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Comparative Analysis of the Efficiency of Sampling Scheme Estimators in Estimating Population Total

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ABSTRACT

Sampling is a fundamental tool in statistical research, providing a practical alternative to complete enumeration where of time, cost, personal and accessibility are constraints Choosing the best estimator for population total estimation is one of the main issues in survey sampling. Even though many different sampling strategies and estimators are available, it is still difficult to assess how effective they are in diverse situations. This study presents a comparative analysis of the efficiency of sampling scheme estimators (Hansen Hurwitz, Horvitz-Thompson, Rao-Hartley-Cochran's and Sen Yates-Grundy) in estimating population total using child birth data) from Ekiti State . Population totals and variances of each estimator were obtained and the most efficient estimator determined in terms of variance. The results revealed that the Rao–Hartley–Cochran estimator consistently produced the lowest population total estimates with the least variance for 2 years, while in the other year, the Sen–Yates–Grundy estimator demonstrated superior efficiency with minimal variance. (The study offers empirical evidence regarding the relative efficiency of the probability proportional to size estimators with replacement (Hansen–Hurwitz) and those without (Horvitz–Thompson, Rao–Hartley–Cochran, and Sen–Yates–Grundy in estimating population totals). The study further showed that' efficiency of estimators is not constant but rather fluctuates over time and across data distributions. The study also closes the gap between theoretical sampling principles and real-world application in demographic and health statistics by using these sample strategies on actual child birth registration data from Ekiti State.

Keyword: Relative efficiency, childbirth, Estimator, Sampling re-arrange for neatness

INTRODUCTION

Comparing the effectiveness of various sampling estimators requires empirical research. Although theoretical characteristics are widely known, actual performance varies based on a number of variables, including sample size limitations, data quality, and demographic variability and operational factors like cost, computational efficiency, and ease of implementation are taken into account when choosing a sample estimator. A number of sampling techniques have been created over time to improve population estimation accuracy Simple random sampling (SRS), in which every unit in the population has an equal chance of being selected, is the most straightforward and widely used technique. Even while SRS offers objective estimations, it frequently produces considerable variability, especially when working with diverse populations (Singh & Mangat, 1996). Alternative sample methods such cluster sampling, stratified sampling, and systematic sampling have been developed to overcome this problem. Regression and ratio estimators have been created to increase the effectiveness of population estimates in addition to these conventional methods. To lower variance and increase precision, these estimators make use of auxiliary data that is connected to the research variable. One well-known example that established the basis for probability-weighted estimate in survey sampling is the Horvitz-Thompson estimator (1952) (Horvitz & Thompson, 1952). Accordingly, Dawodu, Adewara, and Oshungade (2013) confirmed that one of the main justifications for the creation of the Rao-Hartley-Cochran sampling scheme was its shortcomings, such as negative in variance estimates of the Horvitz-Thompson scheme. Claims that the probability proportional to size with replacement is the least efficient scheme are also clear; yet, research like Chaudry and Patra (2023) suggested that the probability proportional to size with replacement is the most efficient estimator. Choosing the best estimator for population total estimation is one of the main issues in survey sampling. Even though there are many different sampling strategies and estimators available, it is still difficult to assess how effective they are in diverse situations (Kish, 1965). Despite being simple to use, simple random sampling (SRS) frequently produces significant variance when estimating population totals, especially in populations that are heterogeneous (Singh & Mangat, 1996). By splitting the population into homogeneous subgroups, stratified sampling, on the other hand, lowers variance; but it necessitates in-depth knowledge of population characteristics, which is not always available (Cochran, 1977).

Even while systematic sampling is straightforward and frequently more effective than SRS, it may produce biased results if the population contains hidden periodic patterns (Thompson, 2012). Compared to stratified or systematic sampling, cluster sampling typically increases variance, but it might be helpful when population elements are naturally grouped (Särndal et al., 2003). The selection of an estimator under probability proportional to size has been controversial. Some of the options include probability proportional to size with replacement estimator, Horvitz-Thompson without replacement estimator, and Rao-Hartley-Cochran's, Yates Grundy, and Midzuno's special cases without replacement estimator. Although each estimator has theoretical explanations from previous research, empirical validation across a variety of datasets is required to create useful guidance for their application (Lohr, 2021).

METHODOLOGY

Probability Proportional to Size

Every unit in the population has an equal chance of being selected in a random sample produced using the basic random sampling scheme. In some cases, giving the units in the population uneven probability of selection yields more effective estimators.

Probability Proportion to Size with Replacement.

PPS sampling with replacement is the term used when the chosen unit with the corresponding size in the sample is reexamined in the sampling frame in the probability proportional to size problem.

Probability Proportion to Size without Replacement.

PPS sampling without sampling occurs when the chosen unit with the corresponding size in the sample is disregarded from the sampling frame.

Probability Proportional to Size with replacement (Hansen and Hurwitz Estimator)

The idea of probability proportional to size sampling was first given by Neyman (1934). Hansen and Hurwitz (1943) developed the general theory of probability proportional to size with replacement. One unit was selected at each of the n draws. They allocated the selection probability to the ith unit of the population given by

$$P_i = \frac{Y_i}{Y} \quad (1)$$

where Y_i is the measure of size (auxiliary variable) for the ith population unit and $Y = \sum_{i=1}^{N} Y_i$.

Unbiased estimate of Population total

If a sample of size n units is drawn with PPS of x_i and with replacement, then

$$\hat{Y}_{HH} = \frac{1}{n} \sum_{i}^{n} \frac{y_i}{p_i} \quad (2)$$

Is an unbiased estimate of the population total Y. where $p_i = \frac{x_i}{X}$ is the probability of selecting the ith unit in the sample.

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Proof: Let t_i be the number of times that the i^{th} unit appears in a specific sample of size n, where t_i may have any of the values 0,1,2,…,n. Consider the joint frequency distribution of the t_i for all N units in the population. The method of drawing the sample is equivalent to the standard probability problem in the n balls are thrown into N boxes, the probability that a ball goes into the i^{th} box being p_i at every throw. Consequently, the joint distribution of the t_i is the multinomial expression

$$\frac{n!}{t_1!\,t_2!\ldots t_n!}{p_1}^{t_1}{p_2}^{t_2}\ldots{p_N}^{t_N} \quad 3$$

For the multinomial, the following properties of the distribution of t_i are well known:

$$E(t_i) = np_i \quad 4$$

$$V(t_i) = np_i\,(1 - p_i) \quad 5$$

$$Cov\left(t_i t_j\right) = -np_i p_j \quad 6$$

We may therefore write (4) as

$$\hat{Y}_{HH} = \frac{1}{n}\left(t_1\frac{y_1}{p_1} + t_2\frac{y_2}{p_2} + \cdots + t_N\frac{y_N}{p_N}\right)$$

$$= \frac{1}{n}\sum_{i}^{n} t_i\,\frac{y_i}{p_i} \quad 7$$

Where the sum extends over all units in the population. In repeated sampling the t_i are the random variables, whereas the y_i and the p_i are a set of fixed numbers.

