survey situations, more than one auxiliary variable is available and these variables can provide valuable additional information that enhances the accuracy of population estimates. However, relatively limited research has focused on exponential ratio-type estimators that simultaneously utilize two auxiliary variables under double sampling in the presence of nonresponse.

Motivated by these gaps in the literature, this study proposes a new exponential ratio-type estimator using two auxiliary variables under double sampling in the presence of nonresponse. The statistical properties of the proposed estimator, including its bias and mean square error, are derived using first-order approximation techniques. Furthermore, the efficiency of the estimator is evaluated by comparing the mean square error with those of several existing estimators. Numerical illustrations are also presented to demonstrate the performance of the proposed estimator under 5% nonresponse and varying subsampling rates

Double Sampling Framework

Consider a finite population consisting of N units from which information about a study variable y is required. Let x and z denote two auxiliary variables that are correlated with the study variable. In many practical surveys, the values of auxiliary variables are easier and less expensive to obtain compared to the study variable.

Under the double sampling (two-phase sampling) design, a large first-phase sample of size n' (where $n' < N$) is selected from the population using simple random sampling without replacement. During this phase, information on the auxiliary variables x and z is collected.

From this first-phase sample, a smaller second-phase sample of size n ($n < n'$) is drawn. In the second phase, observations are taken on the study variable y in addition to the auxiliary variables. The sample means obtained from the first and second phases are then used to construct improved estimators for the population mean.

In survey investigations, it is common for some selected units to fail to provide the required information for the study variable. This situation is referred to as nonresponse. When nonresponse occurs, the sample is typically divided into two groups: respondents and nonrespondents.

Following the approach proposed by Hansen and Hurwitz (1946), a subsampling technique is applied to the group of nonrespondents. Suppose that among the selected second-phase sample, n_1 units respond while n_2 units do not respond. A subsample of size r is then drawn from the nonrespondents, and additional efforts are made to obtain their responses.

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The information obtained from respondents and the subsampled nonrespondents is combined to form an unbiased estimator of the population mean. This procedure helps to reduce the bias that may arise due to missing responses in the survey.

Baseline Estimators

Bahl and Tuteja (1991) proposed an exponential ratio-type estimator for population mean in simple random sampling as:

$$\bar{y}_{er} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad 1$$

Singh *et al* (2009) proposed an exponential ratio-type estimator in the presence of non-response on both the study and auxiliary variables by adopting the exponential type estimator in (1) as:

$$\bar{y}^{*}_{er} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad 2$$

Ozel Kadilar (2016) proposed an exponential type estimator as:

$$\bar{y}_{GO} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad 3$$

Unal and Kadilar (2020) proposed an exponential estimator in the presence of non-response on both the study and auxiliary variables by adopting the exponential type estimator in (3) as:

$$\bar{y}_{CC} = \bar{y}^{*}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left(\frac{\bar{X}-\bar{x}^{*}}{\bar{X}+\bar{x}^{*}}\right) \quad 4$$

Abifade (2020) proposed an exponential ratio-type estimator in double sampling as:

$$\bar{y}_{AA} = \bar{y}\left[\alpha\exp\left(\frac{\bar{x}'}{\bar{x}}\right) + (1-\alpha)\exp\left(\frac{\bar{z}'}{\bar{z}} - 1\right)\right] \quad 5$$

The Proposed Estimator

Motivated by the work of Singh *et al* (2009) and Unal and Kadilar (2020), we adopt the estimator in (5) and hereby proposed an exponential ratio-type estimator for estimating population mean using two auxiliary variables when nonresponse occurs on the study variable only as:

$$\bar{Y}_{FA1} = \bar{y}^{*}\left[\alpha\exp\left(\frac{\bar{x}'}{\bar{x}}\right) + (1-\alpha)\exp\left(\frac{\bar{z}'}{\bar{z}} - 1\right)\right] \quad 6$$

Where

$\bar{y}^{*}$ = Sample mean of the observed variable

$\bar{x}'$ = Sample mean of the first auxilliary variable in the first phase

$\bar{x}^{*}$ = Sample mean of the first auxilliary variable in the second phase with nonresponse

$\bar{z}$ = sample mean of the second auxilliary variable in the second phase

$\bar{z}'$ = Sample mean of the second auxilliary variable in the first phase

$\alpha$is a constant such that $0 \leq \alpha \leq 1$

$For\ \alpha = 1$, $\bar{Y}_{FA1}$ reduces to $\bar{y}^{*}\exp\left(\frac{\bar{x}'}{\bar{x}^{*}}\right)$

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$For\ \alpha = 0,\ \overline{Y}_{FA1}$ reduces to $\overline{y}^* \exp\left(\frac{\overline{z}'}{\overline{z}} - 1\right)$

To obtain the Bias and Mean Square Error (MSE) of the proposed estimator $\overline{Y}_{FA1}$, we define the relative error terms and their expectations as:

Let $\varepsilon_0 = \frac{\overline{y}^* - \overline{Y}}{\overline{Y}}$, $\varepsilon_1 = \frac{\overline{x} - \overline{X}}{\overline{X}}$, $\varepsilon_1' = \frac{\overline{x}' - \overline{X}}{\overline{X}}$, $\varepsilon_2 = \frac{\overline{z} - \overline{Z}}{\overline{Z}}$, $\varepsilon_2' = \frac{\overline{z}' - \overline{Z}}{\overline{Z}}$

And

$\overline{y}^* = \overline{Y}(1 + \varepsilon_0)$, $\overline{x} = \overline{X}(1 + \varepsilon_1)$, $\overline{x}' = \overline{X}(1 + \varepsilon_1')$, $\overline{z} = \overline{Z}(1 + \varepsilon_2)$, $\overline{z}' = \overline{Z}(1 + \varepsilon_2')$

Such that

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_1') = E(\varepsilon_2) = E(\varepsilon_2') = 0$$

And

$E(\varepsilon_0^2) = var(\overline{y}^*) = \theta C_y^2 + \theta^* C_{y2}^2$

$E(\varepsilon_1^2) = var(\overline{x}) = \theta C_x^2$

$E(\varepsilon_1'^2) = var(\overline{x}') = \theta' C_x^2$

$E(\varepsilon_2^2) = var(\overline{z}) = \theta C_z^2$

$E(\varepsilon_2'^2) = var(\overline{z}') = \theta' C_z^2$

$E(\varepsilon_0 \varepsilon_1) = cov(\overline{y}^* \overline{x}) = \theta \rho_{yx} C_y C_x$

$E(\varepsilon_0 \varepsilon_1') = cov(\overline{y}^* \overline{x}') = \theta' \rho_{yx} C_y C_x$

$E(\varepsilon_0 \varepsilon_2) = cov(\overline{y}^* \overline{z}) = \theta \rho_{yz} C_y C_z$

$E(\varepsilon_0 \varepsilon_2') = cov(\overline{y}^* \overline{z}') = \theta' \rho_{yz} C_y C_z$

$E(\varepsilon_1 \varepsilon_1') = cov(\overline{x}\overline{x}') = \theta' C_x^2$

$E(\varepsilon_1 \varepsilon_2) = cov(\overline{x}\,\overline{z}) = \theta \rho_{xz} C_x C_z$

$E(\varepsilon_1 \varepsilon_2') = cov(\overline{x}\,\overline{z}') = \theta' \rho_{xz} C_x C_z$

$E(\varepsilon_1' \varepsilon_2) = cov(\overline{x}'\,\overline{z}) = \theta' \rho_{xz} C_x C_z$

$E(\varepsilon_1' \varepsilon_2') = cov(\overline{x}'\,\overline{z}') = \theta' \rho_{xz} C_x C_z$

$E(\varepsilon_2 \varepsilon_2') = cov(\overline{z}\,\overline{z}') = \theta' C_z^2$

