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Q-Fuzzy Implementations of N- Lie Ideals Structures Over Lie Subalgebas -A New Approach

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ABSTRACT

In this paper, we study the concept of q-fuzzy n-lie sub algebras (ideals) of lie algebras and investigate some of their properties. We discuss the relationship between q-fuzzy lie sub algebra (respectively ideals) and Lie sub algebras (respectively ideals). For a finite number of q-fuzzy n-lie sub algebras, we construct new q-fuzzy n-lie sub algebras on their direct sum. Finally, we analyze the homomorphism concept of q-fuzzy n-lie sub algebra over ideals.

Key words: Q-fuzzy set, sub algebra, q-fuzzy n-ideal, direct sum, homomorphism, Lie algebra, vector space.

AMS subject classification (2000): 04A72, 17B99.

INTRODUCTION

The inception of the concept of Lie algebras was performed by one of the renowned Norwegian mathematicians of the 19th century, Sophus Lie (1842–1899) [3,4]. After him, the idea has undergone many developments, modifications, and additions, by multiple authors who incorporated further advanced and novel types and factors with this concept, one of which is the notion of Hom–Lie algebras. The base of the idea of Hom–Lie algebras was initially formulated by Hartwig, Larsson, and Silvestrov in 2006 [5]. It lies among the generalities of the notion of classical Lie algebras. In recent years, Hom–Lie algebras have turned into a stimulating area of physics and mathematics. For further deliberations concerning Hom–Lie algebras, we refer the reader to [5,6 ,7 ,8 ,9 ,10 ,11].In terms of the case of fuzzy sets, Zadeh [12] was the first one to present this notion. The application of fuzzy set theory extends to various fields including decision theory [13,14], logic [15], soil science [16], computer science [17], social life [18,19], artificial intelligence [20,21 ,22] , management, various disciplines. Originally, the study of fuzzy Lie sub algebras of Lie algebras was first introduced by Yehia [25] in the year 1996. Afterwards, numerous authors [26,27 ,28 ,29 ,30 ,31 ,32 ,33 ,34], as well as the references within) have made contributions to the concept of fuzzy sets (in more general, complex fuzzy sets and intuitionistic fuzzy sets) by applying them in a variety of directions in Lie algebras. Lie algebras was also discovered by him when he attempted to classify certain smooth subgroups of a general linear group. The importance of lie algebras in mathematics and physics has become increasingly evident in recent years. In applied mathematics, Lie theory remains a powerful tool for studying differential equations, special functions and perturbation theory. It is noted that lie theory has application not only in mathematics and physics but also in divergent fields such as continuum mechanics, cosmology and life sciences. Lie algebra has nowadays even been applied by electrical engineers in solving problems in mobile robot control. On the other hand, Zadeh [12.15] introduce the notion of fuzzy subset of a set in 1965. By using fuzzy sets, people have established the theory for study uncertainty. The study of fuzzy Lie algebras was initiated by Yehia [25] in 1996. In this paper, we study the concept of q-fuzzy n-lie sub algebras(ideals) of lie algebras and investigate some of their properties. We discuss the relationship between q-fuzzy lie sub algebra(respectively ideals) and Lie sub algebras (respectively ideals). For a finite number of q-fuzzy n-lie sub algebras, we construct a new q-fuzzy n-lie sub algebras on thir direct sum. Finally, we analyse the homomorphism concept of q-fuzzy n-lie sub algebra over ideals.

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Preliminaries

Definition-2.1: Fuzzy set: Let X be a non-empty set. A fuzzy set A drawn from X is defined as $A = \{(x, \mu_A(x))/x \in X\}$, where $\mu_A: X \to [0,1]$ is the member function of the fuzzy set A.

Definition-2.2: If μ is a fuzzy subset of p, for $\alpha \in [0,1]$, then the set

$\mu_\alpha = \{x \in p/\mu(x) \geq \alpha\}$ is called level subset of p of p with respect to a fuzzy subset μ.

Definition-2.3: A fuzzy set $\mu: p \to [0,1]$ is a non-empty fuzzy subset, if μ is not constant function.

Definition-2.4 (Lie algebra) : Let F be a field. A Lie algebra over F is triple, $(S, [\ ,\ ], \alpha)$ where S is a vector space over F, $\alpha: S \to S$ is a linear map, and $[\ ,\ ]: S \times S \to S$ is a linear map, satisfying the following properties.

