A Mathematical Model for the Dynamics of Banditry and Government Intervention in Kaduna State, Nigeria.

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ABSTRACT

This study developed and analyzes a mathematical model to examine the effect of government policies on the containment of banditry in Kaduna State. Building on the existing model, the model extends existing approaches by incorporating additional compartments such as rehabilitation individuals, security agents and bandit sponsors to better capture real-world dynamics of banditry and intervention mechanisms. A system of nonlinear ordinary differential equations is formulated to describe the interactions among population groups and qualitative analyses including existence and uniqueness of solutions, positivity, invariant region, and banditry-free equilibrium are carried out to ensure the mathematical validity and stability of the model. The analysis reveals that the solutions remain non-negative and bounded within a biological feasible region, confirming the consistency and stability of the system. In addition, the findings show that integrated government policies involving effective security enforcement, rehabilitation programs, rehabilitation programs, intelligence gathering, and disruption of sponsor networks significantly reduce he growth and persistence of banditry. The study highlights the importance of coordinated and sustained intervention strategies in addressing insecurity in Kaduna State and demonstrates the usefulness of mathematical modelling as a predictive and policy-evaluation tool for combating banditry and related criminal activities in Nigeria. and policy-evaluation tool for combating banditry and related criminal activities in Nigeria.

Keywords: Banditry, Mathematical modelling, Government policies, Kaduna State, Nonlinear differential equations, Security intervention, Rehabilitation, Optimal control, Banditry dynamics

INTRODUCTION

Banditry in Kaduna State has become a major security concern, particularly in rural and semi-urban areas where armed groups are involved in kidnapping, cattle rustling, and violent attacks on communities. The crisis is especially pronounced in local government areas such as Birnin Gwari, Igabi, Giwa, and Chikun, resulting in loss of lives, displacement of residents, and disruption of farming and commercial activities. Studies attribute the persistence of banditry in the state to underlying factors such as poverty, unemployment, weak governance, and limited security presence, which create opportunities for criminal networks to flourish (Abdullahi, 2019; International Crisis Group, 2020). Despite efforts by the government through military operations, dialogue, and community-based strategies, the situation remains largely unresolved, highlighting the need for more effective and sustainable interventions (Amao, 2020; Sale & Abubakar, 2025).

Banditry has become a major security threat in Northern Nigeria, significantly undermining socio-economic development and national stability. It encompasses criminal activities such as kidnapping, cattle rustling, and violent attacks, particularly in states like Zamfara, Kaduna, Katsina, and Sokoto. Scholars attribute its rise to structural challenges including poverty, unemployment, weak governance, and environmental pressures like desertification, which intensify competition over limited resources (Akinwale, 2019). The consequences have been severe, leading to widespread displacement, disruption of agricultural livelihoods, and a decline in economic productivity. In response, the government has implemented measures such as military campaigns, peace agreements, amnesty programs, and community policing initiatives, including operations like Hadarin Daji. However, while these interventions have recorded short-term gains, they often fail to address underlying causes, resulting in the persistence of banditry (International Crisis Group, 2020; Amao, 2020).

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More recent strategies have shifted toward non-kinetic approaches, including dialogue, socio-economic reforms, and arms control measures. Policies such as negotiations and amnesty programs have yielded mixed outcomes, with temporary peace in some areas and renewed violence in others. This highlights the complex and dynamic nature of banditry, necessitating more systematic and predictive approaches to policy evaluation. Mathematical modelling has emerged as a useful tool in conflict and policy analysis, allowing researchers to simulate scenarios and assess the potential impact of interventions (Abdullahi & Mukhtar, 2022; Accord, 2022). Despite this, existing studies on insecurity in Nigeria remain largely qualitative, lacking robust predictive frameworks. Consequently, there is a need to integrate mathematical modelling with policy analysis to better understand and manage banditry, providing evidence-based insights for decision-makers (Brigid et al., 2022; Globalsecurity, 2023).

Parallel to banditry, insurgency particularly driven by Boko Haram has evolved into a prolonged and multifaceted crisis in Northern Nigeria. Initially founded as a religious movement, the group became a violent insurgent organization after the death of its leader, Mohammed Yusuf in 2009, engaging in bombings, abductions, and attacks across states such as Borno, Yobe, and Adamawa (Tahir & Bernard, 2021; Rufa'I, 2021). A notable incident was the 2014 Chibok schoolgirls kidnapping, which drew global attention to the region's insecurity (BBC News, 2014; Amnesty International, 2015). The emergence of Islamic State West Africa Province further complicated the security landscape, intensifying violence despite a more strategic focus on military targets (International Crisis Group, 2019; Zenn, 2020). Although government responses, including collaboration through the Multinational Joint Task Force, have achieved some progress, persistent challenges such as poverty, weak governance, and resource constraints continue to sustain the crisis (UNDP, 2017).