By taking the expectation of both sides of (3.4.7), we have

$$E\left(\hat{Y}_{HH}\right) = \frac{1}{n}\sum_{i=1}^{n} E\left(\frac{y_i}{p_i}\right) \quad 8$$

Hence, since $E(t_i) = np_i$ by (3.4.4) we have,

$$E\left(\hat{Y}_{HH}\right) = \frac{1}{n}\sum_{i=1}^{n}(np_i)\,\frac{y_i}{p_i} \quad 9$$

$$= \sum_{i=1}^{N} y_i$$

$$= Y$$

Therefore $\hat{Y}$ is unbiased.

Variance of Hansen Hurwitz estimator

Taking the variance of both sides of the estimator in (3.4.2),

Concerning the variance, we have:

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$$\therefore \quad V(\hat{Y}_{HH}) = \frac{1}{n}\sum_{i=1}^{N} p_i\left(\frac{y_i}{p_i} - Y\right)^2 \qquad 10$$

since $\sum_{i=1}^{N} p_i = 1.$

Unbiased estimator of Variance of Hansen Hurwitz estimator

If a sample of n units is drawn with PPS of x_i and with replacement, an unbiased sample estimate of $V(\hat{Y}_{HH})$ is, for any $n > 1$,

$$v(\hat{Y}_{HH}) = \sum_{i}^{n} \frac{(y_i/p_i - \hat{Y}_{HH})^2}{n(n-1)} \qquad 11$$

By the usual algebraic identity,

$$= n\left(\frac{1}{n}\sum\frac{y_i}{p_i} - Y\right)$$

$$\sum_{i}^{n}\left(\frac{y_i}{p_i} - \hat{Y}_{HH}\right)^2 = \sum\left(\frac{y_i}{p_i} - Y\right)^2 - 2n(\hat{Y}_{HH} - Y)^2 + n(\hat{Y}_{HH} - Y)^2$$

$$= \sum\left(\frac{y_i}{p_i} - Y\right)^2 - n(\hat{Y}_{HH} - Y)^2 \qquad 12$$

$$= \sum\left(\frac{y_i}{p_i} - Y\right)^2 - n\left(\frac{1}{n}\sum\frac{y_i}{p_i} - Y\right)^2$$

$\therefore$ from (3.11) we have

$$n(n-1)v(\hat{Y}_{HH}) = \sum_{i}^{n}\left(\frac{y_i}{p_i} - Y\right)^2 - n(\hat{Y}_{HH} - Y)^2$$

Hence (12) becomes

$$n(n-1)E[v(\hat{Y}_{HH})] = n^2V(\hat{Y}_{HH}) - nV(\hat{Y}_{HH})$$

$$= n(n-1)V(\hat{Y}) \qquad 13$$

$$\therefore E[v(\hat{Y}_{HH})] = V(\hat{Y}_{HH})$$

Hence, an unbiased variance estimator of $V(\hat{Y}_{HH})$ is given by Cochran (1953):

$$Var(y'_{HH}) = \frac{1}{n(n-1)}\sum_{j=1}^{n}\left(\frac{y_i}{p_i} - y'_{HH}\right)^2 \qquad 14$$

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Method II. We have by definition that

$$\hat{v}(\hat{Y}_{HH}) = \frac{1}{n(n-1)}\left[\sum_{i=1}^{n}\frac{{y_i}^2}{{P_i}^2} - n\,\hat{Y}_{HH}^2\right] \quad 15$$

Taking the expectation,

$$E\left[\hat{v}(\hat{Y}_{HH})\right] = E\left[\frac{1}{n(n-1)}\left\{\sum_{i=1}^{n}\frac{{Y_i}^2}{{P_i}^2} - n\hat{Y}_{HH}^2\right\}\right] \quad 16$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n}\frac{{Y_i}^2}{P_i} - Y^2\right) \quad 17$$

Probability Proportional to Size Sampling Without Replacement (PPSWOR) – Horvitz Thompson estimator

Horvitz and Thompson (1952) developed a general theory of sampling without replacement. Horvitz-Thompson estimator of the population total Y is a linear estimator of the sample observations on the basis of n sample observations $y_i, i = 1, 2, \ldots, n$, can be defined as

$$\hat{Y}_{HT} = \sum_{i=1}^{n}\frac{y_i}{\pi_i} = \sum_{i=1}^{N} t_i\frac{y_i}{\pi_i} \quad 18$$

Unbiased estimator of Horvitz Thompson estimator

If a sample of size n units is drawn from population Y without replacement, then we show that Horvitz Thompson estimator

$$y'_{HT} = \sum_{i=1}^{n}\frac{y_i}{\pi_i} = \sum_{i=1}^{N} a_i\frac{y_i}{\pi_i} \quad 19$$

is an unbiased estimate of the population total Y.

$$= \sum_{i=1}^{N} y_i = Y$$

Variance of Horvitz Thompson estimator

From the definition of variance, we have

$$Var(y'_{HT}) = E(y'_{HT})^2 - [E(y'_{HT})]^2 \quad 20$$

Since $y'_{HT} = \sum_{i=1}^{n}\frac{y_i}{\pi_i} = \sum_{i=1}^{N} a_i\frac{y_i}{\pi_i}$ 21

$$Var(y'_{HT}) = E\sum_{i=1}^{N}(a_i\frac{y_i}{\pi_i})^2 - [E(\sum_{i=1}^{N} a_i\frac{y_i}{\pi_i})]^2 \quad 22$$

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By definition of variance and covariance in equation 4 and 5, equation 8 becomes

$$Var(\hat{Y}_{HT}) = \left[\sum_{i=1}^{N} Var(t_i)\frac{{Y_i}^2}{{\pi_i}^2} + \left[\sum_{i\neq j=1}^{N}\sum Cov\,(t_i,t_j)\frac{Y_i}{\pi_i}\frac{Y_j}{\pi_j}\right]\right] \qquad 23$$

$$Var(\hat{Y}_{HT}) = \tfrac{1}{2}\textstyle\sum_{i\neq j=1}^{N}\sum\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2 \qquad 24$$

Unbiased estimator of the Variance of Horvitz Thompson estimator

An unbiased estimator of the variance $V(\hat{Y}_{HT})$ of the Horvitz and Thompson (1952) estimator of the population total Y is given by

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i\in\Omega}\frac{(1-\pi_i)}{\pi_i^2}y^2 + \sum_{i\in\Omega}\sum_{j(\neq i)\epsilon\Omega}\left(\frac{\pi_{ij} - \pi_i\pi_j}{\pi_i\pi_j}\right)\frac{y_iy_j}{\pi_i\pi_j}.$$

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i\in\Omega}\frac{(1-\pi_i)}{\pi_i^2}y^2 + \sum_{i\in\Omega}\sum_{j(\neq i)\epsilon\Omega}\left(\frac{\pi_{ij} - \pi_i\pi_j}{\pi_i\pi_j}\right)\frac{y_iy_j}{\pi_i\pi_j}. \qquad 25$$

Yates-Grundy Draw-by-Draw Procedure

This selection procedure is stated as select the first units with probability proportional to size select the second unit with probability proportional to size of remaining units. This procedure is one of the simplest procedures as it does not impose any restriction on initial probabilities of selection and final probabilities of inclusion.