(i) $[x, y] = -[y, x]$ for all $x, y \in S$ (Skew- symmetry property)
(ii) $[\alpha(x), [y, z]]+[\alpha(y), [z, x]] + [\alpha(z), [x, y]] = 0, x, y, z \in S$ (Jacobi identity).

It is clear that every Lie algebra is a q-fuzzy Lie algebra by setting $\alpha = qI$.

Example 2.5: Let Sbe a vector space over F and $[\ ,\ ]: S \times S \to S$ be any Skew- symmetric bilinear map. If $\alpha: S \to S$ is the zero map, then $(S, [\ ,\ ], \alpha)$ is a Lie algebra.

Definition-2.6: A linear map $\varphi: S_1 \to S_2$ is called a morphismof Lie algebra if the following two identities are satisfied.

(i) $\varphi([x, y]_1) = [\varphi(x), \varphi(y)]_2$, for all $x, y \in S_1$
(ii) $\varphi \circ \alpha_1 = \alpha_2 \circ \varphi$

Note: Throughout this paper S is a Lie algebra over F.

Let $x, y \in [0,1]$. For the sake of simplicity we use the symbols $x\wedge y$ and $x\vee y$ to denote

$\min\{x, y\}$and$\max\{x, y\}$ respectively.

Definition-2.7: A Q-fuzzy set 'δ' on S is a q-fuzzy n-Lie subalgebra if the following conditions are satisfied for all $x, y \in S$ and $\alpha: S \to S$ is a linear map;

(i) $\delta^n(x + y, q) \geq \min\{\delta^n(x, q), \delta^n(y, q)\}$,
(ii) $\delta^n(\alpha(x, q)) \geq \delta^n(x, q)$,
(iii) $\delta^n([x, y], q) \geq \min\{\delta^n(x, q), \delta^n(y, q)\}$.

If the condition (iii) is replaced by $\delta^n([x, y], q) \geq \max\{\delta^n(x, q), \delta^n(y, q)\}$, then 'δ' is called a q-fuzzy n-Lie ideal of S. It is clear that if 'δ' is a q-fuzzy n-Lie ideal of S, then it is a q-fuzzy n-Lie subalgebral of S.

Example-2.8: Let S be a vector space with basis $\{m_1, m_2, m_3\}$. We define the twisted map

$\alpha: S \to S$ by setting $\alpha(m_1) = m_2$ and $\alpha(m_2) = \alpha(m_3) = 0$. Let $[\ ,\ ]: S \times S \to S$ be any Skew- symmetric bilinear map such that $[m_1, m_2] = [m_2, m_3] = 0$, $[m_1, m_3] = m_1$, and

$[m_i, m_i] = 0$for all states i=1,2,3, then $(L, [\ ,\ ], \alpha)$ is a q-fuzzy n-Lie algebra.

Indeed, for each $x, y \in S$, we have $[x, y]$ is a scalar multiple of m_1. Also $\alpha(x)$ is scalar multiple of m_2 for each $x \in S$. Therefore $[\alpha(x), [y, z]] = 0$ for each $x, y, z \in L$. This implies that q-fuzzy n-Jacobi identity is satisfied.

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We define 'δ' as follows $\delta(x) = \begin{cases} 0.9 : & x = 0 \\ 0.6 : x \in \mathrm{span}\{m_1, m_2\} - \{0\} \\ 0.2 : & \text{otherwise} \end{cases}$

Then 'δ' is a q-fuzzy n-Lie ideal of S.

Let V be a vector space and 'δ' be a Q-fuzzy set on it. For $t \in [0,1]$, the set

$\cup(\delta, t) = \{x \in V \text{ and } q \in Q \,/\, \delta(x, q) \geq t\}$is called upper level of δ. The following theorem will show a relation between q-fuzzy n-Lie sub algebra of S and Lie sub algebra of L.

3. Some Results on Q-FUZZY N-LIE Subalgebras

Theorem-3.1: Let 'δ' be a Q-fuzzy subset of S. Then the following statements are equivalent;

(i) 'δ' is a q-fuzzy n-Lie subalgebra of S.

(ii) The non-empty set $\cup(\delta, t)$ is a q-fuzzy n-Lie sub algebra of S for every $t \in \mathrm{Im}(\delta)$. **Proof:**Let $t \in \mathrm{Im}(\delta)$, and let $x, y \in \cup(\delta, t)$.