The mathematical model by Lawal et al. (2023) provides a useful framework for understanding banditry dynamics through five key compartments and the inclusion of control strategies such as job creation and reducing the profitability of banditry. However, the model has limitations as it does not adequately account for important real-world factors such as bandit sponsors, security agents, and rehabilitation processes. Given the increasing complexity of banditry, including organized support systems and reintegration efforts, there is a need for a more comprehensive modelling approach. Therefore, this study seeks to extend the existing model by incorporating additional compartments and policy variables to achieve a more realistic and effective analysis of government interventions in Northern Nigeria.

Bello & Mukhtar present a sociological analysis of kidnapping in Nigeria, linking it to terrorism, poverty, and political instability, and highlighting its connection with insurgent groups such as Boko Haram and Niger Delta militancy (Bello & Mukhtar, 2017). Similarly, Lawal et al. (2023) develop a mathematical framework that conceptualizes banditry as a socio-economic problem driven by poverty, unemployment, weak governance, and illegal mining, aligning with earlier findings (Abdullahi, 2019; Ogbonnaya, 2020). Complementing these perspectives, Gabriel & Nwala provide a qualitative assessment of the broader implications of banditry on national interest, particularly in Northwestern Nigeria (Gabriel & Nwala, 2024).

MATERIALS AND METHOD

In this section, we outline the methodological development of the model were employed. The model developed by Lawal et al. (2023) presented a modeling and optimal control analysis on armed banditry and internal security in Zamfara State. The model will be modified by incorporating some compartmental model due to Lawal et al. (2023).

Development of the Model

The mathematical modeling and optimal control analysis on armed banditry and internal security in Zamfara State developed by Lawal et al. (2023) was formulated. The existing model by Lawal et al. (2023) is divided into five variables, these variables are $S(t)$ stands for Non-informant population, $E(t)$ means Exposed Population, the variable $I(t)$ signifies the Informant Population, the Bandit population indicates $B(t)$ and Removed population refers to $R(t)$.

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The model was modified due to Lawal et al. (2023) by incorporate additional mitigation measures, including individual who undergoing Rehabilitation, Security Agent and Bandit Sponsor.

The variables and parameters of the model is presented in table 2.1 and table 2.2.

Table 2.1: Variables of the Model

Variables	Meaning
$S(t)$	Non-Informant Population
$E(t)$	Exposed Population
$I(t)$	Informant Population
$R_e(t)$	Rehabilitation Individual
$B(t)$	Bandit Population
$B_S(t)$	Bandit Sponsors
$R(t)$	Removed population
$A(t)$	Security Agent

Table 2.2: Parameters of the Model

Parameters	Meaning
Λ	Recruitments
λ_1, λ_2	Force of becoming a bandit
ξ	Proportion of all informers that acquire firearms
$1-\xi$	Remaining proportion that have no firearms
ρ	Movement rate to informants and Bandits population
δ	Movement rate to Informers to Repentant population
σ	Movement rate to Bandit population
μ	Natural death rate
d_1, d_2	Vigilante Penalty Death for being a Bandit
θ	Progression rate of informer to Rehabilitation center
η	death due to torture/life jail in rehabilitation
δ_1	Death rate due to Banditry activities

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α	Progression rate of individuals under rehabilitation back to susceptible
a	Death rate due to Security activity
β	Progression Rate of Bandit to Bandit Sponsor
P	Recruitment rate into security agent
ω	Movement rate of individuals from Bandit Sponsor to rehabilitation
τ	rate of movement of individuals under Bandit to rehabilitation

Assumptions of the model

- The total population is divided into eight mutually exclusive groups (Lawal et al., 2023).
  Individuals in society play different roles in the dynamics of banditry and government intervention. Compartmentalization simplifies the analysis of transition between these roles.

- Susceptible individuals become exposed through interaction with active bandits and criminal networks (Abdullahi, 2019; Ogbonnaya, 220; Mustapha, 2019).
  Banditry recruitment occur through: peer influence, coercion, economic hardship, unemployment, social pressure, and organized criminal networks.

- Individuals do not become active bandits immediately after exposure (Enyinnaya and Olomojobi, 2022). Recruitment into criminal activity usually involves: indoctrination, training, weapons acquisition and gradual participation.

- Bandits or informants undergoing rehabilitation may return to normal societal life instead of rejoining criminal groups (Lawal et al., 2023; Mustapha, 2019).