Variance of Sen-Yates-Grundy estimator

Another form of the $V(\hat{Y}_{HT})$, developed by Sen (1953), and Yates and Grundy (1953) independently, is given by

$$V(\hat{Y}_{HT})_{SYG} = \frac{1}{2}\sum_{i\in s}\sum_{j(\neq i)\epsilon s}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{Y_i}{\pi_i} - \frac{Y_J}{\pi_J}\right)^2. \qquad 26$$

Unbiased estimator of the Variance of Sen-Yates-Grundy estimator

An unbiased estimator of the variance of the Horvitz and Thompson (1952) estimator of the population total Y in the Sen—Yates—Grundy (1953) form is given by

$$\hat{v}_{SYG}(\hat{Y}_{HT}) = \frac{1}{2}\sum_{i\in S}\sum_{j(\neq i)\epsilon S}\left(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2. \qquad 27$$

This implies that ++

$$\frac{1}{2}\sum_{i\in S}\sum_{j(\neq i)\epsilon S}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2 a_{ij}\pi_{ij} = = \frac{1}{2}\sum_{i\in S}\sum_{j(\neq i)\epsilon S}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2 \qquad 28$$

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If $a_{ij}\pi_{ij} = 1, i.e.\, a_{ij} = \frac{1}{\pi_{ij}}$

Rao-Hartley-Cochran Estimator

The estimator for estimating population total is given as

$$y'_{RHC} = \sum_{i=1}^{n} \frac{\pi_i y_{iT}}{p_{iT}} \qquad 29$$

where p_{iT} is the probability of the Tth unit being selected from the ith group.

Table 3.7.1: Rao Hartley Cochran Strategy

Structure of data in RHC-Sampling Strategy										
1st Group (G_1)		**2nd Group** (G_2)			**ith Group** (G_i)			**nth Group** (G_n)	
Value	Prob.	Value	Prob.		Value	Prob.		Value	Prob.
Y_{11}	P_{11}	Y_{21}	P_{21}		Y_{i1}	P_{i1}		Y_{n1}	P_{n1}
Y_{12}	P_{12}	Y_{22}	P_{22}		Y_{i2}	P_{i2}		Y_{n2}	P_{n2}
.	.	.	.		.	.		.	.
.	.	.	.		.	.		.	.
Y_{1N1}	P_{1N1}	Y_{2N2}	P_{2N2}		Y_{iNi}	P_{iNi}		Y_{nNn}	P_{nN1}
	τ_1		τ_2			τ_i			τ_n

Where $= \sum_{j\in G_i} P_{ij}$, $i = 1, 2, \dots, n$. Denotes the sum of selection probability of the i^{th} random group.

Unbiased estimator of Rao Hartley Cochran estimator

The unbiased estimator of population total Y is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{y_{il}}{(P_{il}/\tau_i)} \qquad 30$$

Variance of Rao Hartley Cochran estimator

The variance of the Rao Hartley Cochran estimator, $V(\hat{Y}_{RHC})$ is given by

$$V(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{N(N-1)}\left[\sum_{j=1}^{N} \frac{Y_j^2}{P_j} - Y^2\right]. \qquad 31$$

$$= \sum_{i=1}^{n}\left[\frac{N_i}{N}\sum_{j=1}^{N_i} Y_j^2 + \frac{N_i(N_i-1)}{N(N-1)} \sum_{j\neq l=1}^{N_i} \frac{P_l Y_j^2}{P_j} - E_1(Y_i^2)\right] \qquad 32$$

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Therefore (3.7.4) implies that

$$E_1\left[V_2\left(\sum_{i=1}^{n}\frac{y_{i1}}{(P_{i1}/\tau_i)} \mid G_i\right)\right]$$

$$= V(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{N(N-1)}\left[\sum_{j=1}^{N}\frac{Y_j^2}{P_j} - Y^2\right] \qquad 33$$

Unbiased estimator of the Variance of Rao Hartley Cochran estimator

An unbiased estimator of the variance $V(\hat{Y}_{RHC})$ of Rao Hartley Cochran is given by

$$\hat{v}(\hat{Y}_{RHC}) = \frac{\left(\sum_{i=1}^{n} N_i^2 - N\right)}{\left(N^2 - \sum_{i=1}^{n} N_i^2\right)}\left[\sum_{i=1}^{n}\frac{y_{i1}^2}{(P_{i1}^2/\tau_i)} - \hat{Y}_{RHC}^2\right]. \qquad 34$$

Therefore

$$\hat{v}(\hat{Y}_{RHC}) = \frac{\left(\sum_{i=1}^{n} N_i^2 - N\right)}{N(N-1)}\left[\sum_{i=1}^{n}\frac{y_{i1}^2}{(P_{i1}^2/\tau_i)} - \left\{\left(\sum_{i=1}^{n}\frac{y_{i1}}{(P_{i1}/\tau_i)}\right)^2 - \hat{V}(\hat{Y}_{RHC}^2)\right\}\right]. \qquad 35$$

Or

$$\hat{v}(\hat{Y}_{RHC})\left[1 - \frac{(\sum_{i=1}^{n} N_i^2 - N)}{N(N-1)}\right] = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{N(N-1)}\left[\sum_{i=1}^{n}\frac{y_{i1}^2}{(P_{i1}^2/\tau_i)} - \hat{Y}_{RHC}^2\right]$$

Superiority of Probability Proportional to Size without replacement over Probability Proportional to size with replacement

The Rao Hartley Cochran (RHC) scheme is more efficient than PPSWR sampling if

$$N_i = \frac{N}{n}, \forall\, i = 1, 2, \ldots, n.$$

Without loss of generality, we have

$$V(\hat{Y}_{PPSWR}) = \frac{1}{n}\left[\sum_{j=1}^{N_i}\frac{Y_j^2}{P_j} - Y^2\right], \text{and } V(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} {N_i}^2 - N)}{N(N-1)}\left[\sum_{j=1}^{N_i}\frac{Y_j^2}{P_j} - Y^2\right]$$