As 'δ' is a q-fuzzy n-Lie sub algebra of S, we have

$$\delta^n(x + y, q) \geq \min\{\delta^n(x, q), \delta^n(y, q)\} \geq t,$$
$$\delta^n\big(\alpha(x, q)\big) \geq \delta^n(x, q),$$
$$\delta^n([x, y], q) \geq \min\{\delta^n(x, q), \delta^n(y, q)\} \geq t,$$

and so $x + y$, $\alpha(x)$ and $[x, y]$ are elements in $\cup(\delta, t)$.

Conversely, let $\cup(\delta, t)$ be a q-fuzzy n-Lie sub algebra of S, for every $t \in \mathrm{Im}(\delta)$.

Let $x, y \in S$. We may assume that $\delta^n(y, q) \geq \delta^n(x, q) \geq t_1$. So $x, y \in \cup(\delta, t)$.

As $\cup(\delta, t_1)$ is a sub algebra of S, we have $x + y$ is in $\cup(\delta, t_1)$, and

$\delta^n(x + y, q) \geq t_1 = \min\{\delta^n(x, q), \delta^n(y, q)\}$.

Since $\cup(\delta, t_1)$ is a q-fuzzy n-Lie sub algebra of S, we have $[x, y]$ and $\alpha(x)$ are in $\cup(\delta, t_1)$.

Hence, $\delta^n([x, y], q) \geq t_1 = \min\{\delta^n(x, q), \delta^n(y, q)\}$ and $\delta^n\big(\alpha(x, q)\big) \geq t_1 = \delta^n(x, q)$.

Theorem-3.2: Let 'δ' be a Q-fuzzy subset of S. Then the following statements are equivalent;

(i) 'δ' is a q-fuzzy n-Lie ideal of S.
(ii) The non-empty set $\cup(\delta, t)$ is a q-fuzzy n-Lie ideal of S for every $t \in \mathrm{Im}(\delta)$

Proof: Let 'δ' is a q-fuzzy n-Lie ideal of S.

Then it is a q-fuzzy n-Lie subalgebra of S .

According to the theorem-3.1, every $x, y \in \cup(\delta, t)$, we have $x + y$, $\alpha(x)$ are in $\cup(\delta, t)$.

For $x \in S$and $y \in \cup(\delta, t)$, we find

$\delta^n([x, y], q) \geq \max\{\delta^n(x, q), \delta^n(y, q)\} \geq \delta^n(x, q) \geq t$.

That is $[x, y] \in \cup(\delta, t)$.

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Conversely, assume that every $\cup(\delta, t) \neq \emptyset$ is a Q-fuzzy Lie ideal of S, then $\cup(\delta, t)$

is a Q-fuzzy Lie algebra of S. Thus, we can proceed as in the theorem above and the only difference appears in the proof of the following statement;

$\delta^n([x,y],q) \geq \max\{\delta^n(x,q), \delta^n(y,q)\}$, for all $x, y \in S$.

Let $x, y \in S$. Without loss of generality, we may assume that $\delta^n(x,q) \geq \delta^n(y,q)$.

Set $t_0 = \delta(x,q)$. Hence $x \in \cup(\delta, t_0)$.

As $\cup(\delta, t_0)$ is a q-fuzzy Lie ideal of S, we have $[x,y] \in S$.

This shows that $\delta^n([x,y],q) \geq \max\{\delta^n(x,q), \delta^n(y,q)\}$.

Let V be a vector space. For $t \in [0,1]$ and a Q-fuzzy set δ on V, the set

$\cup(\delta, t) = \{x \in V \text{ and } q \in Q \,/\, \delta(x,q) > t\}$is called strong upper level of δ. We have following result.

Theorem-3.3: Let 'δ' be a Q-fuzzy subset of S. Then the following statements are equivalent;

(i) 'δ' is a q-fuzzy n-Lie subspace of S.
(ii) The strong upper level$\cup(\delta, t)$ is a sub algebra of S for every $t \in Im(\delta)$.

Proof: For$t \in Im(\delta)$, let $x, y \in \cup(\delta^{>}, t)$.

As 'δ' is a q-fuzzy n-Lie subspace of S, we have

$$\delta^n(x+y,q) \geq \min\{\delta^n(x,q), \delta^n(y,q)\} \geq t,$$
$$\delta^n\big(\alpha(x,q)\big) \geq \delta^n(x,q),$$
$$\delta^n([x,y],q) \geq \min\{\delta^n(x,q), \delta^n(y,q)\} \geq t.$$

Consequently, $x+y$, $\alpha(x)$ and $[x,y]$ are elements in $\cup(\delta^{>}, t)$.