  Government rehabilitation programs are designed to: deradicalize offenders, provide counselling, improve employability, and promote reintegration.

- Security agents suppress banditry through: military operations, arrests, intelligence gathering, and deterrence (Lawal et al., 2023; Enyinnaya & Olomojobi, 2022). Increased security presence weakens the operational capacity of armed groups.

- Bandit sponsors provide: funding, arms, logistics, intelligence, or political protection (Ogbonnaya, 2020; Brigid et al., 2022). Armed groups require external support systems to sustain prolonged operations.

- The total population remains finite and evolves within a biologically feasible region (Lawal et al., 2023). No real population grows infinitely within a short period.

- Parameters such as: recruitment rates, death rates, rehabilitation rates, and security effectiveness remain constant during simulation (Lawal et al., 2023). Constant parameters simplify mathematical analysis and numerical computations.

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The model diagram of the modified model is presented in figure 3.2

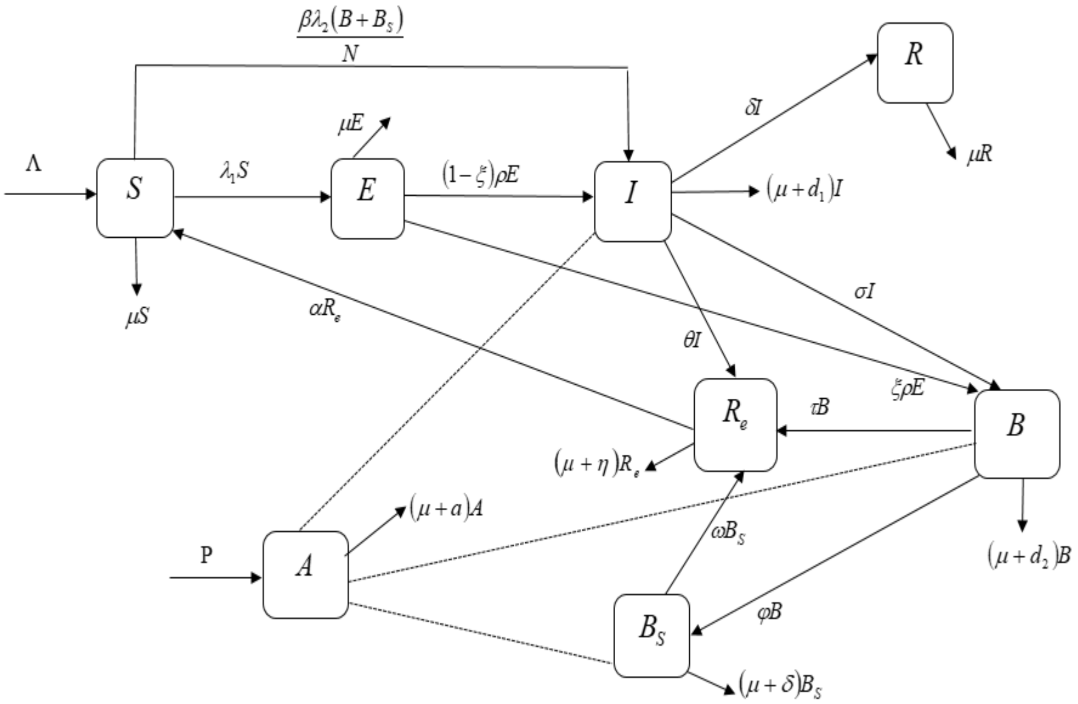

Figure 2.1: Schematic diagram of modified model. The following equation were derived from figure 2.1.

$$
\left.
\begin{aligned}
\frac{dS}{dt} &= \Lambda + \alpha R_e - \frac{\beta\lambda_2(B+B_S)}{N} - (\lambda_1+\mu)S \\
\frac{dE}{dt} &= \lambda_1 S - (\rho - 2\xi\rho + \mu)E \\
\frac{dI}{dt} &= \frac{\beta\lambda_2(B+B_S)}{N} + (1-\xi)\rho E - (\theta+\sigma+\delta+\mu+\delta_1)I \\
\frac{dR_e}{dt} &= \omega B_S + \tau B + \theta I - (\alpha+\mu+\eta)R_e \\
\frac{dB}{dt} &= \xi\rho E + \sigma I - (\tau+\varphi+\mu+d_2)B \\
\frac{dB_S}{dt} &= \varphi B - (\omega+\mu+\delta_1)B_S \\
\frac{dR}{dt} &= \delta I - \mu R \\
\frac{dA}{dt} &= P - (\mu+a)A
\end{aligned}
\right\} \qquad (2.1)
$$

The Mathematical Model Analysis

In this section, the mathematical model analysis of the model (2.1) were analyzed by using relevant theorems and lemmas. This analysis involves various techniques and approaches used to examine and interpret the behavior, characteristics and implications of mathematical or computational models of banditry dynamics. In particular, the discussion focuses on establishing the existence and uniqueness of the solution of the model, positivity of the solution of the model, the banditry-free equilibrium point, feasible region (invariant region) of the model and banditry reproduction number of the model.