Combining these results, we have

$$V(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} {N_i}^2 - N)}{N(N-1)} V(\hat{Y}_{PPSWR}). \qquad 36$$

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On substituting the optimum value of N_i in (3.8.1) we have

$$V(\hat{Y}_{RHC}) = \left[\frac{(\sum_{i=1}^{n} N_i{}^2 - N)}{N(N-1)}\right]\left[nV(\hat{Y}_{PPSWR})\right]$$

$$= \frac{(N-n)}{N-1} V(\hat{Y}_{PPSWR}) \qquad 37$$

Note that $\frac{(N-n)}{N-1} < 1 \, \forall \, n \geq 2,$

Therefore,

$$V(\hat{Y}_{RHC}) < V(\hat{Y}_{PPSWR}).$$

RESULTS

This research presents the findings of four sampling scheme estimators—Hansen Hurwitz, Horvitz-Thompson, Rao-Hartley-Cochran, and Sen Yates Grundy—with regard to population total and variations in live births for the three-year period 2022–2024. Thus, the outcomes are shown below.

Probability Proportional to Size Sampling Scheme

Range and Selection of Samples among the LGAs with cumulative total

Table 4.1.1 Range and Selection of Samples among the LGAs

S/N	LGA	No of Reg. Centre	Cum Total	Ranges	Total Birth (2022)	Total Birth (2023)	Total Birth (2024)	Total Double Birth (2022)	Total Double Birth (2023)	Total Double Birth (2024)
1	Ado*	11	11	1 – 11	10419	9553	10800	331	286	410
2	Efon	4	15	12 – 15	1470	1689	1532	23	30	24
3	Ekiti East*	11	26	16 – 26	2086	3618	1918	77	116	72
4	Ekiti South West	4	30	27 – 30	1112	1360	1106	37	70	46
5	Ekiti West	5	35	31 – 35	1461	1644	1422	41	64	30
6	Emure	4	39	36 – 39	902	1261	849	29	46	34
7	Gbonyin*	6	45	40 – 45	1710	1917	1611	36	59	36
8	Ido Osi	4	49	46 – 49	2190	2173	1730	50	60	56
9	Ijero	6	55	50 – 55	2244	2342	2602	80	133	96
10	Ikere*	5	60	56 – 60	1866	1994	1953	62	128	14
11	Ikole	7	67	61 – 67	2106	2202	1783	49	60	16
12	Ilejemeje	3	70	68 – 70	786	789	814	32	35	15
13	Irepodun/ Ifelodun	6	76	71 – 76	2348	2430	1897	55	64	70
14	Ise Orun*	4	80	77 – 80	1601	1553	1145	34	36	22
15	Moba	6	86	81 – 86	1620	1692	1500	59	108	63
16	Oye	4	90	87 – 90	1753	1677	1610	44	50	41

Note: 5 LGAs were selected based on random sampling with replacement

Source: National Population Commission, Ado Ekiti

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Estimation of Total Population

An unbiased estimator of the population total under probability proportional to size with replacement, Y is given by

$$\hat{Y}_{PPS} = \frac{1}{n}\sum_{i=1}^{N} {y_i}/{p_i}$$

Table 4.1.2 Probabilities and Yearly data of child's birth for the sampled LGAs

LGAs	Ado	Ekiti East	Gbonyin	Ikere	Ise Orun
p_i	${11}/{90}$= 0.1222	${11}/{90}$ = 0.1222	${6}/{90}$ = 0.0667	${5}/{90}$ = 0.0555	${4}/{90}$= 0.0444
$y_{i(2022)}$	331	77	36	62	34
$y_{i(2023)}$	286	116	59	128	36
$y_{i(2024)}$	410	72	36	14	22

Total Population Estimate for 2022

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{331}{11} + \frac{77}{11} + \frac{36}{6} + \frac{62}{5} + \frac{34}{4}\right]$$

$$\hat{Y}_{PPS} = 18(30.09 + 7 + 6 + 12.4 + 8.5)$$

$$\hat{Y}_{PPS} = 18(63.99) = 1151.82 \sim 1152\ \textit{double births}$$

Total Population Estimate for 2023

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{286}{11} + \frac{116}{11} + \frac{59}{6} + \frac{128}{5} + \frac{36}{4}\right]$$

$$\hat{Y}_{PPS} = 18(26 + 10.55 + 9.83 + 25.6 + 9)$$

$$\hat{Y}_{PPS} = 18(80.98) = 1457.64\ \sim\ 1458\ \textit{double births}$$

Total Population Estimate for 2024

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{410}{11} + \frac{72}{11} + \frac{36}{6} + \frac{14}{5} + \frac{22}{4}\right]$$

$$\hat{Y}_{PPS} = 18(37.27 + 6.55 + 6 + 2.8 + 5.5)$$

$$\hat{Y}_{PPS} = 18(58.12) = 1046.16 \sim\ 1046\ \textit{double births}$$

Estimation of Variance

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left({y_i}/{p_i} - \hat{Y}_{pps}\right)^2$$

Variance Estimate for 2022

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[\left(\frac{331}{0.1222}\right)^2 + \left(\frac{77}{0.1222}\right)^2 + \left(\frac{36}{0.0667}\right)^2 + \left(\frac{62}{0.0555}\right)^2 + \left(\frac{34}{0.0444}\right)^2 - 5(1151.82)^2\right]$$

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Standard Error (SE) is

$$SE\left(\hat{Y}_{pps}\right) = \sqrt{161308.54}$$

$$SE\left(\hat{Y}_{pps}\right) = 401.63$$

Variance Estimate for 2023

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{5(4)}\left[\left(\frac{286}{0.1222}\right)^2 + \left(\frac{116}{0.1222}\right)^2 + \left(\frac{59}{0.0667}\right)^2 + \left(\frac{128}{0.0555}\right)^2 + \left(\frac{36}{0.0444}\right)^2 - 5(1457.64)^2\right]$$

Standard Error (SE) is

$$SE\left(\hat{Y}_{pps}\right) = \sqrt{125701.32}$$

$$SE\left(\hat{Y}_{pps}\right) = 354.54$$

Variance Estimate for 2024

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{5(4)}\left[\left(\frac{410}{0.1222}\right)^2 + \left(\frac{72}{0.1222}\right)^2 + \left(\frac{36}{0.0667}\right)^2 + \left(\frac{14}{0.0555}\right)^2 + \left(\frac{22}{0.0444}\right)^2 - 5(1046.16)^2\right]$$

Standard Error (SE) is

$$SE\left(\hat{Y}_{pps}\right) = \sqrt{336621.23}$$

$$SE\left(\hat{Y}_{pps}\right) = 580.19$$

Horvitz-Thompson Sampling Scheme estimator

The first number, which is random number 10 falls within the range of 1 – 11 which is for Ado, the second number 28 lies in range 27 – 30 which stand for Ekiti South West, the third number, 46 falls within 46 – 49 which is for Ido/Osi, 64 number falls in the range of 61 – 67 is for Ikole and lastly, 82 falls within 81 – 86 which is for Moba. Hence, the five (5) selected Local Government Areas under Horvitz-Thompson sampling scheme are Ado, Ekiti South West, Ido/Osi, Ikole and Moba.

