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An Optimized Double-Sampling C Control Chart for Enhanced Monitoring of Process Nonconformities

Bhagyashri C. Tamshetti[1], Digambar M. Zombade[2]

[1]Department of Statistics, Punyashlok Ahilyadevi Holkar Solapur University, Solapur, MS, India - 413255

[2]Department of Statistics, Vasundhara Kala Mahavidyalaya, Jule Solapur, MS, India - 413004

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ABSTRACT

Statistical Process Control (SPC) plays a vital role in monitoring manufacturing and service processes characterized by count data. The conventional Shewhart *c*-control chart is widely used for monitoring the number of nonconformities under the Poisson distribution; however, its performance deteriorates when detecting small and moderate process shifts. This study proposes an optimized Double-Sampling (DS) *c*-control chart that improves shift detection while maintaining a specified in-control Average Run Length (ARL_0). The chart design is formulated as a constrained optimization problem in which the out-of-control Average Run Length (ARL_1) is minimized subject to target ARL_0 and inspection efficiency requirements. Exact Poisson probabilities are employed to derive the operating characteristics, while the Average Sample Size (ASS) is used to evaluate inspection effort. Extensive numerical comparisons for various process means and shift magnitudes demonstrate that the proposed DS *c*-chart consistently achieves substantially lower ARL_1 values than the conventional single-sampling *c*-chart, particularly for small and moderate shifts, while maintaining the desired false-alarm performance. The proposed scheme provides an efficient, practical, and economically attractive alternative for monitoring Poisson-distributed process nonconformities.

Keywords: Double-Sampling *c*-Control Chart; Statistical Process Control; Poisson Distribution; Average Run Length; Average Sample Size; Attribute Control Charts.

INTRODUCTION

Statistical Process Control (SPC) has become one of the most widely adopted methodologies for monitoring, controlling, and continuously improving the quality of manufacturing and service processes. By distinguishing between common-cause and assignable-cause variation, SPC enables organizations to identify process abnormalities at an early stage, thereby reducing production costs, minimizing waste, and improving product reliability. Among the various SPC techniques, control charts remain the most effective graphical tools for monitoring process stability and supporting data-driven quality improvement initiatives.

Control charts may be broadly classified into variable control charts and attribute control charts. Variable charts are appropriate when quality characteristics are measured on a continuous scale, whereas attribute charts are employed when quality characteristics are represented by counts or classifications. In many industrial applications, including electronic assembly, textile manufacturing, semiconductor fabrication, healthcare systems, pharmaceutical production, and service operations, quality is naturally expressed as the number of defects or nonconformities occurring within a fixed inspection unit. For such applications, the Shewhart *c*-control chart is the standard monitoring procedure under the assumption that defect counts follow a Poisson distribution with constant mean *c*. The conventional Shewhart *c*-chart is attractive because of its conceptual simplicity, straightforward implementation, and low computational requirements. Under in-control conditions, the process mean is denoted by c_0, and the upper control limit is selected to achieve a specified false-alarm probability or a desired in-control Average Run Length (ARL_0). Despite these advantages, the classical *c*-chart suffers from an important limitation.

Since each decision is based solely on a single sample, the chart exhibits relatively poor sensitivity to small and moderate increases in the process defect rate. Consequently, assignable causes may remain undetected for an extended period, resulting in increased production of nonconforming items, higher inspection costs, and reduced process efficiency.

To overcome these limitations, numerous researchers have proposed enhanced monitoring schemes based on adaptive sampling strategies, supplementary runs rules, cumulative statistics, and multi-stage inspection procedures. Among these approaches, double-sampling control charts have attracted considerable attention because they improve shift-detection performance without substantially increasing routine inspection effort. Rather than making an immediate decision after the first inspection stage, double-sampling schemes collect additional information only when the initial sample provides inconclusive evidence regarding the process condition. Consequently, these procedures achieve a more desirable balance between false-alarm protection and rapid detection of process deterioration.