Conversely, assume that for every $t \in Im(\delta)$, we have $\cup(\delta^{>}, t)$ is a Lie-subalgebra of S. Let $x, y \in S$. We need to show that the conditions of definition ** are satisfied .

If $\delta^n(x,q) = 0$or $\delta^n(y,q) = 0$, then $\delta^n(x+y,q) \geq 0 = \min\{\delta^n(x,q), \delta^n(y,q)\}$.

Suppose that $\delta^n(x,q) \neq 0$and $\delta^n(y,q) \neq 0$.

Suppose to the contrary that $\delta^n(x+y,q)$ and $\delta^n([x,y],q)$ are less than the value of

$\min\{\delta^n(x,q), \delta^n(y,q)\}$. Let t_0 be the greatest lower bound of the set

$\big\{t/t < \min\{\delta^n(x,q), \delta^n(y,q)\}\big\}$and less than the value of $\min\{\delta^n(x,q), \delta^n(y,q)\}$.

We have $x+y$, $[x,y]$ $\in\cup(\delta^{>}, t_0)$, and hence $\delta^n(x,q), \delta^n(y,q) > t_0$.

This contradicts that there are no element $a \in S$with $t_0 < \delta^n(a,q) < \{\delta^n(x,q), \delta^n(y,q)\}$.

This shows that $\delta^n(x+y,q), \delta^n([x,y],q) \geq \min\{\delta^n(x,q), \delta^n(y,q)\}$.

Again let 't_0' be the largest number of [0,1] such that $t_0 < \delta^n(x,q)$ and there are no $a \in S$with $t_0 < \delta^n(a,q) < \delta^n(x,q)$. As $\cup(\delta^{>}, t_0)$ is q-fuzzy n-Lie subspace, we have $\alpha(x)$ and $\cup(\delta^{>}, t_0)$ and so $\delta^n\big(\alpha(x,q)\big) > t_0$. Thus $\delta^n\big(\alpha(x,q)\big)$ is greater than or equal to $\delta^n(x,q)$.

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Using almost the same argument one can show the following result.

Construction of Direct Sum of Q-FUZZY N-LIE Algebras

Given q-fuzzy m-Lie algebras $(S_i, [\,,], \alpha_i)$, i=1,2,…,m, then

$(S_1 \oplus S_2 \oplus \ldots \oplus S_m, [\,,], \alpha_1 \oplus \alpha_2 \oplus \ldots \oplus \alpha_m)$is a q-fuzzy Lie algebra by setting

$$[\,,]: (S_1 \oplus S_2 \oplus \ldots \oplus S_m) \times (S_1 \oplus S_2 \oplus \ldots \oplus S_m) \to (S_1 \oplus S_2 \oplus \ldots \oplus S_m)$$

$\big((x_1, x_2, \ldots, x_m), (y_1, y_2, \ldots, y_m)\big) \to ([x_1, y_1], [x_2, y_2], \ldots, [x_m, y_m]),$

and the linear map

$$(\alpha_1 \oplus \alpha_2 \oplus \ldots \oplus \alpha_m): (S_1 \oplus S_2 \oplus \ldots \oplus S_m) \to (S_1 \oplus S_2 \oplus \ldots \oplus S_m)$$

$$(x_1, x_2, \ldots, x_m) \to \big(\alpha_1(x_1), \alpha_2(x_2), \ldots, \alpha_m(x_m)\big)$$

In the special case when $m = 2$, we obtain[ ].

Let $(S_1, [\,,], \alpha_1)$, $(S_2, [\,,], \alpha_2)$, …….,$(S_m, [\,,], \alpha_m)$ be q-fuzzy Lie algebras.