Existence and Uniqueness of the Banditry Model

The existence and uniqueness of solutions of the model equation (3.2) are mathematical proved following the theorem according to Abah et al. (2024).

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Theorem 3.1

Consider

$$y_1(t)=S,\ y_2(t)=E,\ y_3(t)=I,\ y_4(t)=R_e,\ y_5(t)=B,\ y_6(t)=B_S,\ y_7(t)=R,\ y_8(t)=A$$

so that the of the bandit model equations (3.2) can be re-written in a complex form as

$$\left.\begin{array}{l}\frac{dy}{dt}=f(t,\ y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6,\ y_7,\ y_8),\ y_1(t_0)=y_{10},\ y_2(t_0)=y_{20}, y_3(t_0)=y_{30},\\ y_4(t_0)=y_{40},\ y_5(t_0)=y_{50},\ y_6(t_0)=y_{60},\ y_7(t_0)=y_{70},\ y_8(t_0)=y_{80}\end{array}\right\} \quad (3.1)$$

Suppose that the function $f(t, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8)$ in the model equation given by system (2.1) satisfies Lipchitz condition in the region $\Delta=\{(t,\ y): 0\le|t-t_0|\le\|y-y_0\|\le\vartheta\}$ for some $a>0,\ \vartheta>0,\ \partial,\ \vartheta\in\Omega$, then, there exist a natural constant number $\vartheta>0$, such that a unique continuous vector solution $y(t)$ of the model equation given by equation (3.1) exists in the interval $|t-t_0|<\vartheta$ (Lawal et al. 2023).

Proof

From the model equation given by equation (2.1) let

$$\left.\begin{array}{l}f_1(t,\ y_1)=\frac{dS}{dt}=\Lambda+\alpha R_e-\frac{\beta\lambda_2(B+B_S)}{N}-(\lambda_1+\mu)S\\ f_1(t,\ y_1)=\frac{dE}{dt}=\lambda_1 S-(\rho-2\xi\rho+\mu)E\\ f_1(t,\ y_1)=\frac{dI}{dt}=\frac{\beta\lambda_2(B+B_S)}{N}+(1-\xi)\rho E-(\theta+\sigma+\delta+\mu+\delta_1)I\\ f_1(t,\ y_1)=\frac{dR_e}{dt}=\omega B_S+\tau B+\theta I-(\alpha+\mu+\eta)R_e\\ f_1(t,\ y_1)=\frac{dB}{dt}=\xi\rho E+\sigma I-(\tau+\varphi+\mu+d_2)B\\ f_1(t,\ y_1)=\frac{dB_S}{dt}=\varphi B-(\omega+\mu+\delta_1)B_S\\ f_1(t,\ y_1)=\frac{dR}{dt}=\delta I-\mu R\\ f_1(t,\ y_1)=\frac{dA}{dt}=P-(\mu+a)A\end{array}\right\} \quad (3.2)$$

According to theorem 3.1, for the functions given by the equation (3.2) to satisfy Lipchitz condition.

To show that $\frac{\partial f_i}{\partial y_j},\ i, j=1, 2, 3, 4, 5, 3, 7, 8$ are continuous and bounded in the region Ω.

Now, consider the partial derivatives of the first equation (3.2)

$$f_1(t,\ y_1)=\frac{dS}{dt}=\Lambda+\alpha R_e-\frac{\beta\lambda_2(B+B_S)}{N}-(\lambda_1+\mu)S$$

$$\left.\begin{array}{l}\left|\frac{\partial f_1}{\partial S}\right|=-(\lambda_1+\mu)<\infty,\ \left|\frac{\partial f_1}{\partial E}\right|=0<\infty,\ \left|\frac{\partial f_1}{\partial I}\right|=0<\infty,\left|\frac{\partial f_1}{\partial R_e}\right|=\alpha<\infty,\\ \left|\frac{\partial f_1}{\partial B}\right|=0<\infty,\left|\frac{\partial f_1}{\partial B_S}\right|=0<\infty,\ \left|\frac{\partial f_1}{\partial R}\right|=0<\infty,\ \left|\frac{\partial f_1}{\partial A}\right|=0<\infty\end{array}\right\} \quad (3.3)$$

Using the partial derivatives of the second equation of (3.2)