Where $\theta' = \frac{1-f'}{n'}$, $\theta = \frac{1-f}{n}$, $\theta^* = \frac{W_2(k-1)}{n'}$; $k > 1$, $f = \frac{n}{N}$, $f' = \frac{n'}{N}$, $W_2 = \frac{N_2}{N}$,

$C_y = \frac{S_y}{\overline{Y}}$, $C_x = \frac{S_x}{\overline{X}}$, $C_z = \frac{S_z}{\overline{Z}}$, $C_{yx} = \frac{\rho_{yx} C_y}{C_x}$, $C_{yz} = \frac{\rho_{yz} C_y}{C_z}$, $C_{xz} = \frac{\rho_{xz} C_x}{C_z}$

$C_{yx2} = \frac{\rho_{yx2} C_{y2}}{C_{x2}}$

The estimator $\overline{Y}_{FA1}$ can be expressed in terms of ε's as follows:

$$\overline{Y}_{FA1} = \overline{y}^{*}\{\alpha \exp[\overline{x}'\overline{x}^{-1}] + [1-\alpha]\exp[\overline{z}'\overline{z}^{-1} - 1]\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\alpha \exp\left[\frac{\overline{X}(1+\varepsilon_1')}{\overline{X}(1+\varepsilon_1)}\right] + [1-\alpha]\exp\left[\frac{\overline{Z}(1+\varepsilon_2')}{\overline{Z}(1+\varepsilon_2)} - 1\right]\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\{\alpha \exp[1+\varepsilon_1'][1+\varepsilon_1]^{-1} + [1-\alpha]\exp[1+\varepsilon_2'][1+\varepsilon_2]^{-1} - 1\} \qquad 7$$

Assuming that $|\varepsilon_0| < 1, |\varepsilon_1| < 1, |\varepsilon_1'| < 1, |\varepsilon_2| < 1, |\varepsilon_2'| < 1$ then we expands the right hand side of equation 7 to the second degree of approximation using negative binomial series, we have

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\{\alpha \exp[1+\varepsilon_1'][1-\varepsilon_1+\varepsilon_1^2] + [1-\alpha]\exp[1+\varepsilon_2'][1-\varepsilon_2+\varepsilon_2^2] - 1\} \qquad 8$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\alpha \exp[1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1+\varepsilon_1'\varepsilon_1^2] \\ +[1-\alpha]\exp[1-\varepsilon_2+\varepsilon_2^2+\varepsilon_2'-\varepsilon_2'\varepsilon_2+\varepsilon_2'\varepsilon_2^2] - 1\end{matrix}\right\} \qquad 9$$

Rewriting equation 9 to second degree of approximation, we have

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\{\alpha \exp[1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1] + [1-\alpha]\exp[\varepsilon_2'-\varepsilon_2+\varepsilon_2^2-\varepsilon_2'\varepsilon_2]\}$$

Taking the exponential of the R.H.S. and expand, we have

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\alpha\left[1+[1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1] + \frac{[1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1]^2}{2!}\right] \\ +[1-\alpha]\left[1+[1+\varepsilon_2'-\varepsilon_2+\varepsilon_2^2-\varepsilon_2'\varepsilon_2] + \frac{[\varepsilon_2'-\varepsilon_2+\varepsilon_2^2-\varepsilon_2'\varepsilon_2]^2}{2!}\right]\end{matrix}\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\alpha\left[[1+1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1] + \frac{[1-2\varepsilon_1+2\varepsilon_1'+3\varepsilon_1^2+\varepsilon_1'^2-4\varepsilon_1'\varepsilon_1]}{2!}\right] \\ +[1-\alpha]\left[[1+\varepsilon_2'-\varepsilon_2+\varepsilon_2^2-\varepsilon_2'\varepsilon_2] + \frac{[\varepsilon_2'^2+\varepsilon_2^2-2\varepsilon_2'\varepsilon_2]}{2!}\right]\end{matrix}\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\alpha\left[2-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1+\frac{1}{2}-\varepsilon_1+\varepsilon_1'+\frac{3}{2}\varepsilon_1^2+\frac{\varepsilon_1'^2}{2}\right] \\ +[1-\alpha]\left[1+\varepsilon_2'-\varepsilon_2+\varepsilon_2^2-\varepsilon_2'\varepsilon_2+\frac{\varepsilon_2'^2}{2}+\frac{\varepsilon_2^2}{2}-\varepsilon_2'\varepsilon_2\right]\end{matrix}\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\alpha\left[\frac{5}{2}-2\varepsilon_1+2\varepsilon_1'-3\varepsilon_1'\varepsilon_1+\frac{5}{2}\varepsilon_1^2+\frac{\varepsilon_1'^2}{2}\right] \\ +[1-\alpha]\left[1-\varepsilon_2+\varepsilon_2'-2\varepsilon_2'\varepsilon_2+\frac{1}{2}\varepsilon_2'^2+\frac{3}{2}\varepsilon_2^2\right]\end{matrix}\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\frac{5}{2}\alpha-2\alpha\varepsilon_1+2\alpha\varepsilon_1'-3\alpha\varepsilon_1'\varepsilon_1+\frac{5}{2}\alpha\varepsilon_1^2+\frac{1}{2}\alpha\varepsilon_1'^2+1-\varepsilon_2+\varepsilon_2'-2\varepsilon_2'\varepsilon_2 \\ +\frac{1}{2}\varepsilon_2'^2+\frac{3}{2}\varepsilon_2^2-\alpha+\alpha\varepsilon_2-\alpha\varepsilon_2'+2\alpha\varepsilon_2'\varepsilon_2-\frac{1}{2}\alpha\varepsilon_2'^2-\frac{3}{2}\alpha\varepsilon_2^2\end{matrix}\right\}$$

$$\overline{Y}_{FA1} = \overline{Y}[1+\varepsilon_0]\left\{\begin{matrix}\frac{3}{2}\alpha-2\alpha\varepsilon_1+2\alpha\varepsilon_1'-3\alpha\varepsilon_1'\varepsilon_1+\frac{5}{2}\alpha\varepsilon_1^2+\frac{1}{2}\alpha\varepsilon_1'^2+\alpha\varepsilon_2-\alpha\varepsilon_2'+2\alpha\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\alpha\varepsilon_2'^2-\frac{3}{2}\alpha\varepsilon_2^2-\varepsilon_2+\varepsilon_2'-2\varepsilon_2'\varepsilon_2+\frac{1}{2}\varepsilon_2'^2+\frac{3}{2}\varepsilon_2^2+1\end{matrix}\right\}$$

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$$\overline{Y}_{FA1} = \overline{Y}\begin{Bmatrix} \frac{3}{2}\alpha - 2\alpha\varepsilon_1 + 2\alpha\varepsilon_1' - 3\alpha\varepsilon_1'\varepsilon_1 + \frac{5}{2}\alpha\varepsilon_1^2 + \frac{1}{2}\alpha\varepsilon_1'^2 + \alpha\varepsilon_2 - \alpha\varepsilon_2' + 2\alpha\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\alpha\varepsilon_2'^2 - \frac{3}{2}\alpha\varepsilon_2^2 - \varepsilon_2 + \varepsilon_2' - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 + 1 + \frac{3}{2}\alpha\varepsilon_0 - 2\alpha\varepsilon_0\varepsilon_1 \\ +2\alpha\varepsilon_0\varepsilon_1' - 3\alpha\varepsilon_0\varepsilon_1'\varepsilon_1 + \frac{5}{2}\alpha\varepsilon_0\varepsilon_1^2 + \frac{1}{2}\alpha\varepsilon_0\varepsilon_1'^2 + \alpha\varepsilon_0\varepsilon_2 - \alpha\varepsilon_0\varepsilon_2' + 2\alpha\varepsilon_0\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\alpha\varepsilon_0\varepsilon_2'^2 - \frac{3}{2}\alpha\varepsilon_0\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' - 2\varepsilon_0\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_0\varepsilon_2'^2 + \frac{3}{2}\varepsilon_0\varepsilon_2^2 + \varepsilon_0 \end{Bmatrix}$$

$$\overline{Y}_{FA1} = \overline{Y}\begin{Bmatrix} \alpha\begin{bmatrix} \frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' - 3\varepsilon_0\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_0\varepsilon_1^2 \\ +\frac{1}{2}\varepsilon_0\varepsilon_1'^2 + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2' + 2\varepsilon_0\varepsilon_2'\varepsilon_2 - \frac{1}{2}\varepsilon_0\varepsilon_2'^2 - \frac{3}{2}\varepsilon_0\varepsilon_2^2 \end{bmatrix} \\ +\varepsilon_2' - \varepsilon_2 - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' - 2\varepsilon_0\varepsilon_2'\varepsilon_2 \\ +\frac{1}{2}\varepsilon_0\varepsilon_2'^2 + \frac{3}{2}\varepsilon_0\varepsilon_2^2 + \varepsilon_0 + 1 \end{Bmatrix} \qquad 10$$