Suppose that $\delta_1,\delta_2,\ldots,\delta_m$ are Q-fuzzy subsets of S_1, $S_2,\ldots,S_m$ respectively. Then the generalized Cartesian sum of Q-fuzzy sets induced by $\delta_1,\delta_2,\ldots,\delta_m$ on $S_1 \oplus S_2 \oplus \ldots \oplus S_m$ is

$\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m: S_1 \oplus S_2 \oplus \ldots \oplus S_m \to [0,1];$

$(x_1, x_2, \ldots, x_m) \to \min\{\delta_1{}^n(x_1, q), \delta_2{}^n(x_2, q), \ldots, \delta_m{}^n(x_m, q)\}.$

Theorem-4.1: Let $(S_1, [\,,], \alpha_1)$, $(S_2, [\,,], \alpha_2)$, …….,$(S_m, [\,,], \alpha_m)$ be q-fuzzy Lie algebras. Let $\delta_1,\delta_2,\ldots,\delta_m$ be Q-fuzzy n-Lie sub algebras of $S_1,S_2,\ldots,S_m$ respectively, then

$\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m$is Q-fuzzy n-Lie sub algebras of $S_1 \oplus S_2 \oplus \ldots \oplus S_m$.

Proof: Let $(x_1, x_2, \ldots, x_m), (y_1, y_2, \ldots, y_m) \in S_1 \oplus S_2 \oplus \ldots \oplus S_m$.

$\delta^n([(x_1, x_2, \ldots, x_m), (y_1, y_2, \ldots, y_m)])$

$= (\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)([x_1, y_1], [x_2, y_2], \ldots, [x_m, y_m])$

$= min\{\delta_1{}^n([x_1, y_1], q), \delta_2{}^n([x_2, y_2], q), \ldots, \delta_m{}^n([x_m, y_m], q)\}$

$\geq min\{\delta_1{}^n(x_1, q), \delta_1{}^n(y_1, q), \delta_2{}^n(x_2, q), \delta_2{}^n(y_2, q), \ldots, \delta_m{}^n(x_m, q), \delta_m{}^n(y_m, q)\}$

$= min\big\{(\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)\big((x_1, x_2, \ldots, x_m), q\big), (\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)\big((y_1, y_2, \ldots, y_m), q\big)\big\}.$

Also,

$(\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)\big((\alpha_1 + \alpha_2 + \ldots + \alpha_m)(x_1, x_2, \ldots, x_m), q\big)$

$= (\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)\big(\alpha_1(x_1, q), \alpha_2(x_2, q), \ldots, \alpha_m(x_m, q)\big)$

$= min\{\delta_1{}^n\big(\alpha_1(x_1, q)\big), \delta_2{}^n\big(\alpha_2(x_2, q)\big), \ldots, \delta_m{}^n\big(\alpha_m(x_m, q)\big)\}$

$\geq min\{\delta_1{}^n(x_1, q), \delta_2{}^n(x_2, q), \ldots, \delta_m{}^n(x_m, q)\}$

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$= \big((\delta_1 \oplus \delta_2 \oplus \ldots \oplus \delta_m)\big)\big((x_1, x_2, \ldots, x_m), q\big).$

The rest of the proof is similar. so we omit it.

However the direct sum of q-fuzzy n-Lie ideals of q-fuzzy Lie sub algebrasS_1 and S_2 is not necessary to be a q-fuzzy n-Lie ideal of the q-fuzzy Lie aubalgebra $S_1 \oplus S_2$.

5. Q-FUZZY N-LIE Algebras and Q-FUZZY N-LIE Algebras Morphisms

Suppose $\varphi: X \times Q \to Y$ is a function. If δ_B is a Q-fuzzy set on Y, then we can define Q-fuzzy set on X induced by φ and " δ_A" by setting

${\delta_{\varphi^{-1}(B)}}^n(x,q) = {\delta_B}^n\big(\varphi(x,q)\big)$for only $x \in X$. Also if δ_A is a Q-fuzzy set on X, then

$${\delta_{\varphi(A)}}^n(y,q) = \begin{cases} \underset{x \in \varphi^{-1}(y)}{Sup} \{{\delta_A}^n(x,q)\}: y \in \varphi(X) \\ 0 \qquad\qquad\qquad\qquad : \ y \notin \varphi(X) \end{cases}$$

is actually set on Y induced by φ and δ_A in the setting of Lie algebras.

We extend it to q-fuzzy Lie algebra case.

Theorem-5.1: Let $\varphi: (S_1, [\,,], \alpha_1) \to (S_2, [\,,], \alpha_2)$ be a morphism of q-fuzzy Lie algebras. If

$B_= = \delta_B$is a q-fuzzy n-lie sub algeba (respectively ideal) of S_2, then the Q-fuzzy set $\varphi^{-1}(B)$

is also a q-fuzzy n-lie sub algeba (respectively ideal) of S_1.