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$$f_2(t, y_2) = \frac{dE}{dt} = \lambda_1 S - (\rho - 2\xi\rho + \mu)E$$

$$\left.\begin{aligned}&\left|\frac{\partial f_2}{\partial S}\right| = \lambda_1 < \infty,\ \left|\frac{\partial f_2}{\partial E}\right| = -(\rho - 2\xi\rho + \mu) < \infty,\ \left|\frac{\partial f_2}{\partial I}\right| = 0 < \infty, \left|\frac{\partial f_2}{\partial R_e}\right| = 0 < \infty,\\ &\left|\frac{\partial f_2}{\partial B}\right| = 0 < \infty, \left|\frac{\partial f_2}{\partial B_S}\right| = 0 < \infty,\ \left|\frac{\partial f_2}{\partial R}\right| = 0 < \infty,\ \left|\frac{\partial f_2}{\partial A}\right| = 0 < \infty\end{aligned}\right\} \tag{3.4}$$

Taking the partial derivatives of the third equation of (3.2)

$$f_3(t, y_3) = \frac{dI}{dt} = \frac{\beta\lambda_2(B + B_S)}{N} + (1-\xi)\rho E - (\theta + \sigma + \delta + \mu + \delta_1)I$$

$$\left.\begin{aligned}&\left|\frac{\partial f_3}{\partial S}\right| = 0 < \infty,\ \left|\frac{\partial f_3}{\partial E}\right| = (1-\xi) < \infty,\ \left|\frac{\partial f_3}{\partial I}\right| = -(\theta + \sigma + \delta + \mu + \delta_1) < \infty,\\ &\left|\frac{\partial f_3}{\partial R_e}\right| = 0 < \infty, \left|\frac{\partial f_3}{\partial B}\right| = 0 < \infty, \left|\frac{\partial f_3}{\partial B_S}\right| = 0 < \infty,\ \left|\frac{\partial f_3}{\partial R}\right| = 0 < \infty,\ \left|\frac{\partial f_3}{\partial A}\right| = 0 < \infty\end{aligned}\right\} \tag{3.5}$$

Taking the partial derivatives of the fourth equation of (3.2)

$$f_4(t, y_4) = \frac{dR_e}{dt} = \omega B_S + \tau B + \theta I - (\alpha + \mu + \eta)R_e$$

$$\left.\begin{aligned}&\left|\frac{\partial f_4}{\partial S}\right| = 0 < \infty,\ \left|\frac{\partial f_4}{\partial E}\right| = 0 < \infty,\ \left|\frac{\partial f_4}{\partial I}\right| = \theta < \infty, \left|\frac{\partial f_4}{\partial R_e}\right| = -(\alpha + \mu + \eta) < \infty,\\ &\left|\frac{\partial f_4}{\partial B}\right| = \tau < \infty, \left|\frac{\partial f_4}{\partial B_S}\right| = \omega < \infty,\ \left|\frac{\partial f_4}{\partial R}\right| = 0 < \infty,\ \left|\frac{\partial f_4}{\partial A}\right| = 0 < \infty\end{aligned}\right\} \tag{3.6}$$

the partial derivatives of the fifth equation of (3.2) is consider as

$$f_5(t, y_5) = \frac{dB}{dt} = \xi\rho E + \sigma I - (\tau + \varphi + \mu + d_2)B$$

$$\left.\begin{aligned}&\left|\frac{\partial f_5}{\partial S}\right| = 0 < \infty,\ \left|\frac{\partial f_5}{\partial E}\right| = \xi\rho < \infty,\ \left|\frac{\partial f_5}{\partial I}\right| = \sigma < \infty, \left|\frac{\partial f_5}{\partial R_e}\right| = 0 < \infty,\\ &\left|\frac{\partial f_5}{\partial B}\right| = -(\tau + \varphi + \mu + d_2) < \infty, \left|\frac{\partial f_5}{\partial B_S}\right| = 0 < \infty,\ \left|\frac{\partial f_5}{\partial R}\right| = 0 < \infty,\ \left|\frac{\partial f_5}{\partial A}\right| = 0 < \infty\end{aligned}\right\} \tag{3.7}$$

From the sixth equation of (3.2), take the partial derivatives

$$f_6(t, y_6) = \frac{dB_S}{dt} = \varphi B - (\omega + \mu + \delta_1)B_S$$

$$\left.\begin{aligned}&\left|\frac{\partial f_6}{\partial S}\right| = 0 < \infty,\ \left|\frac{\partial f_6}{\partial E}\right| = 0 < \infty,\ \left|\frac{\partial f_6}{\partial I}\right| = 0 < \infty, \left|\frac{\partial f_6}{\partial R_e}\right| = 0 < \infty,\\ &\left|\frac{\partial f_1}{\partial B}\right| = \varphi < \infty, \left|\frac{\partial f_6}{\partial B_S}\right| = -(\omega + \mu + \delta_1) < \infty,\ \left|\frac{\partial f_6}{\partial R}\right| = 0 < \infty,\ \left|\frac{\partial f_6}{\partial A}\right| = 0 < \infty\end{aligned}\right\} \tag{3.8}$$