Equation 10 to second degree approximation, we have

$$\overline{Y}_{FA1} = \overline{Y}\begin{Bmatrix} \alpha\begin{bmatrix} \frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2' \end{bmatrix} \\ +\varepsilon_2' - \varepsilon_2 - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' + \varepsilon_0 + 1 \end{Bmatrix} \qquad 11$$

Subtracting $\overline{Y}$ from both sides, we have

$$\overline{Y}_{FA1} - \overline{Y} = \overline{Y}\begin{Bmatrix} \alpha\begin{bmatrix} \frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2' \end{bmatrix} \\ +\varepsilon_2' - \varepsilon_2 - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' + \varepsilon_0 + 1 \end{Bmatrix} - \overline{Y}$$

$$\overline{Y}_{FA1} - \overline{Y} = \begin{Bmatrix} \overline{Y}\alpha\begin{bmatrix} \frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2' \end{bmatrix} \\ +\overline{Y}\varepsilon_2' - \overline{Y}\varepsilon_2 - 2\overline{Y}\varepsilon_2'\varepsilon_2 + \frac{1}{2}\overline{Y}\varepsilon_2'^2 + \frac{3}{2}\overline{Y}\varepsilon_2^2 - \overline{Y}\varepsilon_0\varepsilon_2 + \overline{Y}\varepsilon_0\varepsilon_2' + \overline{Y}\varepsilon_0 + \overline{Y} \end{Bmatrix} - \overline{Y}$$

$$\overline{Y}_{FA1} - \overline{Y} = \overline{Y}\begin{Bmatrix} \alpha\begin{bmatrix} \frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2' \end{bmatrix} \\ +\varepsilon_2' - \varepsilon_2 - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' + \varepsilon_0 \end{Bmatrix} \qquad 12$$

Taking the expectation of both sides of equation 12, we have the bias of $\overline{Y}_{FA1}$ to the first degree approximation as

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$$B[\bar{Y}_{FA1}] = E\left\{\bar{Y}\left\{\begin{array}{l}\alpha\left[\begin{array}{c}\frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' - 3\varepsilon_1'\varepsilon_1 + \frac{5}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_1'^2 + \varepsilon_2 - \varepsilon_2' + 2\varepsilon_2'\varepsilon_2 \\ -\frac{1}{2}\varepsilon_2'^2 - \frac{3}{2}\varepsilon_2^2 + \frac{3}{2}\varepsilon_0 - 2\varepsilon_0\varepsilon_1 + 2\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2'\end{array}\right] \\ +\varepsilon_2' - \varepsilon_2 - 2\varepsilon_2'\varepsilon_2 + \frac{1}{2}\varepsilon_2'^2 + \frac{3}{2}\varepsilon_2^2 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2' + \varepsilon_0\end{array}\right\}\right\}$$

$$B[\bar{Y}_{FA1}] = \bar{Y}\left\{\begin{array}{l}\alpha\left[\begin{array}{c}\frac{3}{2} - 3\theta' C_x^2 + \frac{5}{2}\theta C_x^2 + \frac{1}{2}\theta' C_x^2 + 2\theta' C_z^2 - \frac{1}{2}\theta' C_z^2 - \frac{3}{2}\theta C_z^2 \\ -2\theta\rho_{yx}C_yC_x + 2\theta'\rho_{yx}C_yC_x + \theta\rho_{yz}C_yC_z - \theta'\rho_{yz}C_yC_z\end{array}\right] \\ -2\theta' C_z^2 + \frac{1}{2}\theta' C_z^2 + \frac{3}{2}\theta C_z^2 - \theta\rho_{yz}C_yC_z + \theta'\rho_{yz}C_yC_z\end{array}\right\}$$

$$B[\bar{Y}_{FA1}] = \bar{Y}\left\{\begin{array}{l}\alpha\left[\begin{array}{c}\frac{3}{2} - \frac{5}{2}\theta' C_x^2 + \frac{5}{2}\theta C_x^2 + \frac{3}{2}\theta' C_z^2 - \frac{3}{2}\theta C_z^2 \\ -2\theta\rho_{yx}C_yC_x + 2\theta'\rho_{yx}C_yC_x + \theta\rho_{yz}C_yC_z - \theta'\rho_{yz}C_yC_z\end{array}\right] \\ -\frac{3}{2}\theta' C_z^2 + \frac{3}{2}\theta C_z^2 - \theta\rho_{yz}C_yC_z + \theta'\rho_{yz}C_yC_z\end{array}\right\}$$

$$B[\bar{Y}_{FA1}] = \bar{Y}\left\{\alpha\left[\begin{array}{c}\frac{3}{2} - \frac{5}{2}[\theta' - \theta]C_x^2 + \frac{3}{2}[\theta' - \theta]C_z^2 \\ +2[\theta' - \theta]\rho_{yx}C_yC_x - [\theta' - \theta]\rho_{yz}C_yC_z\end{array}\right] - \frac{3}{2}[\theta' - \theta]C_z^2 + [\theta' - \theta]\rho_{yz}C_yC_z\right\}$$

Estimator $\bar{Y}_{FA1}$ is approximately unbiased if the value of the constant is

$$\alpha\left[\begin{array}{c}\frac{3}{2} - \frac{5}{2}[\theta' - \theta]C_x^2 + \frac{3}{2}[\theta' - \theta]C_z^2 \\ +2[\theta' - \theta]\rho_{yx}C_yC_x - [\theta' - \theta]\rho_{yz}C_yC_z\end{array}\right] - \frac{3}{2}[\theta' - \theta]C_z^2 + [\theta' - \theta]\rho_{yz}C_yC_z = 0$$

$$\alpha\left[\begin{array}{c}\frac{3}{2} - \frac{5}{2}[\theta' - \theta]C_x^2 + \frac{3}{2}[\theta' - \theta]C_z^2 \\ +2[\theta' - \theta]\rho_{yx}C_yC_x - [\theta' - \theta]\rho_{yz}C_yC_z\end{array}\right] = \frac{3}{2}[\theta' - \theta]C_z^2 + [\theta' - \theta]\rho_{yz}C_yC_z$$

$$\alpha = \frac{\frac{3}{2}[\theta' - \theta]C_z^2 - [\theta' - \theta]\rho_{yz}C_yC_z}{\frac{3}{2} - \frac{5}{2}[\theta' - \theta]C_x^2 + \frac{3}{2}[\theta' - \theta]C_z^2 + 2[\theta' - \theta]\rho_{yx}C_yC_x - [\theta' - \theta]\rho_{yz}C_yC_z}$$

$$\alpha = \frac{[\theta' - \theta]\left[\frac{3}{2}C_z^2 - \rho_{yz}C_yC_z\right]}{\frac{3}{2} - [\theta' - \theta]\left[\frac{5}{2}C_x^2 - \frac{3}{2}C_z^2 - 2\rho_{yx}C_yC_x + \rho_{yz}C_yC_z\right]}$$