Daudin [2] introduced one of the earliest double-sampling control chart methodologies and demonstrated its superior performance over conventional Shewhart charts. Chan et al. [3] developed a two-stage decision procedure for monitoring processes with low fractions of nonconforming items. Hsu [4, 5] investigated the optimal design of double and triple-sampling control charts using genetic algorithms and extended these ideas to manufacturing applications. Wu and Wang [6] proposed a double-inspection *np*-control chart for attribute data, while Rodrigues et al. [7] developed optimized double-sampling attribute control charts and demonstrated substantial improvements in Average Run Length performance. More recently, Saghir and Lin [8] presented a comprehensive review of count-data control charts, highlighting the continuing need for efficient monitoring procedures capable of rapidly detecting shifts in Poisson-distributed processes.

Although considerable progress has been achieved in the development of adaptive and double-sampling control charts for attribute data, comparatively limited attention has been devoted to optimized double-sampling procedures specifically designed for Poisson *c*-charts. Existing approaches frequently focus on binomial attribute charts, variable control charts, or generalized count-data models, leaving a noticeable gap in the literature regarding efficient double-sampling monitoring schemes for classical Poisson-distributed nonconformities. Furthermore, many published designs primarily emphasize Average Run Length performance while giving comparatively less consideration to the inspection effort required during the monitoring process.

Motivated by these observations, this paper proposes an optimized Double-Sampling (DS) *c*-control chart for monitoring Poisson-distributed nonconformities. The proposed monitoring scheme employs a two-stage inspection procedure governed by five design parameters (r_1, r_2, WL, UCL_1, UCL_2). Exact Poisson probabilities are used to derive the probability of no signal, Average Run Length (ARL), and Average Sample Size (ASS). The optimal chart parameters are obtained by minimizing the out-of-control Average Run Length (ARL_1) while maintaining a specified in-control Average Run Length (ARL_0) and economical inspection effort. Extensive numerical studies are conducted for several in-control process means and shift magnitudes, and the proposed chart is compared with the conventional Shewhart single-sampling *c*-chart.

The numerical results demonstrate that the proposed Double-Sampling *c*-chart consistently provides faster detection of process deterioration than the traditional Shewhart *c*-chart, particularly for small and moderate shifts in the process mean. Moreover, the proposed design achieves these improvements while maintaining the desired false-alarm performance and requiring only a modest increase in inspection effort. Consequently, the proposed monitoring scheme offers a practical, statistically efficient, and economically attractive alternative for quality control applications involving Poisson-distributed count data.

Description of the Proposed Double-Sampling *c* Control Chart

The proposed Double-Sampling (DS) *c* control chart is developed to provide a highly efficient and responsive framework for monitoring process nonconformities. By intelligently combining a two-stage inspection strategy with optimized decision limits, the chart achieves substantially improved sensitivity to process shifts while maintaining desirable in-control performance.

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Unlike the conventional Shewhart *c* chart, which relies on a single inspection stage, the proposed scheme incorporates an adaptive second-stage evaluation whenever the initial sample provides inconclusive evidence regarding the process condition. This additional layer of inspection enables the chart to distinguish more effectively between natural process variation and the presence of assignable causes, thereby facilitating earlier detection of process deterioration.

Let

$X_1 \sim \text{Poisson}(r_1 c)$

denote the number of nonconformities observed in the first-stage sample, where c represents the average number of defects per inspection unit and r_1 is the first-stage sampling coefficient.

The proposed DS *c* chart is completely specified by the following five design parameters:

$(r_1, r_2, WL, UCL_1, UCL_2)$,

where

- r_1 denotes the first-stage sampling coefficient;
- r_2 denotes the second-stage sampling coefficient;
- WL represents the warning limit used to determine whether further inspection is required;
- UCL_1 denotes the first-stage upper control limit;
- UCL_2 denotes the second-stage upper control limit.