The partial derivatives of the seventh equation of (3.2) is taking as

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$f_7(t, y_7) = \frac{dR}{dt} = \delta I - \mu R$

$$\left.\begin{array}{l} \left|\frac{\partial f_1}{\partial S}\right| = 0 < \infty, \ \left|\frac{\partial f_1}{\partial E}\right| = 0 < \infty, \ \left|\frac{\partial f_1}{\partial I}\right| = \delta < \infty, \left|\frac{\partial f_1}{\partial R_e}\right| = 0 < \infty, \\ \left|\frac{\partial f_1}{\partial B}\right| = 0 < \infty, \left|\frac{\partial f_1}{\partial B_S}\right| = 0 < \infty, \ \left|\frac{\partial f_1}{\partial R}\right| = -\mu < \infty, \ \left|\frac{\partial f_1}{\partial A}\right| = 0 < \infty \end{array}\right\} \quad (3.9)$$

Therefore, consider the partial derivatives of the eight equation of (3.2)

$f_8(t, y_8) = \frac{dA}{dt} = P - (\mu + a)A$

$$\left.\begin{array}{l} \left|\frac{\partial f_8}{\partial S}\right| = 0 < \infty, \ \left|\frac{\partial f_8}{\partial E}\right| = 0 < \infty, \ \left|\frac{\partial f_8}{\partial I}\right| = 0 < \infty, \left|\frac{\partial f_8}{\partial R_e}\right| = 0 < \infty, \\ \left|\frac{\partial f_8}{\partial B}\right| = 0 < \infty, \left|\frac{\partial f_8}{\partial B_S}\right| = 0 < \infty, \ \left|\frac{\partial f_8}{\partial R}\right| = 0 < \infty, \ \left|\frac{\partial f_8}{\partial A}\right| = -(\mu + a) < \infty \end{array}\right\} \quad (3.10)$$

It can be observed from equations (3.3) to (3.10) that all the partial derivatives of the model equation are continues and bounded in the interval, $0 < \Omega < \infty$ by the theorem 3.1. The functions given in by equation (2.1) satisfy Lipchitz condition and hence, there exists a unique solution of model equation (2.1) in the region Ω.

Positivity of the Solution of the Model

The positivity of the solution of the model (2.1) ensure banditry dynamics realism. We are to show that every path starting from the non-negative region $\Re_+^8$ will ultimately converge to and stay within the feasible area ϑ. To prove this, we will establish that the set ϑ is positively invariant and is the system's global attractor.

Theorem 3.2:

Consider that the initial conditions of (2.1) are nonnegative, then the solutions for the different groups S, E, I, R_e, B, B_S, R and A of equation (2.1) remain nonnegative $\forall\, t > 0$.

Let the initial solution set be

$$\{S_0 \geq 0, E_0 \geq 0, I_0 \geq 0, R_{e0} \geq 0, B_0 \geq 0, B_{S0} \geq 0, R_0 \geq 0, A_0 \geq 0\} \in R_+^8$$

Then the solution set $\{S(t), E(t), I(t), R_e(t), B(t), B_S(t), R(t), A(t)\}$ is positive for all $t > 0$.

Proof

From the first compartment equation of (2.1),

$$\frac{dS}{dt} = \Lambda + \alpha R_e - \frac{\beta \lambda_2 (B + B_S)}{N} - (\lambda_1 + \mu) S$$

Since we are considering only the negative terms of susceptible population S, then

$$\frac{dS}{dt} \geq -(\lambda_1 + \mu) S \quad (3.11)$$

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Using separation of variable on (3.11), we have

$$\frac{dS}{S} \geq -(\lambda_1 + \mu)dt \tag{3.12}$$

Integrate both side of equation (3.12) to obtained

$$\ln(S) \geq -(\lambda_1 + \mu)t + z \tag{3.13}$$

$$S(t) \geq ce^{-(\lambda_1+\mu)t} \tag{3.14}$$

At $t = 0$

$$S(0) \geq Z_1$$

In a similar way, we can text the positivity of the remaining compartments of equation (2.1).