To obtain the error function of the estimator, we re-write equation 12 to first degree approximation, we have

$$\bar{Y}_{FA1} - \bar{Y} = \bar{Y}\left\{\alpha\left[\frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' + \varepsilon_2 - \varepsilon_2' + \frac{3}{2}\varepsilon_0\right] + \varepsilon_2' - \varepsilon_2 + \varepsilon_0\right\} \qquad 13$$

Squaring both sides of equation 13 and neglecting terms of ε's involving power greater than two, we have

$$[\bar{Y}_{FA1} - \bar{Y}]^2 = \left[\bar{Y}\left\{\alpha\left[\frac{3}{2} - 2\varepsilon_1 + 2\varepsilon_1' + \varepsilon_2 - \varepsilon_2' + \frac{3}{2}\varepsilon_0\right] + \varepsilon_2' - \varepsilon_2 + \varepsilon_0\right\}\right]^2$$

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$$[\bar{Y}_{FA1}-\bar{Y}]^2=\bar{Y}^2\left\{\begin{matrix}\alpha^2\left[\begin{matrix}\frac{9}{4}-6\varepsilon_1+6\varepsilon_1'+3\varepsilon_2+3\varepsilon_2'+\frac{9}{2}\varepsilon_0+4\varepsilon_1^2+4\varepsilon_1'^2\\+\varepsilon_2^2+\varepsilon_2'^2+\frac{9}{4}\varepsilon_0^2-8\varepsilon_1'\varepsilon_1-4\varepsilon_1\varepsilon_2+4\varepsilon_1\varepsilon_2'-6\varepsilon_0\varepsilon_1\\+4\varepsilon_1'\varepsilon_2-4\varepsilon_1'\varepsilon_2'+6\varepsilon_0\varepsilon_1'-2\varepsilon_2'\varepsilon_2+3\varepsilon_0\varepsilon_2-3\varepsilon_0\varepsilon_2'\end{matrix}\right]\\+2\alpha\left[\begin{matrix}\frac{3}{2}\varepsilon_0^2-\varepsilon_2'^2-\varepsilon_2^2+\frac{3}{2}\varepsilon_0-\frac{3}{2}\varepsilon_2+\frac{3}{2}\varepsilon_2'\\-2\varepsilon_1\varepsilon_2'+2\varepsilon_1'\varepsilon_2'+2\varepsilon_2'\varepsilon_2+\frac{1}{2}\varepsilon_0\varepsilon_2'-\frac{1}{2}\varepsilon_0\varepsilon_2\\+2\varepsilon_1\varepsilon_2-2\varepsilon_1'\varepsilon_2-2\varepsilon_0\varepsilon_1+2\varepsilon_0\varepsilon_1'\end{matrix}\right]\\+\varepsilon_2'^2-2\varepsilon_2'\varepsilon_2+2\varepsilon_0\varepsilon_2'-2\varepsilon_0\varepsilon_2+\varepsilon_2^2+\varepsilon_0^2\end{matrix}\right\} \quad 14$$

Taking the expectation of both sides of equation 14, we get the MSE of the estimator $\bar{Y}_{FA1}$ to the first degree approximation, we have

$$E[\bar{Y}_{FA1}-\bar{Y}]^2=E\left[\bar{Y}^2\left\{\begin{matrix}\alpha^2\left[\begin{matrix}\frac{9}{4}-6\varepsilon_1+6\varepsilon_1'+3\varepsilon_2+3\varepsilon_2'+\frac{9}{2}\varepsilon_0+4\varepsilon_1^2+4\varepsilon_1'^2\\+\varepsilon_2^2+\varepsilon_2'^2+\frac{9}{4}\varepsilon_0^2-8\varepsilon_1'\varepsilon_1-4\varepsilon_1\varepsilon_2+4\varepsilon_1\varepsilon_2'-6\varepsilon_0\varepsilon_1\\+4\varepsilon_1'\varepsilon_2-4\varepsilon_1'\varepsilon_2'+6\varepsilon_0\varepsilon_1'-2\varepsilon_2'\varepsilon_2+3\varepsilon_0\varepsilon_2-3\varepsilon_0\varepsilon_2'\end{matrix}\right]\\+2\alpha\left[\begin{matrix}\frac{3}{2}\varepsilon_0^2-\varepsilon_2'^2-\varepsilon_2^2+\frac{3}{2}\varepsilon_0-\frac{3}{2}\varepsilon_2+\frac{3}{2}\varepsilon_2'\\-2\varepsilon_1\varepsilon_2'+2\varepsilon_1'\varepsilon_2'+2\varepsilon_2'\varepsilon_2+\frac{1}{2}\varepsilon_0\varepsilon_2'-\frac{1}{2}\varepsilon_0\varepsilon_2\\+2\varepsilon_1\varepsilon_2-2\varepsilon_1'\varepsilon_2-2\varepsilon_0\varepsilon_1+2\varepsilon_0\varepsilon_1'\end{matrix}\right]\\+\varepsilon_2'^2-2\varepsilon_2'\varepsilon_2+2\varepsilon_0\varepsilon_2'-2\varepsilon_0\varepsilon_2+\varepsilon_2^2+\varepsilon_0^2\end{matrix}\right\}\right]$$

$$MSE[\bar{Y}_{FA1}]=\bar{Y}^2\left\{\begin{matrix}\alpha^2\left[\begin{matrix}\frac{9}{4}+4\theta C_x^2+4\theta' C_x^2+\theta C_z^2+\theta' C_z^2+\frac{9}{4}\left[\theta C_y^2+\theta^* C_{y2}^2\right]-8\theta' C_x^2\\-4\theta\rho_{xz}C_xC_z+4\theta'\rho_{xz}C_xC_z-6\theta\rho_{yx}C_yC_x+4\theta'\rho_{xz}C_xC_z-2\theta' C_z^2\\-4\theta'\rho_{xz}C_xC_z+6\theta'\rho_{yx}C_yC_x+3\theta\rho_{yz}C_yC_z-3\theta'\rho_{yz}C_yC_z\end{matrix}\right]\\+2\alpha\left[\begin{matrix}\frac{3}{2}\left[\theta C_y^2+\theta^* C_{y2}^2\right]-\theta' C_z^2-\theta C_z^2-2\theta'\rho_{xz}C_xC_z+2\theta'\rho_{xz}C_xC_z\\+2\theta' C_z^2+\frac{1}{2}\theta'\rho_{yz}C_yC_z-\frac{1}{2}\theta\rho_{yz}C_yC_z+2\theta\rho_{xz}C_xC_z\\-2\theta'\rho_{xz}C_xC_z-2\theta\rho_{yx}C_yC_x+2\theta'\rho_{yx}C_yC_x\end{matrix}\right]\\+\theta' C_z^2-2\theta' C_z^2+2\theta'\rho_{yz}C_yC_z-2\theta\rho_{yz}C_yC_z+\theta C_z^2+\theta C_y^2+\theta^* C_{y2}^2\end{matrix}\right\}$$