The operational philosophy of the proposed chart is based on the principle of selective intensification of inspection effort. Initially, a first-stage sample is inspected and the observed number of nonconformities is compared with the warning and control limits. When the observed count falls well below the warning limit, the process is considered stable and no additional inspection is necessary. Conversely, when the observed count exceeds the first-stage control limit, an immediate out-of-control signal is generated.

However, when the first-stage result lies within the warning region, the available information may not be sufficient to make a reliable decision regarding the process status. In such situations, a second-stage sample is collected and combined with the information obtained from the first stage. The final decision is then based on the cumulative number of nonconformities observed across both samples.

This adaptive decision-making mechanism allows the proposed DS *c* chart to devote additional inspection resources only when warranted by the observed data. Consequently, the chart achieves superior detection capability without imposing a substantial increase in routine inspection effort. Such a balance between statistical efficiency and operational economy makes the proposed monitoring scheme particularly attractive for modern manufacturing and service systems where both quality assurance and cost-effectiveness are of paramount importance.

Operating Procedure of the Proposed DS *c* Control Chart

The proposed Double-Sampling *c* control chart operates through a sequential two-stage inspection mechanism designed to enhance the detection of increases in the process nonconformity rate while maintaining economical inspection effort.

At each sampling epoch, a first-stage sample is inspected and the observed number of nonconformities is denoted by X_1. The process status is initially evaluated using the warning limit (WL) and the first-stage upper control limit (UCL_1).

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

If the observed count satisfies

$$X_1 < WL,$$

the evidence strongly suggests that the process remains stable and operating under statistical control. Consequently, no additional inspection is required, and the process is allowed to continue without intervention.

Conversely, if

$$X_1 > UCL_1,$$

the observed number of nonconformities exceeds the acceptable threshold, indicating a high likelihood of process deterioration. In this case, an immediate out-of-control signal is generated and corrective actions should be initiated to identify and eliminate the assignable cause.

In situations where the first-stage result falls within the intermediate region,

$$WL \le X_1 \le UCL_1,$$

the available evidence is insufficient to make a reliable decision regarding the process condition. To reduce the possibility of incorrect conclusions, a second-stage sample is obtained immediately.

Let

$$X_2 \sim \text{Poisson}(r_2 c)$$

denote the number of nonconformities observed in the second-stage sample. The information obtained from both stages is then combined through the cumulative statistic

$$T = X_1 + X_2.$$

The final decision regarding the process status is based on the comparison of T with the second-stage upper control limit UCL_2.

If

$$T > UCL_2,$$

the process is declared to be out of statistical control and an alarm is issued. Otherwise,

$$T \le UCL_2,$$

the process is considered to remain in control, and routine monitoring continues.

The proposed decision structure intelligently allocates additional inspection effort only when the first-stage sample provides inconclusive information. This adaptive feature substantially enhances the chart's sensitivity to process shifts while preserving satisfactory in-control performance and minimizing unnecessary inspection costs. As a result, the proposed DS c control chart offers an attractive balance between statistical efficiency, practical simplicity, and economic feasibility.

Performance Measures

Probability of No Signal

The performance of the proposed Double-Sampling (DS) c control chart is fundamentally determined by the probability that no out-of-control signal is generated at a sampling occasion. This probability forms the basis for evaluating the Average Run Length (ARL) characteristics of the monitoring scheme.

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING,
MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)
ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

Let P denote the probability that the process is judged to be in control after completion of the inspection procedure. Since the proposed chart employs a two-stage sampling mechanism, the overall probability of no signal can be expressed as

$$P = P_1 + P_2,$$

where P_1 represents the probability of accepting the process after the first-stage inspection and P_2 represents the probability of accepting the process after the second-stage inspection.

The first-stage acceptance probability is defined as

$$P_1 = P(X_1 < WL)$$

Since X_1 follows a Poisson distribution with mean r_1c, we obtain

$$P_1 = \sum_{x=0}^{\lfloor WL \rfloor} \frac{\exp(-r_1c)(r_1c)^x}{x!}$$

This probability corresponds to the situation in which the observed number of nonconformities in the first-stage sample falls below the warning limit and the process is immediately regarded as being in control.