Probability of Inclusion

The numbers of Registration Centre under each of the selected Local Government Area are 11, 4, 4, 7, and 6, based on the presentation in table 4.1.2, as well as k which is 18, the probability for each of the selected Local Government Area is given below;

Table 4.2.1: Probability of the selected samples among all the Local Government Areas.

Registration Centre	Ado	Ekiti South West	Ido/Osi	Ikole	Moba
$\boldsymbol{\pi_i}$	$^{11}/_{18}$	$^{4}/_{18}$	$^{4}/_{18}$	$^{7}/_{18}$	$^{6}/_{18}$
Range	1 – 11	27 – 30	46 – 49	61 – 67	81 – 86
Samples of 5	18	36	54	72	90

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Estimation of Total Population by Horvitz-Thompson Sampling Scheme

An unbiased estimator of the population total for PPSWOR sampling as given by Horvitz-Thompson is

$$\hat{Y}_{HT} = \sum_{i=1}^{N} {y_i}/{\pi_i}$$

Table 4.2.2: Probability of the selected samples among all the LGAs with annual enrolment data

Sample LGA	Nos. of Reg. Centres	Serial no	π_i	Total Double Birth (2022)	Total Double Birth (2023)	Total Double Birth (2024)
Ado	11	1	${11}/{18} = 0.6111$	331	286	410
Ekiti South West	4	2	${4}/{18} = 0.2222$	37	70	46
Ido/Osi	4	3	${4}/{18} = 0.2222$	50	60	56
Ikole	7	4	${7}/{18} = 0.3889$	49	60	16
Moba	6	5	${6}/{18} = 0.3333$	59	108	63

Based on the Horvitz-Thompson sampling scheme criteria, Table 4.2.2 reveals the probability of selecting each of the Local Government which are sampled, Ado, Ekiti South West, Ido/Osi, Ikole and Moba based on the number 10 selected from the random. In the Table 4.2.2 probabilities stood at ${11}/{18}$, ${4}/{18}$, ${4}/{18}$, ${7}/{18}$ and ${6}/{18}$ for Ado, Ekiti South West, Ido/Osi, Ikole and Moba respectively, in regards to k and the number of units (Registration Centres) in each of the LGA.

Estimation of Variance by Horvitz-Thompson Sampling Scheme

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\, y_i^2\right) + 2\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\,\frac{y_i}{\pi_i}\frac{y_j}{\pi_j}$$

Rao-Hartley-Cochran's (RHC) Sampling Scheme

Table 4.3.1: Total Registered Live Births by Type of Birth by Registration Centre Across All LGA in Ekiti State for 2022, 2023 & 2024

S/N	LGA	No of Registration Centres	Total Double Birth (2022)	Total Double Birth (2023)	Total Double Birth (2024)
1	Ado	11	331	286	410
2	Efon	4	23	30	24
3	Ekiti East	11	77	116	72
4	Ekiti South West	4	37	70	46
5	Ekiti West	5	41	64	30
6	Emure	4	29	46	34
7	Gbonyin	6	36	59	36
8	Ido Osi	4	50	60	56
9	Ijero	6	80	133	96

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10	Ikere	5	62	128	14
11	Ikole	7	49	60	16
12	Ilejemeje	3	32	35	15
13	Irepodun/ Ifelodun	6	55	64	70
14	Ise Orun	4	34	36	22
15	Moba	6	59	108	63
16	Oye	4	44	50	41
	Total		1039	1345	1045

Following the numbers of Local Government Areas in the population collected above, we are to select a sample of size 4 by using RHC scheme. Thus, the population (LGAs) will be divided into four (4) random groups. To do this, we selected 16 distinct random numbers between 1 and 16 from 36th to 43rd rows

This random numbers came in the sequence; 10, 11, 01, 02, 08, 16, 05, 12, 09, 13, 04, 14, 06, 15, 03, 07.

The LGAs bearing the serial numbers corresponding to the first four selected random numbers constitute the first random group, whereas, the next four random numbers form the second random group and so on.

Let $X_{ij} = Numbers\ of\ Registration\ centres\ \ for\ the\ jth\ LGA\ in\ the\ ith\ random\ group$

$$Y_{ij} = Registration\ of\ double\ birthfor\ the\ jth\ LGA\ in\ the\ ith\ random\ group$$

$$P_{ij} = The\ initial\ selection\ probability\ of\ the\ jth\ unit\ in\ the\ ith\ random\ group$$

$$\sum Y_{ij\ (2022)} = 1039, \sum Y_{ij\ (2023)} = 1345, \sum Y_{ij\ (2024)} = 1045, \sum X_{ij} = 90$$

Therefore, the following are the 4 random groups of units along with the initial selection of probability.

S/N	**RANDOM NO.**	**LGA**	X_{ij}	$Y_{ij(2022)}$	$Y_{ij(2023)}$	$Y_{ij(2024)}$	P_{ij}
1	10	Ikere	5	62	128	14	0.0556
2	11	Ikole	7	49	60	16	0.0778
3	01	Ado	11	331	286	410	0.1222
4	02	Efon	4	23	30	24	0.0444
		Total	27	**465**	**504**	**464**	**0.3000**

Table 4.3.2: first random group by RHC selection

S/N	**Random No.**	**LGA**	X_{ij}	$Y_{ij(2022)}$	$Y_{ij(2023)}$	$Y_{ij(2024)}$	P_{ij}
1	08	Ido/Osi	4	50	60	56	0.0444
2	16	Oye	4	44	50	41	0.0444
3	05	Ekiti West	5	41	64	30	0.0556
4	12	Ilejemeje	3	32	35	15	0.0333
		Total	16	**167**	**209**	**142**	**0.1778**

Table 4.3.3: second random group by RHC selection

S/N	**Random No.**	**LGA**	X_{ij}	$Y_{ij(2022)}$	$Y_{ij(2023)}$	$Y_{ij(2024)}$	P_{ij}
1	09	Ijero	6	80	133	96	0.0667

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2	13	Irepodun/ Ifelodun	6	55	64	70	0.0667
3	04	Ekiti South West	4	37	70	46	0.0444
4	14	Ise/Orun	4	34	36	22	0.0444
		Total	20	**206**	**303**	**234**	**0.2222**

Table 4.3.4: third random group by RHC selection

S/N	**Random No.**	**LGA**	X_{ij}	$Y_{ij(2022)}$	$Y_{ij(2023)}$	$Y_{ij(2024)}$	P_{ij}
1	06	Emure	4	29	46	34	0.0444
2	15	Moba	6	59	108	63	0.0667
3	03	Ekiti East	11	77	116	72	0.1222
4	07	Gbonyin	6	36	59	36	0.0667
		Total	27	**201**	**329**	**205**	**0.3000**

Table 4.3.5: fourth random group by RHC selection

The next thing is to select one unit independently in each group using a method of selection of sample (we make use of Lahiri's method).