Proof: Let $x_1, x_2 \in S_1$. Then

${\delta_{\varphi^{-1}(B)}}^n\big((x_1 + x_2), q\big) = {\delta_B}^n \Big(\varphi\big((x_1 + x_2), q\big)\Big)$

$= {\delta_B}^n\big(\varphi(x_1,q) + \varphi(x_2,q)\big)$ (Since φ is linear)

$$\geq min\{{\delta_B}^n\big(\varphi(x_1,q)\big), {\delta_B}^n\big(\varphi(x_2,q)\big)\}$$

(Since δ_B is a q-fuzzy n-lie sub algebra)

$$= min\{{\delta_{\varphi^{-1}(B)}}^n(x_1,q), {\delta_{\varphi^{-1}(B)}}^n(x_2,q)\}$$and

${\delta_{\varphi^{-1}(B)}}^n([x_1,x_2],q) = {\delta_B}^n\big(\varphi([x_1,x_2],q)\big)$

$= {\delta_B}^n([\varphi(x_1,q), \varphi(x_2,q)])$ (Since φ is homomorphism)

$$\geq min\{{\delta_B}^n\big(\varphi(x_1,q)\big), {\delta_B}^n\big(\varphi(x_2,q)\big)\}$$

(Since δ_B is a q-fuzzy n-lie sub algebra)

$$= min\{{\delta_{\varphi^{-1}(B)}}^n(x_1,q), {\delta_{\varphi^{-1}(B)}}^n(x_2,q)\}.$$

Let $x \in S_1$. Then ${\delta_{\varphi^{-1}(B)}}^n\big(\alpha(x,q)\big) = {\delta_B}^n \Big(\varphi\big(\alpha(x)\big)\Big)$

$$= {\delta_B}^n \Big(\alpha\big(\varphi(x)\big)\Big) \text{ (Since } \varphi \text{ is homomorphism)}$$

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$$\geq {\delta_B}^n(\varphi(x,q)) \quad \text{(Since } \delta_B \text{ is a q-fuzzy n-lie sub algebra)}$$

$$= {\delta_{\varphi^{-1}(B)}}^n(x,q)$$

The case of q-fuzzy n-lie ideal is similar to show.

If $\varphi: S_1 \to S_2$ is a Lie algebra homomorphism and $A = \delta_A$ is a Q-fuzzy sub algebra of S_1, then the image of A, $\varphi(A)$ is a Q-fuzzy subalgebra of $\varphi(C_1)([\ \ ])$. In the following theorem we consider an analogue result for the case of q-fuzzy Lie algebras.

Theorem-5.2: Let $\varphi: (S_1, [\,,], \alpha_1) \to (S_2, [\,,], \alpha_2)$ be a morphism from $S_1$to S_2. If $A = \delta_A$ is a q-fuzzy n-Lie sub algebras of S_1, then $\varphi(A)$ is also a q-fuzzy n-Lie subalgebras of S_2.

Proof: Let $y_1, y_2 \in S_2$. As φ is onto, there are $x_1, x_2 \in S_1$ such that $(x_1) = y_1$,

$\varphi(x_2) = y_2$.

We get,$\{x_1 + x_2/x_1 \in \varphi^{-1}(y_1) and x_2 \in \varphi^{-1}(y_2)\} \subseteq \{x/x \in \varphi^{-1}(y_1 + y_2)\}$, and

$\{[x_1, x_2]/x_1 \in \varphi^{-1}(y_1) and x_2 \in \varphi^{-1}(y_2)\} \subseteq \{x/x \in \varphi^{-1}([y_1, y_2])\}$.

Now, we find

$${\delta_{\varphi(A)}}^n((y_1 + y_2), q) = \underset{x\in\varphi^{-1}(y_1+y_2)}{Sup}\{{\delta_A}^n(x,q)\}$$

$$\geq \{{\delta_A}^n((x_1 + x_2), q)/x_1 \in \varphi^{-1}(y_1) and x_2 \in \varphi^{-1}(y_2)\}$$

$$\geq min\left\{\underset{x\in\varphi^{-1}(y_1)}{Sup}\{{\delta_A}^n(x_1,q)\}, \underset{x\in\varphi^{-1}(y_2)}{Sup}\{{\delta_A}^n(x_2,q)\}\right\}$$