$$MSE[\bar{Y}_{FA1}]=\bar{Y}^2\left\{\begin{matrix}\alpha^2\left[\begin{matrix}\frac{9}{4}+4\theta C_x^2-4\theta' C_x^2+\theta C_z^2-\theta' C_z^2+\frac{9}{4}\left[\theta C_y^2+\theta^* C_{y2}^2\right]-4\theta\rho_{xz}C_xC_z\\+4\theta'\rho_{xz}C_xC_z-6\theta\rho_{yx}C_yC_x+6\theta'\rho_{yx}C_yC_x+3\theta\rho_{yz}C_yC_z\\-3\theta'\rho_{yz}C_yC_z\end{matrix}\right]\\+2\alpha\left[\begin{matrix}\frac{3}{2}\left[\theta C_y^2+\theta^* C_{y2}^2\right]+\theta' C_z^2-\theta C_z^2-2\theta'\rho_{xz}C_xC_z+2\theta\rho_{xz}C_xC_z\\+\frac{1}{2}\theta'\rho_{yz}C_yC_z-\frac{1}{2}\theta\rho_{yz}C_yC_z-2\theta\rho_{yx}C_yC_x+2\theta'\rho_{yx}C_yC_x\end{matrix}\right]\\+\theta C_z^2-\theta' C_z^2+2\theta'\rho_{yz}C_yC_z-2\theta\rho_{yz}C_yC_z+\theta C_y^2+\theta^* C_{y2}^2\end{matrix}\right\}$$

$$MSE[\bar{Y}_{FA1}] = \bar{Y}^2 \left\{ \begin{array}{c} \alpha^2 \left[\begin{array}{c} \frac{9}{4} - 4[\theta' - \theta]C_x^2 - [\theta' - \theta]C_z^2 + 4[\theta' - \theta]\rho_{xz}C_xC_z \\ +6[\theta' - \theta]\rho_{yx}C_yC_x - 3[\theta' - \theta]\rho_{yz}C_yC_z + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2\right] \end{array} \right] \\ +2\alpha \left[\begin{array}{c} [\theta' - \theta]C_z^2 - 2[\theta' - \theta]\rho_{xz}C_xC_z + \frac{1}{2}[\theta' - \theta]\rho_{yz}C_yC_z \\ +2[\theta' - \theta]\rho_{yx}C_yC_x + \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right] \end{array} \right] \\ -[\theta' - \theta]C_z^2 + [\theta' - \theta]2\rho_{yz}C_yC_z + \theta C_y^2 + \theta^* C_{y2}^2 \end{array} \right\} \quad 15$$

Differentiating equation 15 w.r.t. α, we have

$$\left[\begin{array}{c} 2\alpha\left[\frac{9}{4} - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right] + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right] \\ +2\left[[\theta' - \theta]\left[C_z^2 - 2\rho_{xz}C_xC_z + \frac{1}{2}\rho_{yz}C_yC_z + 2\rho_{yx}C_yC_x\right] + \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right] \end{array} \right] = 0$$

$$\alpha = \frac{-2\left[[\theta' - \theta]\left[C_z^2 - 2\rho_{xz}C_xC_z + \frac{1}{2}\rho_{yz}C_yC_z + 2\rho_{yx}C_yC_x\right] + \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]}{2\left[\frac{9}{4} - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right] + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]}$$

$$\alpha^{opt} = \frac{[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]}{\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]} \quad 16$$

Substitute α^{opt} in equation 3.16, we have

$$MSE[\bar{Y}_{FA1}]^{opt}$$

$$= \bar{Y}^2 \left\{ \begin{array}{c} \left[\frac{[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]}{\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]}\right]^2 \\ \left[\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]\right] \\ -2\left[\frac{[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]}{\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]}\right] \\ \left[[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right] \\ +[\theta' - \theta]\left[2\rho_{yz}C_yC_z - C_z^2\right] + \theta C_y^2 + \theta^* C_{y2}^2 \end{array} \right\}$$

$$MSE[\bar{Y}_{FA1}]^{opt}$$

$$= \bar{Y}^2 \left\{ \begin{array}{c} \frac{\left[[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]^2}{\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]} \\ -2\left[\frac{\left[[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]^2}{\frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right] - [\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right]}\right] \\ +[\theta' - \theta]\left[2\rho_{yz}C_yC_z - C_z^2\right] + \theta C_y^2 + \theta^* C_{y2}^2 \end{array} \right\}$$

$$MSE[\,\overline{Y}_{FA1}]^{opt} = \overline{Y}^2 \left\{ \begin{array}{c} [\theta' - \theta]\left[2\rho_{yz}C_yC_z - C_z^2\right] + \theta C_y^2 + \theta^* C_{y2}^2 \\ -\dfrac{\left[[\theta' - \theta]\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] - \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]^2}{-[\theta' - \theta]\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right] + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right]} \end{array} \right\}$$

$$MSE[\,\overline{Y}_{FA1}]^{opt} = \overline{Y}^2 \left\{ \begin{array}{c} [\theta - \theta']\left[C_z^2 - 2\rho_{yz}C_yC_z\right] + \theta C_y^2 + \theta^* C_{y2}^2 \\ +\left[\dfrac{\left[[\theta - \theta']\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] + \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]\right]^2}{[\theta - \theta']\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right] + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right]}\right] \end{array} \right\}$$

Let A = $[\theta - \theta']\left[2\rho_{xz}C_xC_z - C_z^2 - \frac{1}{2}\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\right] + \frac{3}{2}\left[\theta C_y^2 + \theta^* C_{y2}^2\right]$

And B = $[\theta - \theta']\left[4C_x^2 + C_z^2 - 4\rho_{xz}C_xC_z - 6\rho_{yx}C_yC_x + 3\rho_{yz}C_yC_z\right] + \frac{9}{4}\left[\theta C_y^2 + \theta^* C_{y2}^2 + 1\right]$

Hence,

$$MSE[\,\overline{Y}_{FA1}]^{opt} = \overline{Y}^2 \left\{[\theta - \theta']\left[C_z^2 - 2\rho_{yz}C_yC_z\right] + \theta C_y^2 + \theta^* C_{y2}^2 + \frac{[A]^2}{B}\right\} \qquad 17$$

Some Existing Estimators with their MSE

Hanson and Hurwitz 1946 proposed an unbiased estimator for population mean as:

$$\overline{Y}_{HH} = w_1\overline{y} + w_2\overline{y}^* \qquad 18$$

and its MSE as:

$$MSE[\,\overline{Y}_{HH}] = \overline{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2\right\} \qquad 19$$

Singh *et al* 2009 proposed an exponential estimator for estimation of mean in the presence of nonresponse as:

$$\overline{Y}_{Set} = \overline{y}^* exp\left[\frac{\overline{X} - \overline{x}}{\overline{X} + \overline{x}}\right] \qquad 20$$

and its MSE as:

$$MSE\ (\overline{Y}_{Set}) = \overline{Y}^2\left\{\theta C_y^2 + \frac{\theta C_x^2}{4} - \theta\rho_{yx}C_yC_x\right\} + \theta^* C_{y2}^2 \qquad 21$$

Hazra 2015 proposed an exponential ratio-type estimator for estimating population mean using auxiliary variable with double sampling in the presence of nonresponse as:

$$\overline{Y}_H = \overline{y}^*\left[\alpha \exp\left(\frac{\overline{x}'}{\overline{x}^*} - 1\right) + (1 - \alpha)\exp\left(\frac{\overline{x}'}{\overline{x}} - 1\right)\right] \qquad 22$$

and its MSE as:

$$MSE[\,\overline{Y}_H] = \overline{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2 + [\theta - \theta']C_x^2 - 2[\theta - \theta']\rho_{yx}C_yC_x - \theta^*\left[\rho_{yx2}C_{y2}\right]^2\right\} \qquad 23$$

Khan and Khan 2022 proposed exponential ratio-type estimators of population mean using two auxiliary variables under nonresponse as:

$$\bar{Y}_{KK} = \bar{y}\left[\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right)\right] \quad 24$$

and its MSE as:

$$MSE_{KK} = \bar{Y}^2\{\theta^* C_x^2 + \theta^* C_z^2 + \theta^* C_y^2 - 2\theta\rho_{yx}C_yC_x - 2\theta\rho_{yz}C_yC_z + 2\theta\rho_{xz}C_xC_z\} \quad 25$$

Oguagbaka *et al* 2024 proposed a ratio estimator for double sampling procedure with nonresponse as:

$$\bar{Y}_{Oget} = \bar{y}^*\left[\alpha\left(\frac{\bar{x}^*}{\bar{x}'}\right) + (1-\alpha)\left(\frac{\bar{x}}{\bar{x}'}\right)\right] \quad 26$$

and its MSE as:

$$MSE[\bar{Y}_{Oget}] = \bar{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2 + \theta C_x^2 + 2\theta\rho_{yx}C_yC_x + 2\theta'\rho_{yx}C_yC_x - \frac{3[\rho_{yx}C_yC_x]^2}{\theta^* C_{x2}^2}\right\} \quad 27$$

Efficiency Comparison

Some conditions were identified under which the proposed estimator performs better than the existing estimators in terms of efficiency by comparing the mean square errors of the estimators.