The first random group consists of $N_1 = 4$ units and maximum value of the variable $X_{1j} = 11$. Choosing $X_0 = 15$, we select random number $1 \leq R_i \leq 4$ by starting with 4th and 5th columns and another random number $1 \leq R_j \leq 15$ by starting from 23rd and 24th columns of the Random Number Table. Then the first effective pair of random number is (04, 05) as shown in the table of effective pairs as shown in the table below. Thus, from the first random group, **Ikere LGA** will be included in the sample.

Trial No.	Group S/N ($1 \leq R_i \leq N$)	LGA	$1 \leq R_j \leq N$	X_{1j}	Decision R = Rejected S = Selected
1	04	Efon	05	4	R
2	03	Ado	14	11	R
3	02	Ikole	12	7	R
4	01	Ikere	04	5	S

Table 4.3.6: selected LGA for group 1 by RHC selection

The second random group consists of $N_2 = 4$ units and maximum value of the variable $X_{2j} = 5$. Choosing $X_0 = 10$, we select random number $1 \leq R_i \leq 4$ by starting with 13th and 14th columns and another random number $1 \leq R_j \leq 10$ by starting from 22nd to 24th columns of the Random Number Table. Then the first effective pair of random number is (02, 04) as shown in the table of effective pairs as shown in the table below. Thus, from the first random group, **Oye LGA** will be included in the sample.

Trial No.	Group S/N ($1 \leq R_i \leq N$)	LGA	$1 \leq R_j \leq N$	X_{2j}	Decision R = Rejected S = Selected
1	01	Ido/Osi	09	4	R
2	04	Ilejemeje	07	3	R
3	02	Oye	02	4	S
4	03	Ekiti West	06	5	R

Table 4.3.7: selected LGA for group 2 by RHC selection

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The third random group consists of $N_3 = 4$ units and maximum value of the variable $X_{3j} = 6$. Choosing $X_0 = 10$, we select random number $1 \leq R_i \leq 4$ by starting with 19th and 20th columns and another random number $1 \leq R_j \leq 10$ by starting from 15th and 16th columns of the Random Number Table. Then the first effective pair of random number is (03, 01) as shown in the table of effective pairs as shown in the table below. Thus, from the first random group, **Ekiti South West LGA** will be included in the sample.

Trial No.	Group S/N ($1 \leq R_i \leq N$)	LGA	$1 \leq R_j \leq N$	X_{3j}	Decision R = Rejected S = Selected
1	03	Ekiti South West	01	4	S
2	02	Irepodun/ Ifelodun	08	6	R
3	01	Ijero	10	6	R
4	04	Ise/ Orun	06	4	R

Table 4.3.8: selected LGA for group 3 by RHC selection

The fourth random group consists of $N_4 = 4$ units and maximum value of the variable $X_{4j} = 11$. Choosing $X_0 = 15$, we select random number $1 \leq R_i \leq 4$ by starting with 41st and 42nd columns and another random number $1 \leq R_j \leq 15$ by starting from 31st to 32nd columns of the Random Number Table. Then the first effective pair of random number is (03, 11) as shown in the table of effective pairs as shown in the table below. Thus, from the first random group, **Ekiti East LGA** will be included in the sample

Trial No.	Group S/N ($1 \leq R_i \leq N$)	LGA	$1 \leq R_j \leq N$	X_{4j}	Decision R = Rejected S = Selected
1	03	Ekiti East	07	11	S
2	02	Moba	04	6	S
3	04	Gbonyin	14	6	R
4	01	Emure	10	4	R

Table 4.3.9: selected LGA for group 4 by RHC selection

Result presented in *Table 4.3.6 – Table 4.3.9* reveals that there are four random groups for the Local Government Areas covered in the study. For the first random group, there are Ikere, Ikole, Ado and Efon. In the second random group we have Ido/Osi, Oye, Ekiti West and Ilejemeje.

The third random group consist of Ijero, Irepodun/Ifelodun, Ekiti South West, Ise/Orun while the last random group is made up of Emure, Moba, Ekiti East and Gbonyin. In line with selection of one LGA from each of the groups, based on random sampling and Lahiri's selection method, the LGAs selected were Ikere, Oye, Ekiti South West and Ekiti East for first, second, third and fourth group respectively.

Probability of Selection under Rao-Hartley-Cochran's estimator

Table 4.3.10: Sampled LGAs and Selection Probabilities for double birth in 2022

S/N	LGA	Y_{i1}	X_{i1}	P_{i1}	τ_i	$\frac{P_{i1}}{\tau_i}$	$\frac{Y_{i1}}{(P_{i1}/\tau_i)}$	${Y_{i1}}^2$	${P_{i1}}^2$	$\frac{{P_{i1}}^2}{\tau_i}$	$\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$
1	Ikere	5	62	0.0556	0.3000	0.1852	334.80	3844	0.0031	0.0103	373636.80
2	Oye	4	44	0.0444	0.1778	0.2500	176.02	1936	0.0020	0.0111	174261.78

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3	Ekiti South West	4	37	0.0444	0.2222	0.2000	184.98	1369	0.0020	0.0089	153997.10
4	Ekiti East	11	77	0.1222	0.3000	0.4074	189.00	5929	0.0149	0.0498	119070.00
	Total						**884.80**				**820965.68**

Note that

$$\tau_i = \sum_{j \in G_i} P_{ij}\ ,\ i = 1,2,3,4$$

Population Total for RHC is given as

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{Y_{i1}}{(P_{i1}/\tau_i)}$$

The Total population estimate of double births for the year 2022 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{Y_{i1}}{(P_{i1}/\tau_i)} = \mathbf{884.80}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)} \left[\sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2 \right]$$

$$= 12696.15$$

Table 4.3.11: Sampled LGAs and Selection Probabilities for double birth in 2023

S/N	LGA	Y_{i1}	X_{i1}	P_{i1}	τ_i	$\frac{P_{i1}}{\tau_i}$	$\frac{Y_{i1}}{(P_{i1}/\tau_i)}$	${Y_{i1}}^2$	${P_{i1}}^2$	$\frac{{P_{i1}}^2}{\tau_i}$	$\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$
1	Ikere	5	128	0.0556	0.3000	0.1852	691.20	16384	0.0031	0.01028807	1592524.8
2	Oye	4	50	0.0444	0.1778	0.2500	200.03	2500	0.0020	0.01110972	225028.125
3	Ekiti South West	4	70	0.0444	0.2222	0.2000	349.97	4900	0.0020	0.00888978	551194.875
4	Ekiti East	11	116	0.1222	0.3000	0.4074	284.73	13456	0.0149	0.04979424	270232.0661
	Total						**1525.92**				**2638979.87**