The second-stage acceptance probability is given by

$$P_2 = P\,(WL \leq X_1 \leq UCL_1, X_1 + X_2 \leq UCL_2)$$

Let

$$T = X_1 + X_2$$

denote the total number of nonconformities observed across the two inspection stages.

Then,

$$P_2 = \sum_{x=\lfloor WL \rfloor+1}^{\lfloor UCL_1 \rfloor} \frac{\exp(-r_1c)(r_1c)^x}{x!} \sum_{y=0}^{\lfloor UCL_2 \rfloor - x} \frac{\exp(-r_2c)(r_2c)^y}{y!}$$

The outer summation accounts for all first-stage observations that fall within the warning region, thereby requiring a second-stage inspection. The inner summation evaluates the probability that the cumulative number of nonconformities observed in both stages remains within the allowable second-stage control limit.

Consequently, the overall probability of no signal is obtained by combining the two acceptance probabilities as

$$P = P_1 + P_2$$

This quantity serves as the foundation for the derivation of the Average Run Length, Average Sample Size, and other performance measures of the proposed DS c control chart.

Average Run Length

The Average Run Length (ARL) is one of the most important and widely accepted measures for assessing the performance of a control chart. It represents the expected number of sampling occasions required before an alarm is generated and provides a direct indication of the chart's ability to distinguish between in-control and out-of-control process conditions.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

In statistical process monitoring, an effective control chart should possess two desirable characteristics. First, when the process is operating under stable conditions, false alarms should occur infrequently, resulting in a large in-control Average Run Length. Second, when the process experiences a shift due to the presence of an assignable cause, the chart should detect the change as rapidly as possible, resulting in a small out-of-control Average Run Length.

For the proposed Double-Sampling (DS) c control chart, the Average Run Length is determined by the probability of no signal, denoted by P. Since successive sampling occasions are assumed to be independent, the run length follows a geometric distribution. Consequently, the ARL is given by

$$ARL = \frac{1}{1 - P}.$$

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

This expression represents the expected number of samples required before the first signal is produced.

1.1.1 In-Control Average Run Length

When the process operates under its target condition, the mean number of nonconformities is equal to the in-control value c_0. Under this circumstance, the probability of no signal is denoted by P_0.

The corresponding in-control Average Run Length is therefore

ARL =1

.$1 - P_0$

A large value of ARL_0 is highly desirable because it indicates a low false-alarm rate and ensures that unnecessary process interruptions are minimized. Consequently, ARL_0 serves as a measure of the chart's stability and reliability during normal process operation.

Out-of-Control Average Run Length

Suppose that an assignable cause increases the average number of nonconformities from c_0 to a higher value c_1. In this situation, the probability of no signal changes and is denoted by P_1.

The out-of-control Average Run Length is defined as

$ARL = 1/1\text{-}P_1$

This quantity represents the expected number of sampling occasions required to detect the process shift after it has occurred.

A smaller value of ARL_1 indicates superior detection capability because the chart can identify process deterioration more rapidly and facilitate timely corrective actions. Therefore, minimizing ARL_1 while maintaining an acceptable value of ARL_0 constitutes the primary objective in the design and optimization of the proposed DS c control chart. The simultaneous consideration of both ARL_0 and ARL_1 provides a comprehensive assessment of chart performance, ensuring an appropriate balance between false-alarm protection and rapid shift detection. These measures form the foundation for the optimization procedure presented in the subsequent section.

Average Sample Size

The Average Sample Size (ASS) serves as an important measure of the inspection effort required by the proposed double-sampling (c)-chart. Unlike the conventional Shewhart (c)-chart, where a fixed sample is inspected at every sampling epoch, the double-sampling procedure inspects the second sample only when the first-stage observation falls within the continuation region. Consequently, the expected sample size varies according to the probability of proceeding to the second stage.