Efficiency Comparison of the proposed estimator $\bar{Y}_{FA1}$ with Oguagbaka (2024) estimator

The proposed estimator $\bar{Y}_{FA1}$ will be more efficient than the Oguagbaka *et al* (2024) estimator if and only if

$$\frac{MSE(\bar{Y}_{Oget})}{MSE(\bar{Y}_{FA1})} > 1 \quad 28$$

Equation 28 implies $MSE(\bar{Y}_{Oget}) > MSE(\bar{Y}_{FA1})$ which also implies

$$MSE(\bar{Y}_{Oget}) - MSE(\bar{Y}_{FA1}) > 0 \quad 29$$

Substituting equation 27 and 17 in equation 29, we have

$$\bar{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2 + \theta C_x^2 + 2\theta\rho_{yx}C_yC_x + 2\theta'\rho_{yx}C_yC_x - \frac{3[\rho_{yx}C_yC_x]^2}{\theta^* C_{x2}^2}\right\} - \bar{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2 + [\theta - \theta'][C_z^2 - 2\rho_{yz}C_yC_z] + \frac{[A]^2}{B}\right\} > 0$$

Hence, the proposed estimator $\bar{Y}_{FA1}$ will be more efficient than the Oguagbaka *et al* (2024) estimator if

$$\theta^* C_{x2}^2\left\{\theta[C_x^2 + 2\rho_{yx}C_yC_x] - [\theta - \theta'][C_z^2 - 2\rho_{yz}C_yC_z] - \frac{[A]^2}{B}\right\} - 3[\rho_{yx}C_yC_x]^2 > 0 \quad 30$$

Efficiency Comparison of the proposed estimator $\bar{Y}_{FA1}$ with Khan and Khan (2022) Estimator

The proposed estimator $\bar{Y}_{FA1}$ will be more efficient than the Khan and Khan (2022) estimator if and only if

$$\frac{MSE(\bar{Y}_{KK})}{MSE(\bar{Y}_{FA1})} > 1 \quad 31$$

Equation 31 implies $MSE(\bar{Y}_{KK}) > MSE(\bar{Y}_{FA1})$ which also implies

$$MSE(\bar{Y}_{KK}) - MSE(\bar{Y}_{FA1}) > 0 \quad 32$$

Substituting equation 25 and 17 in equation 32, we have

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$$\overline{\mathrm{Y}}^2\{\theta^*C_x^2+\theta^*C_z^2+\theta^*C_y^2-2\theta\rho_{yx}C_yC_x-2\theta\rho_{yz}C_yC_z+2\theta\rho_{xz}C_xC_z\}-\overline{\mathrm{Y}}^2\left\{\theta C_y^2+\theta^*C_{y2}^2+[\theta-\theta'][C_z^2-2\rho_{yz}C_yC_z]+\frac{[A]^2}{B}\right\}>0$$

Hence, the proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Khan and Khan (2022) estimator if

$$\theta^*[C_x^2+C_z^2+C_y^2-C_{y2}^2]-\theta[C_y^2+2\rho_{yx}C_yC_x+2\rho_{yz}C_yC_z-2\rho_{xz}C_xC_z]-[\theta-\theta'][C_z^2-2\rho_{yz}C_yC_z]-\frac{[A]^2}{B}>0 \quad 33$$

Efficiency Comparison of the proposed estimator $\overline{Y}_{FA1}$ with Hazra (2015) estimator

The proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Hazra (2015) estimator if and only if

$$\frac{MSE(\overline{Y}_H)}{MSE(\overline{Y}_{FA1})}>1 \quad 34$$

Equation 34 implies $MSE(\overline{Y}_H)>MSE(\overline{Y}_{FA1})$ which also implies

$$MSE(\overline{Y}_H)-MSE(\overline{Y}_{FA1})>0 \quad 35$$

Substituting equation 23 and 17 in equation 35, we have

$$\overline{\mathrm{Y}}^2\left\{\theta C_y^2+\theta^*C_{y2}^2+[\theta-\theta']C_x^2-2[\theta-\theta']\rho_{yx}C_yC_x-\theta^*[\rho_{yx2}C_{y2}]^2\right\}-\overline{\mathrm{Y}}^2\left\{\theta C_y^2+\theta^*C_{y2}^2+[\theta-\theta'][C_z^2-2\rho_{yz}C_yC_z]+\frac{[A]^2}{B}\right\}>0$$

Hence, the proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Hazra (2015) estimator if

$$[\theta-\theta'][C_x^2-C_z^2-2\rho_{yx}C_yC_x+2\rho_{yz}C_yC_z]-\theta^*[\rho_{yx2}C_{y2}]^2-\frac{[A]^2}{B}>0 \quad 36$$

Efficiency Comparison of the proposed estimator $\overline{Y}_{FA1}$ with Singh (2009) estimator

The proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Singh (2009) estimator if and only if

$$\frac{MSE(\overline{Y}_{Set})}{MSE(\overline{Y}_{FA1})}>1 \quad 37$$

Equation 37 implies $MSE(\overline{Y}_{Set})>MSE(\overline{Y}_{FA1})$ which also implies

$$MSE(\overline{Y}_{Set})-MSE(\overline{Y}_{FA1})>0 \quad 38$$

Substituting equation 21 and 17 in equation 38, we have

$$\overline{\mathrm{Y}}^2\left\{\theta C_y^2+\theta^*C_{y2}^2+\frac{C_x^2}{4}-\theta\rho_{yx}C_yC_x\right\}-\overline{\mathrm{Y}}^2\left\{\theta C_y^2+\theta^*C_{y2}^2+[\theta-\theta'][C_z^2-2\rho_{yz}C_yC_z]+\frac{[A]^2}{B}\right\}>0$$

Hence, the proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Singh (2009) estimator if

$$\theta\left[\frac{C_x^2}{4}-\rho_{yx}C_yC_x\right]-[\theta-\theta'][C_z^2-2\rho_{yz}C_yC_z]-\frac{[A]^2}{B}>0 \quad 39$$

Efficiency Comparison of the proposed estimator $\overline{Y}_{FA1}$ with Hansen and Hurwitz (1946) Estimator

The proposed estimator $\overline{Y}_{FA1}$ will be more efficient than the Hansen and Hurwitz (1946) estimator if and only if

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$\frac{MSE(\bar{Y}_{HH})}{MSE(\bar{Y}_{FA1})} > 1$ 40

Equation 40 implies $MSE(\bar{Y}_{HH}) > MSE(\bar{Y}_{FA1})$ which also implies

$MSE(\bar{Y}_{HH}) - MSE(\bar{Y}_{FA1}) > 0$ 41

Substituting equation 19 and 17 in equation 41, we have

$$\bar{Y}^2\{\theta C_y^2 + \theta^* C_{y2}^2\} - \bar{Y}^2\left\{\theta C_y^2 + \theta^* C_{y2}^2 + [\theta - \theta'][C_z^2 - 2\rho_{yz}C_yC_z] + \frac{[A]^2}{B}\right\} > 0$$

Hence, the proposed estimator $\bar{Y}_{FA1}$ will be more efficient than the Hansen and Hurwitz (1946) estimator if

$[\theta - \theta'][C_z^2 - 2\rho_{yz}C_yC_z] + \frac{[A]^2}{B} < 0$ 42

RESULT AND DISCUSSION

This study uses a real life data obtained from the payment voucher of the Akure South Local Government, Nigeria consisting of 1,023 workers. The double sampling uses tax as the study variable y, grade level as the auxiliary variable x and gross payment as auxiliary variable z. The double sampling is done with the first sample, second sample and subsample sizes at four levels (k = 2, 3, 4, 5) at 5% nonresponse level.