$$= min\{{\delta_{\varphi(A)}}^n(y_1,q), {\delta_{\varphi(A)}}^n(y_2,q)\}.$$

Also,

$${\delta_{\varphi(A)}}^n([y_1, y_2], q) = \underset{x\in\varphi^{-1}([y_1,y_2])}{Sup}\{{\delta_A}^n(x,q)\}$$

$$\geq \{{\delta_A}^n([x_1, x_2], q)/x_1 \in \varphi^{-1}(y_1) and x_2 \in \varphi^{-1}(y_2)\}$$

$$\geq sup\, min\{{\delta_A}^n(x_1,q), {\delta_A}^n(x_2,q)/x_1 \in \varphi^{-1}(y_1) and x_2 \in \varphi^{-1}(y_2)\}$$

$$= min\left\{\underset{x\in\varphi^{-1}(y_1)}{Sup}\{{\delta_A}^n(x_1,q)\}, \underset{x\in\varphi^{-1}(y_2)}{Sup}\{{\delta_A}^n(x_2,q)\}\right\}.$$

$$= min\{{\delta_{\varphi(A)}}^n(y_1,q), {\delta_{\varphi(A)}}^n(y_2,q)\}.$$

Also,

$${\delta_{\varphi(A)}}^n(\alpha(y,q)) = \underset{x\in\varphi^{-1}(\alpha(y))}{Sup}\{{\delta_A}^n(x,q)\} \geq \{{\delta_A}^n(\alpha(y,q))/x \in \varphi^{-1}(\alpha(y))\}$$

$$\geq \{{\delta_A}^n(x,q)/x \in \varphi^{-1}(y)\} = {\delta_{\varphi(A)}}^n(y,q).$$

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Chung-Gook Kim and Dong-Soo Lee ([ ]) proved if $f: S \to S'$ is a surjective Lie algebra homomorphism and If $A = \delta_A$ is a fuzzy ideal of L, then $f(A)$is a fuzzy ideal of L'.

We will extend the result to q-fuzzy n-Lie subalgebra case.

Theorem-5.3: Let $\varphi: (S_1, [\ ,\ ], \alpha_1) \to (S_2, [\ ,\ ], \alpha_2)$ be an ontomorphism of q-fuzzy Lie algebras. If $A = \delta_A$ is a q-fuzzy n-Lie ideal ofS_1 , then $\varphi(A)$ is also a q-fuzzy n-Lie ideal of S_2.

Proof: The proof is similar to the proof of theorem above.

We only need to claim that

$\delta_{\varphi(A)}{}^n([y_1, y_2], q) \geq max\{\delta_{\varphi(A)}{}^n(y_1, q), \delta_{\varphi(A)}{}^n(y_2, q)\}, \forall y_1, y_2 \in S_2$.

Let $y_1, y_2 \in S_2$ and assume by contradiction, that

$$\delta_{\varphi(A)}{}^n([y_1, y_2], q) \leq max\{\delta_{\varphi(A)}{}^n(y_1, q), \delta_{\varphi(A)}{}^n(y_2, q)\}$$

Then $\delta_{\varphi(A)}{}^n([y_1, y_2], q) < \delta_{\varphi(A)}{}^n(y_1, q)$or $\delta_{\varphi(A)}{}^n([y_1, y_2], q) < \delta_{\varphi(A)}{}^n(y_2, q)$.

We may assume, without loss of generality, that $\delta_{\varphi(A)}{}^n([y_1, y_2], q) <$.

Choose a number $t \in [0,1]$ such that $\delta_{\varphi(A)}{}^n([y_1, y_2], q) < t < \delta_{\varphi(A)}{}^n(y_1, q)$.

There is $a \in \varphi^{-1}(y_1)$with $\delta_A{}^n(a, q) > t$.

As φ is onto, there exists $b \in \varphi^{-1}(y_2)$, we note that

$\varphi([a, b]_1) = [\varphi(a), \varphi(b)]_2 = [y_1, y_2]_2$.

Thus $\delta_{\varphi(A)}{}^n([y_1, y_2]_2, q) \geq \delta_A{}^n([a, b]_1, q)$

$$\geq \max\{\delta_A{}^n(a, q), \delta_A{}^n(b, q)\}$$

$$\geq t$$

$>\delta_{\varphi(A)}{}^n([y_1, y_2], q)$which is contradiction.

CONCLUSION

We discussed q-fuzzy n-Lie sub algebra over ideal structures in this article. We constructed direct sum of q-fuzzy n-ideals and homomorphism of q-fuzzy lie sub algebra is analysed.

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