The Banditry-Free Equilibrium Point

In Banditry modeling, steady states refer to situations where the number of people in each group remains constant over time. This happens when the rates at which people move between groups are balanced. It means the number of infected individuals remains stable over time for a specific value of R_0, no matter how many people were initially infected. Here, we discuss free equilibrium point of the model as follows:

To determine the Banditry -Free Equilibrium Point (BFE) P_0 of System (2.1), each equation on the right-hand side of system (2.1) is set to zero. That is

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR_e}{dt} = \frac{dB}{dt} = \frac{dB_S}{dt} = \frac{dR}{dt} = \frac{dA}{dt} = 0 \tag{3.15}$$

Theorem 3.3

The banditry free equilibrium of the model exists at the point

$$P_0 = (S_0, E_0, I_0, R_{e0}, B_0, B_{S0}, R_0, A_0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0, 0\right) \tag{3.16}$$

Proof:

Let

$$(S, E, I, R_e, B, B_S, R_S, A) = (S^0, E^0, I^0, R_e{}^0, B^0, B_S{}^0, R_S{}^0, A^0) \tag{3.17}$$

be at equilibrium state.

From the first equation of (2.1)

$$\frac{dS}{dt} = \frac{dS}{dt} = \Lambda + \alpha R_e - \frac{\beta\lambda_2(B + B_S)}{N} - (\lambda_1 + \mu)S = 0$$
$$\Lambda + \alpha R_e - \frac{\beta\lambda_2(B + B_S)}{N} - (\lambda_1 + \mu)S = 0$$
$$\Lambda - (\lambda_1 + \mu)S = 0$$
$$\Lambda = (\lambda_1 + \mu)S$$
$$S^0 = \frac{\Lambda}{(\lambda_1 + \mu)}$$

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From the second equation of (2.1), we have

$$\frac{dE}{dt} = \lambda_1 S - (\rho - 2\xi\rho - \mu)E = 0$$
$$\lambda_1 S - (\rho - 2\xi\rho - \mu)E = 0$$
$$\lambda_1 S = (\rho - 2\xi\rho - \mu)E$$
$$E^0 = \frac{0}{(\rho - 2\xi\rho - \mu)} = 0$$

From the third equation of (2.1), we get

$$\frac{dI}{dt} = \frac{\beta\lambda_2(B + B_S)}{N} + (1-\xi)\rho E - (\theta + \sigma + \delta + \mu + \delta_1)I = 0$$
$$\frac{\beta\lambda_2(B + B_S)}{N} + (1-\xi)\rho E - (\theta + \sigma + \delta + \mu + \delta_1)I = 0$$
$$\frac{\beta\lambda_2(B + B_S)}{N} + (1-\xi)\rho E = (\theta + \sigma + \delta + \mu + \delta_1)I$$
$$\frac{0}{(\theta + \sigma + \delta + \mu + \delta_1)} = I$$
$$I^0 = 0$$

Similarly, $R_e = B = B_S = R = K_C = K_L = R = P = F = A = 0$.

Therefore, the Disease free equilibrium point is

$$P_0 = (S_0, E_0, I_0, R_{e0}, B_0, B_{S0}, R_0, A_0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0, 0\right).$$

Feasible Region (Invariant Region) of the Model

The population size can be determined by summing the nonlinear differential equation (2.1). The boundedness and feasibility of the invariant region for the model (2.1) are established in the following theorem by Lawal et al. (2023).

Theorem 3.3

The solution of the model (2.1) is feasible/bounded for $t < 1$ in the closed set, if they inter the invariant area X as

$$\mathrm{X} = \left\{(S, E, I, R_e, B, B_S, R, A) \in \Re_+^8 \leq \frac{\Lambda}{\mu}\right\} \tag{3.18}$$

Furthermore, the set X is positively invariant and attracting with respect to the model (2.1).

Proof

To find the feasible region (also known as the invariant region) for the model equations (2.1), we identify the region in which the total population remains bounded and positive over time.

The total population at any time t can be represented as:

$$N(t) = S(t) + E(t) + I(t) + R_e(t) + B(t) + B_S(t) + R(t) + A \tag{3.19}$$

Differentiating equation (4.22) with respect to time t we have

$$\frac{dN}{dt}=\frac{dS}{dt}+\frac{dE}{dt}+\frac{dI}{dt}+\frac{dR_e}{dt}+\frac{dB}{dt}+\frac{dB_S}{dt}+\frac{dR}{dt}+\frac{dA}{dt} \tag{3.20}$$