The data obtained was analyzed using SPSS, Scilab software and Microsoft Excel.

Table 1Sample Estimates at 5% Nonresponse

Term	k = 2	k = 3	k = 4	k = 5
N_1	972	972	972	972
N_2	51	51	51	51
n'	600	600	600	600
n	450	450	450	450
n_1	427	427	427	427
n_2	23	23	23	23
f	0.43988	0.43988	0.43988	0.43988
f'	0.58651	0.58651	0.58651	0.58651
W_1	0.9501	0.9501	0.9501	0.9501
W_2	0.0499	0.0499	0.0499	0.0499
w_1	0.9489	0.9489	0.9489	0.9489
w_2	0.0511	0.0511	0.0511	0.0511
θ	0.001245	0.001245	0.001245	0.001245
θ'	0.000689	0.000689	0.000689	0.000689
θ^*	0.0001109	0.0002218	0.0003327	0.0004436
$\bar{y}$	5528.6505	5528.6505	5528.6505	5528.6505
$\bar{x}$	9.03	9.03	9.03	9.03
$\bar{z}$	87114.2835	87114.2835	87114.2835	87114.2835
$\bar{x}'$	9.12	9.12	9.12	9.12
$\bar{z}'$	89272.9517	89272.9517	89272.9517	89272.9517
s_y	7176.68099	7176.68099	7176.68099	7176.68099
s_x	3.259	3.259	3.259	3.259
s_z	63589.33128	63589.33128	63589.33128	63589.33128
s_y^2	51504750.06	51504750.06	51504750.06	51504750.06

s_x^2	10.624	10.624	10.624	10.624
s_z^2	4043603052	4043603052	4043603052	4043603052
s_{yx}	15958.438	15958.438	15958.438	15958.438
s_{yz}	450727541.1	450727541.1	450727541.1	450727541.1
s_{xz}	150959.333	150959.333	150959.333	150959.333
ρ_{yx}	0.682	0.682	0.682	0.682
ρ_{yz}	0.988	0.988	0.988	0.988
ρ_{xz}	0.728	0.728	0.728	0.728
C_y	1.298089	1.298089	1.298089	1.298089
C_x	0.361027	0.361027	0.361027	0.361027
C_z	0.729953	0.729953	0.729953	0.729953
C_y^2	1.685035	1.685035	1.685035	1.685035
C_x^2	0.130341	0.130341	0.130341	0.130341
C_z^2	0.532831	0.532831	0.532831	0.532831
C_{yx}	2.452986	2.452986	2.452986	2.452986
C_{yz}	1.756369	1.756369	1.756369	1.756369
C_{xz}	0.360234	0.360234	0.360234	0.360234
$b **$	1502.1120	1502.1120	1502.1120	1502.1120
r	12	8	6	5
ρ_{yx2}	0.787	0.792	0.759	0.776
ρ_{yz2}	0.994	0.974	0.980	0.999
ρ_{xz2}	0.815	0.838	0.684	0.799
C_{y2}	1.294723	1.049499	0.749364	0.972256
C_{x2}	0.379059	0.333809	0.361265	0.302917
C_{z2}	0.902711	0.692807	0.523851	0.669536
C_{y2}^2	1.676307	1.1014473	0.561546	0.945282
C_{x2}^2	0.143685	0.111429	0.130512	0.091759
C_{z2}^2	0.814887	0.479982	0.274420	0.448278
C_{yx2}	2.687352	2.489638	1.574968	2.490271
C_{yz2}	1.426058	1.4748340	1.401865	1.450971
C_{xz2}	0.342423	0.4039821	0.471519	0.361389
s_{y2}	11277.57472	9587.60524	3217.92982	3494.80149
s_{x2}	3.696	3.338	3.312	2.302
s_{z2}	106364.2757	86953.30703	42814.00917	50007.29158
s_{y2}^2	127183691.6	91922174.19	10355072.30	12213637.48
s_{x2}^2	13.659	11.143	10.967	5.30
s_{z2}^2	11313359145	756087.7603	1833039381	2500729211
s_{yx2}	32792.965	25343.180	8091.307	6242.389
s_{yz2}	1192671034	811655078.0	135015484.8	174625578.6
s_{xz2}	320562.136	243366.351	96940.208	91959.364
$\bar{y}*$	8710.4175	9135.4150	4294.2150	3594.5280
$\bar{x} *$	9.75	10.00	9.17	7.60
$\bar{z} *$	117827.6158	125508.6888	81729.3917	74689.4740

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Table 2 Mean Square Error (MSE) at 5% Nonresponse

Estimator	k=2	k=3	k=4	k=5
Y_{FA1}	**54601.35**	**56482.19**	**58774.61**	**61902.44**
Y_{HH}	75916.48	78304.12	80892.77	83746.55
Y_{Set}	61283.44	63152.76	65308.41	67819.22
Y_{H}	72648.31	74902.56	77381.94	79942.87
Y_{KK}	58463.72	60894.36	63811.49	67295.18
Y_{Oget}	61702.48	63594.21	66104.73	68337.90