Note that

$$\boldsymbol{\tau_i = \sum_{j \in G_i} P_{ij}\ , \qquad i = 1, 2, 3, 4}$$

The Total population estimate of double births for the year 2023 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{Y_{i1}}{(P_{i1}/\tau_i)} = \mathbf{1525.92}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)}\left[\sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2\right]$$

$$= 103518.78$$

Table 4.3.12: Sampled LGAs and Selection Probabilities for double birth in 2024

S/N	LGA	Y_{i1}	X_{i1}	P_{i1}	τ_i	$\frac{P_{i1}}{\tau_i}$	$\frac{Y_{i1}}{(P_{i1}/\tau_i)}$	${Y_{i1}}^2$	${P_{i1}}^2$	$\frac{{P_{i1}}^2}{\tau_i}$	$\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$
1	Ikere	5	14	0.0556	0.3000	0.1852	75.60	196	0.0031	0.01028807	19051.2
2	Oye	4	41	0.0444	0.1778	0.2500	164.02	1681	0.0020	0.01110972	151308.9113
3	Ekiti South West	4	46	0.0444	0.2222	0.2000	229.98	2116	0.0020	0.00888978	238026.195
4	Ekiti East	11	72	0.1222	0.3000	0.4074	176.73	5184	0.0149	0.04979424	104108.4298
	Total						**646.32**				**512494.74**

Note that

$$\boldsymbol{\tau_i = \sum_{j \in G_i} P_{ij}\ , \qquad i = 1, 2, 3, 4}$$

The Total population estimate of double births for the year 2024 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{Y_{i1}}{(P_{i1}/\tau_i)} = \mathbf{646.32}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)}\left[\sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2\right]$$

$$= 31586.34$$

Sen – Yates – Grundy Sampling Scheme

Estimation of Variance by Sen – Yates – Grundy Estimator

Sen – Yates – Grundy form of the variance of the Horvitz and Thompson estimator of the population total Y is given by

$$\hat{V}_{(SYG)}(\hat{Y}_{HT}) = \frac{1}{2}\sum_{i \in S}\sum_{j(\neq i) \in S} (\pi_i\pi_j - \pi_{ij})\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2$$

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Comparison of Estimated Population and Variance for the four PPS estimators

Table 4.5.1: Comparison of Estimated Population Total for HH, HT and RHC

	Estimated Population Total		
YEAR	HH	HT	RHC
2022	1152	1236	885
2023	1458	1531	1526
2024	1046	1360	646

Note: HH= Hansen Hurwitz; HT= Horvitz-Thompson; RHC= Rao-Hartley-Cochran's

Bar Chart Visualizations for PPSWR and PPSWOR Estimates (2022–2024)

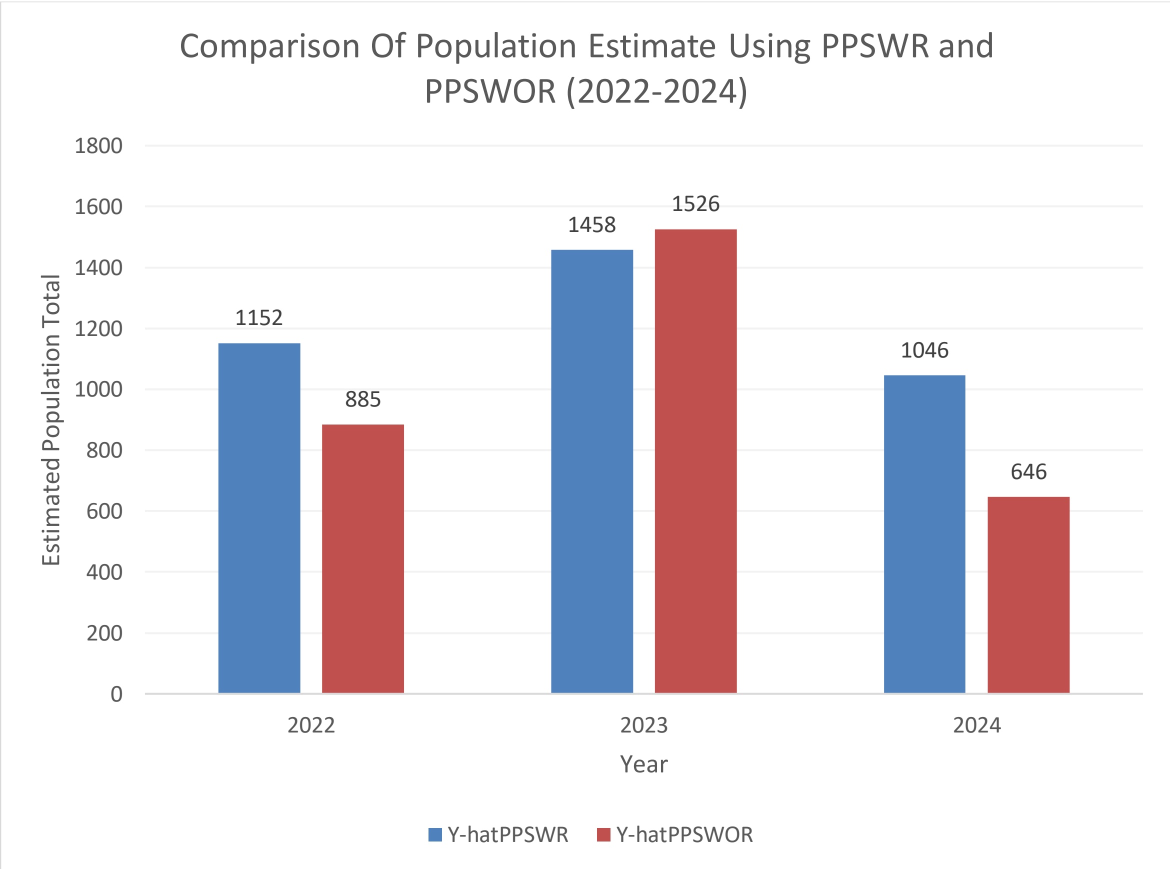

Figure 4.5.1: Estimated Population Total for PPS, HT and RHC

Table 4.5.1 and Figure 4.5.1 present the estimated population total of double child birth enrollments in all LGAs in Ekiti State under the Rao-Hartley-Cochran's sampling scheme, the Horvitz-Thompson sampling scheme, and the probability proportional to size sampling scheme (Hansen Hurwitz) for the years 2022, 2023, and 2024. The results show that the Horvitz-Thompson sampling scheme without replacement has the highest estimated population total of double child births enrolled in Ekiti State over the period covered, the Hansen Hurwitz sampling scheme with replacement has the lowest estimated population total for 2023, and the Rao-Hartley-Cochran sampling scheme without replacement has the lowest estimated population total for 2022 and 2024.