We sum all equation to get the equation for the total population $N(t)$ using (3.20) we have

$$\left.\begin{aligned}&\frac{dN}{dt}=\frac{dS}{dt}+\frac{dE}{dt}+\frac{dI}{dt}+\frac{dR_e}{dt}+\frac{dB}{dt}+\frac{dB_S}{dt}+\frac{dR}{dt}+\frac{dA}{dt}\\&=\Lambda+\alpha R_e-\frac{\beta\lambda_2(B+B_S)}{N}-(\lambda_1+\mu)S+\lambda_1 S-(\rho-2\xi\rho+\mu)E+\frac{\beta\lambda_2(B+B_S)}{N}+\\&(1-\xi)\rho E-(\theta+\sigma+\delta+\mu+\delta_1)I+\omega B_S+\tau B+\theta I-(\alpha+\mu+\eta)R_e+\xi\rho E+\sigma I-\\&(\tau+\varphi+\mu+d_2)B+\varphi B-(\omega+\mu+\delta)B_S+\delta I-\mu R+P-(\mu+a)A\end{aligned}\right\} \tag{3.21}$$

Simplifying (3.21),

$$\frac{dN}{dt}=\Lambda-(\lambda_1+\mu)S-\mu E-(\mu+\delta_1)I-(\mu+\eta)R_e-(\mu+d_2)B-(\mu+\delta)B_S-\mu R-(\mu+a)A \tag{3.22}$$

terms involving E, I, R_e, B, B_S, R, A cancel out, leaving us with:

$$\frac{dN}{dt}=\Lambda-\mu S=0$$
$$\Lambda-\mu S=0$$

$$\Lambda+Z=\mu S \tag{3.23}$$

$$S=\frac{\Lambda}{\mu} \tag{3.24}$$

Therefore, the invariant region of the kidnapped model is:

$$\mathrm{X}=\left\{(S, E, I, R_e, B, B_S, R, A)\in\Re_+^8\leq\frac{\Lambda}{\mu}\right\}.$$

This region ensures that the total population remains non-negative and bounded above by $\frac{\Lambda}{\mu}$, guaranteeing the feasibility and sustainability of the population dynamics modeled by the kidnapped model equations (2.1).

DISCUSSION OF RESULTS

The Mathematical Modelling the Effect of Government Policies on the Containment of Banditry in Kaduna State based on the framework developed by Lawal et al. (2023), which originally consists of five interacting compartments: non-informants (S), exposed individuals (E), informants (I), bandits (B), and removed individuals (R). In the modified formulation, the model is expanded to incorporate additional realistic components that capture intervention and support structures within the system. These include rehabilitation individuals, who represent persons undergoing recovery and reintegration; security agents, representing enforcement and counter-banditry operations; and bandit sponsors, who model the logistical and financial backbone sustaining bandit activities. The inclusion of these compartments allows the model to better reflect real-life security complexities, including post-crime rehabilitation and indirect support systems. The variables and parameters governing these transitions are defined in Tables 2.1 and 2.2, while the schematic diagram in Figure 2.1 illustrates the flow of individuals between compartments, forming the basis for the system of nonlinear differential equations in Equation (2.1).

The model is demonstrated to be mathematically well-posed through the establishment of existence and uniqueness of solutions. By expressing the system in vector form and applying results from nonlinear differential equation theory, it is shown that the model satisfies the Lipschitz condition within a defined region. This guarantees that, for given initial conditions, a unique and continuous solution exists, ensuring that the model

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produces consistent and non-contradictory results. , making the model suitable for analytical and predictive purposes.

Furthermore, the positivity and boundedness of the model are established to ensure realistic behavior of the system. All state variables are proven to remain non-negative over time, preserving their interpretation as population groups, while the feasible (invariant) region confirms that the total population remains bounded. This implies that the system evolves within a closed and biologically meaningful region that is both positively invariant and attracting. Consequently, the model is not only mathematically consistent but also structurally stable, providing a dependable framework for studying banditry dynamics and assessing the impact of intervention strategies.

SUMMARY AND CONCLUSION

This study developed a comprehensive mathematical model to analyze the dynamics of banditry and the impact of government policies in Kaduna State. By extending the framework of Lawal et al. (2023), the model incorporated additional realistic compartments, including rehabilitation individuals, security agents, and bandit sponsors, to better capture the complexity of banditry operations and interventions. The model was shown to be mathematically well-posed through the establishment of existence, uniqueness, positivity of solutions, and a bounded invariant region.

In conclusion, the results of the foregoing analysis underscore the importance of integrated and sustained policy measures in controlling banditry. Strategies that combine prevention, rehabilitation, and enhanced security enforcement were found to be most effective in reducing bandit activities and promoting long-term stability. The inclusion of bandit sponsors and rehabilitation processes in the model highlights the need for policies that not only suppress criminal activities but also disrupt support networks and facilitate reintegration.

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