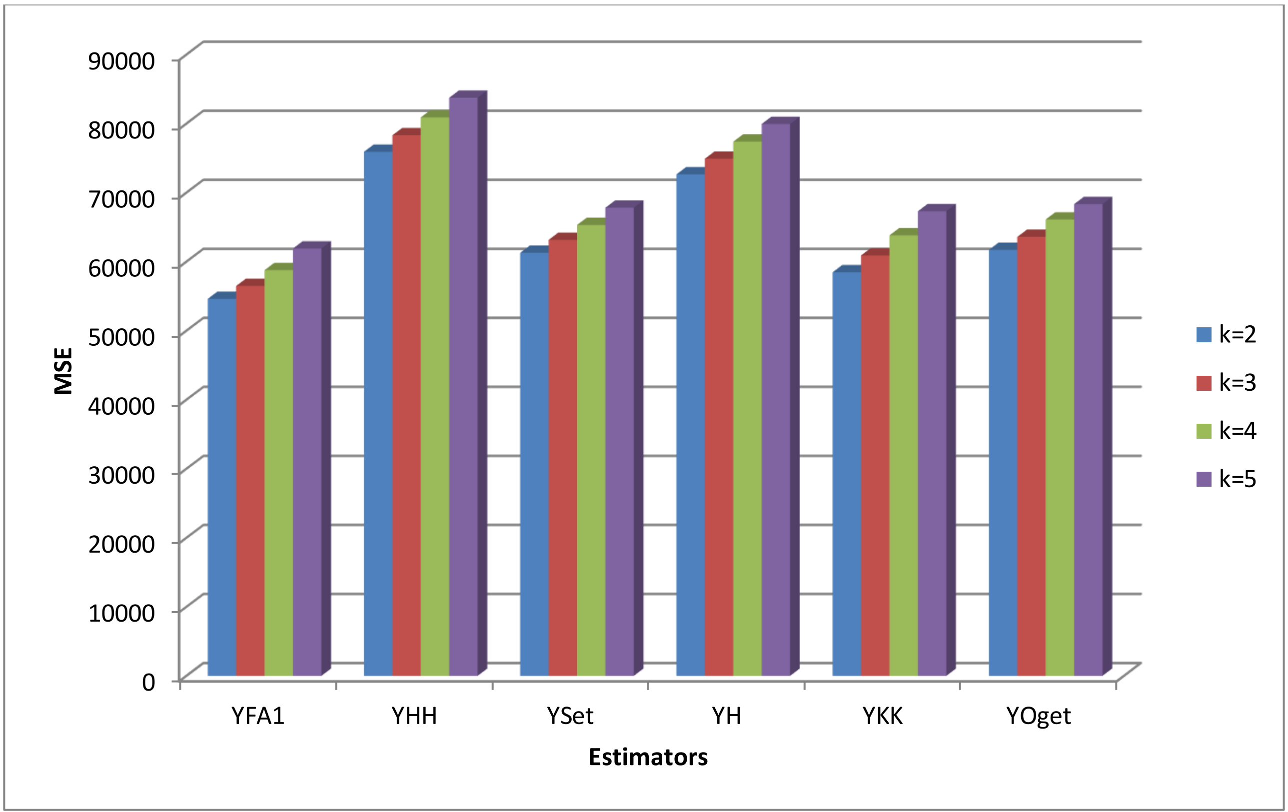

Figure 1 Bar chart showing the Mean Square Error at 5% Nonresponse

Table 2 and Figure 1 show that at 5% nonresponse, the MSE of all estimators increases as the value of k increases. The proposed estimators, Y_{FA1} achieves the minimum MSE at all levels of k respectively.

Table 3 Coefficient of Variation (CV) at 5% Nonresponse

Estimator	k=2	k=3	k=4	k=5
Y_{FA1}	**2.68**	**2.60**	**5.64**	**6.93**
Y_{HH}	3.16	3.06	6.63	8.03
Y_{Set}	2.84	2.74	5.94	7.20
Y_{H}	3.09	2.99	6.48	7.83
Y_{KK}	2.76	2.69	5.83	7.09
Y_{Oget}	2.85	2.75	5.98	7.26

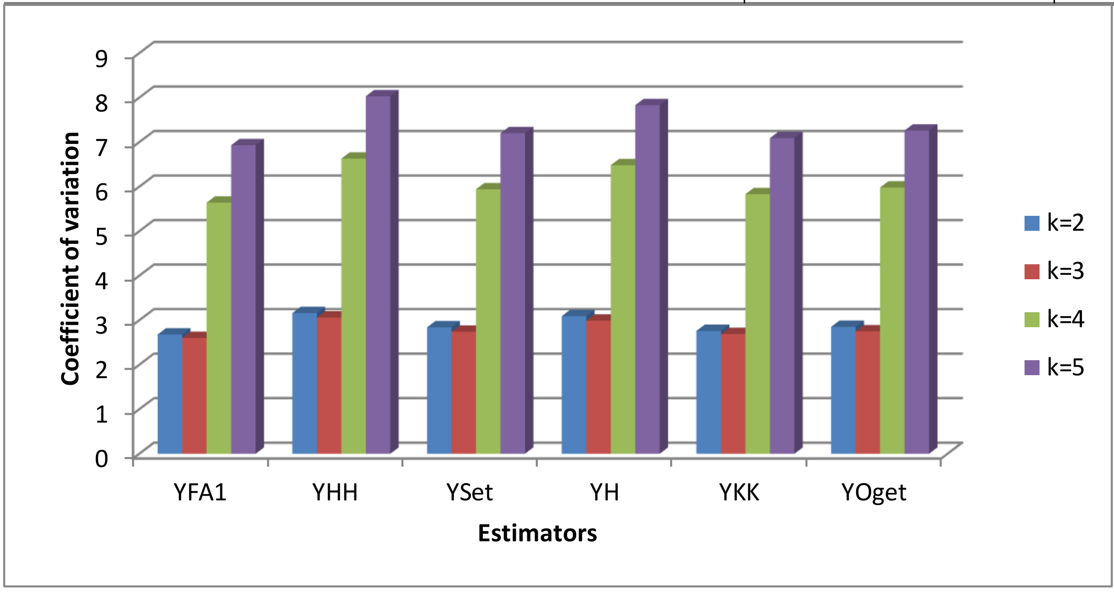

Figure 2Bar chart showing the Coefficient of Variation at 5% nonresponse

Table 3 and figure 2 show that the CV increases as k increases and it was observed that the CV values are generally low across all estimators, however, the proposed estimator outperform the existing ones.

Table 4 Relative Efficiency (RE) at 5% Nonresponse

Estimator	k=2	k=3	k=4	k=5
Y_{FA1}	139.04	138.64	137.63	135.29
Y_{HH}	100	100	100	100
Y_{Set}	123.88	123.99	123.86	123.48
Y_{H}	104.50	104.54	104.54	104.76
Y_{KK}	129.85	128.59	126.77	124.45
Y_{Oget}	123.04	123.13	122.37	122.55

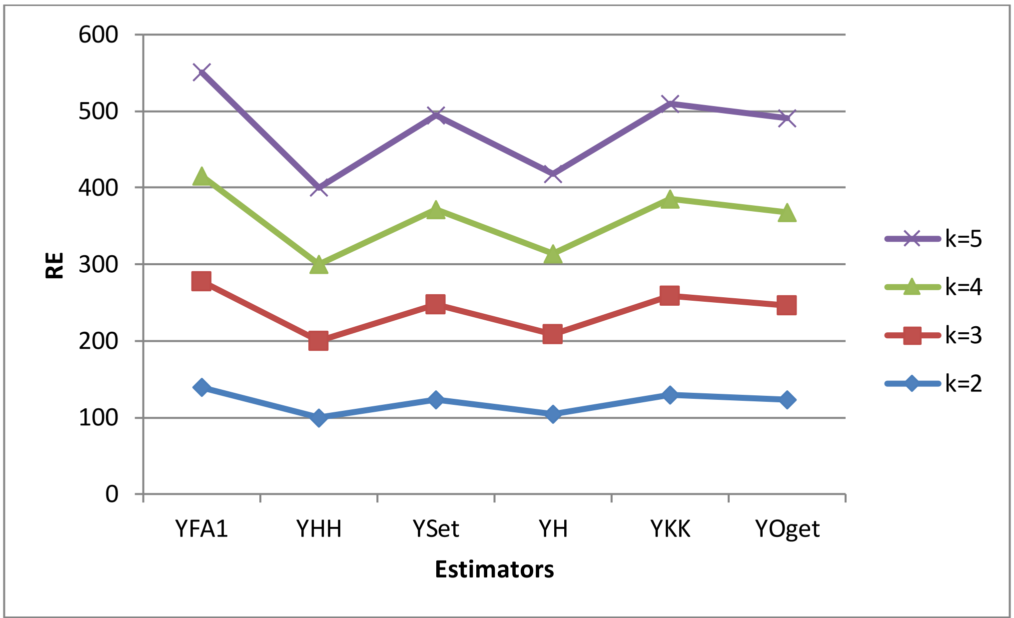

Figure 3 Graph showing the Relative Efficiency at 5% Nonresponse

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Table 4 and figure 3 show that the proposed estimator had the highest RE value across all subsampling rate k which implies that the proposed estimator is more efficient than other estimators.

DISCUSSION

The performance of the proposed estimator was evaluated using numerical illustrations and compared with several existing estimators, including estimators by Hansen–Hurwitz, Singh et al., Hazra, Khan and Khan, and Oguagbaka et al.

Summarily, the mean square error (MSE) values obtained from the numerical study demonstrate that the proposed estimator consistently produces smaller MSE values, lower CV values than competing estimators across different values of (k).

In conclusion, the theoretical and empirical results demonstrate that the proposed estimator provides a more efficient alternative for estimating the population mean in double sampling in the presence of nonresponse. The reduction in MSE can be attributed to the effective use of two auxiliary variables within an exponential framework, which helps capture additional information about the population and reduce estimation variability.

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