Table 4.5.2: Comparison of Estimated Variance for HH, HT, RHC and SYG

	Estimated Variance			
YEAR	HH	HT	RHC	SYG
2022	161308.54	391184.02	12696.15	40356.96

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2023	125701.32	791623.40	103518.78	16102.79
2024	336621.23	431017.39	31586.34	79283.25

Comparison of Variance Estimates Using PPSWR and PPSWOR (2022–2024)

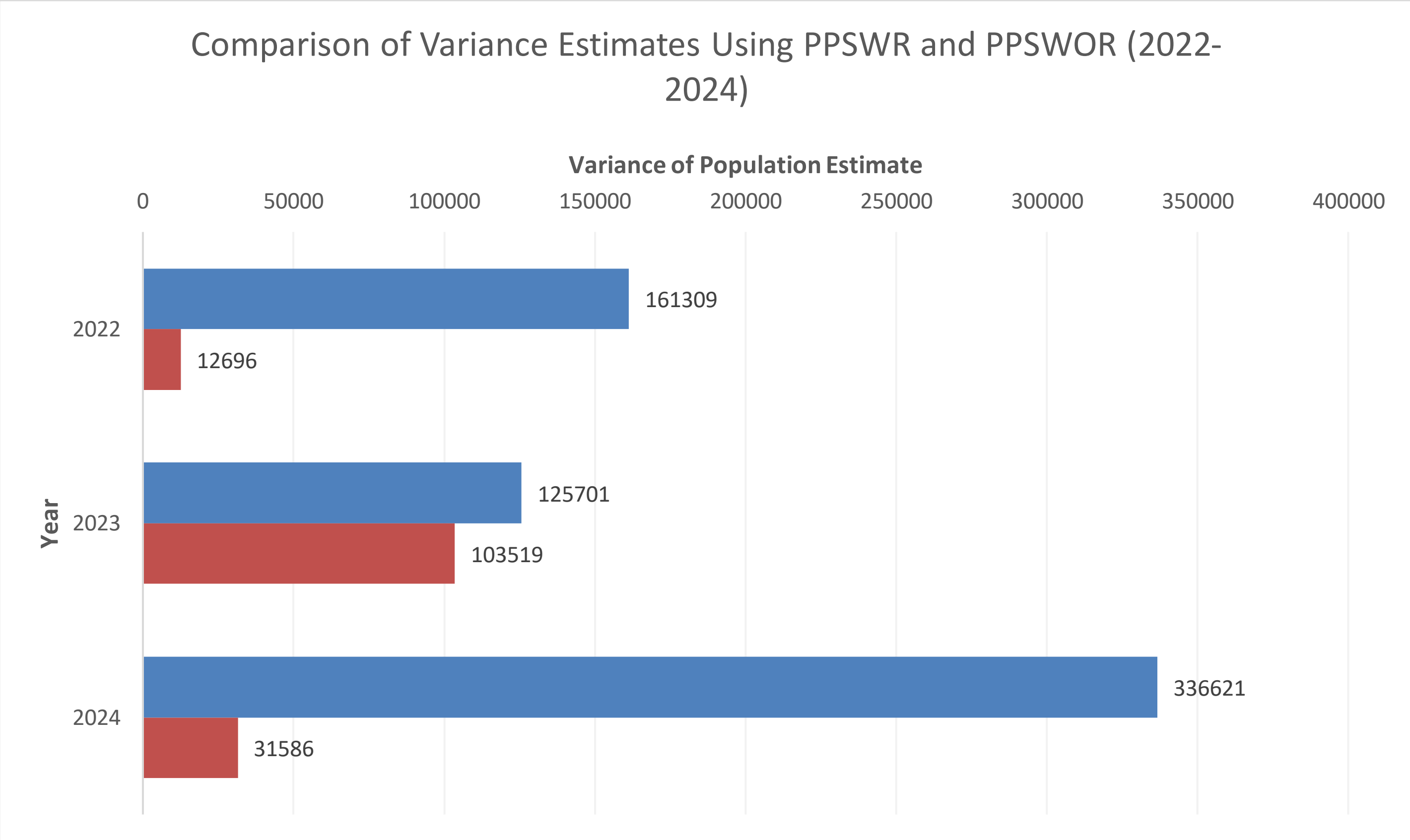

Figure 4.5.2: Estimated Variance for HH, HT, RHC and SYG

Estimated variance of double child birth enrollment in all LGAs in Ekiti State under probability proportional to size sampling schemes with replacement (Hansen Hurwitz) and probability proportional to size sampling schemes without replacement (Horvitz-Thompson, Rao-Hartley-Cochran, and Sen – Yates – Grundy) for 2022, 2023, and 2024 is shown in Table 4.5.2 and Figure 4.5.2. It turns out that Sen-Yates-Grundy has the lowest projected variance for 2023, while Rao-Hartley-Cochran's has the lowest estimated variance for 2022 and 2024. Additionally, it was discovered that throughout the entire time under review, Horvitz-Thompson had the biggest variance in double childbirth enrollment among all Ekiti State LGAs.

CONCLUSION

In order to estimate the population total of double child birth registrations across the Local Government Areas of Ekiti State, this study conducted a comparative investigation of the efficiency of sampling scheme estimators. Hansen-Hurwitz's probability proportional to size with replacement and Horvitz-Thompson, Rao-Hartley-Cochran, and Sen-Yates-Grundy's probability proportional to size without replacement estimators were the main subjects of the analysis.

The results showed that in 2022 and 2024, the Rao–Hartley–Cochran estimator consistently yielded the lowest estimated population totals and achieved the least variance in those years. However, by producing the least variance in 2023, the Sen–Yates–Grundy estimate demonstrated greater efficiency. These findings imply that efficiency differed by year and the underlying data distribution, rather than any one estimator being consistently better over all time periods.

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The study concludes that, in terms of variance reduction, probability proportional to size sampling without replacement estimators—specifically, Rao–Hartley–Cochran and Sen–Yates–Grundy—generally performs better than the conventional Hansen–Hurwitz estimator. Because it has a substantial impact on the precision and dependability of population total estimates, this conclusion emphasizes the significance of carefully choosing an estimator based on data feature

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