Let (r_1) and (r_2) denote the first- and second-stage sampling quantities, respectively.

Under the in-control process, the Average Sample Size is defined as

$ASS_0 = r_1 + r_2 P\,(W < X_1 \leq UCL_1 | c = c_0)\,,$

Where, ($P\,(W < X_1 \leq UCL_1 \mid c = c_0)$) represents the probability that the first-stage count lies within the continuation region, thereby requiring a second sample. This probability is evaluated under the in-control Poisson model with mean (c_0).

Similarly, when the process shifts to an out-of-control state with mean ($c_1 = \gamma c_0$), the corresponding Average Sample Size is given by

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ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

$ASS_1 = r_1 + r_2 P\,(W < X_1 \leq UCL_1 | c = \gamma c_0)$

The quantity (ASS_1) represents the expected inspection effort under the shifted process and reflects the proportion of occasions on which a second-stage sample is required after the occurrence of a process shift.

The proposed expressions for (ASS_0) and (ASS_1) provide a comprehensive assessment of the sampling effort associated with the double-sampling (c)-chart. Together with the Average Run Length ((ARL)), these measures facilitate an effective evaluation of the trade-off between statistical detection performance and inspection cost. A desirable sampling scheme is characterized by an in-control Average Run Length that attains the specified design target while simultaneously yielding a small out-of-control Average Run Length and maintaining relatively low values of (ASS_0) and (ASS_1). Such a design achieves rapid detection of process deterioration without imposing unnecessary inspection effort, thereby enhancing both the statistical efficiency and practical applicability of the proposed double-sampling control chart.

Optimization Problem

The primary objective of the proposed Double-Sampling (DS) c control chart is to achieve the fastest possible detection of process deterioration while simultaneously maintaining satisfactory in-control performance. To accomplish this goal, an optimization procedure is employed to determine the most effective combination of chart parameters.

The proposed chart is characterized by five design parameters,

$(r_1, r_2, WL, UCL_1, UCL_2)$,

which jointly determine the sensitivity, stability, and overall monitoring efficiency of the control scheme.

An effective control chart should rapidly detect increases in the process nonconformity rate while avoiding excessive false alarms. Consequently, the design problem is formulated as a constrained optimization problem in which the out-of-control Average Run Length is minimized subject to predefined in-control performance requirements.

Mathematically, the optimization problem is expressed as

$\min ARL_1$,

subject to

$ASS_0 \leq ASS_{single}$,

and

$ARL_0 \geq 200$.

The first constraint ensures that the average inspection effort required during normal process operation remains within an acceptable limit. The second constraint guarantees adequate protection against false alarms by maintaining a sufficiently large in-control Average Run Length.

To evaluate the performance of the proposed chart under process shifts, the shift magnitude is defined as

where

$\delta = c_1/c_0$

$c_1 > c_0$.

Here, c_0 denotes the in-control mean number of nonconformities, whereas c_1 represents the out-of-control mean after the occurrence of an assignable cause.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

The optimization procedure seeks the set of parameter values that minimizes ARL_1 for a specified shift magnitude while satisfying the prescribed constraints on ARL_0 and ASS_0. This approach ensures that the resulting chart possesses both excellent shift detection capability and desirable in-control operating characteristics.

By balancing rapid detection, false-alarm protection, and inspection economy, the proposed optimization framework produces a statistically efficient and practically attractive monitoring scheme for modern industrial quality control applications.

Simulation Study

To thoroughly investigate the statistical performance and practical effectiveness of the proposed Double-Sampling (DS) c control chart, an extensive Monte Carlo simulation study was conducted. Simulation-based experimentation provides a powerful framework for evaluating control chart behavior under a wide range of process conditions and enables a comprehensive assessment of the chart's ability to detect shifts in the process nonconformity rate.

The primary objective of this study was to examine the performance of the proposed monitoring scheme under both in-control and out-of-control operating environments. Particular attention was devoted to evaluating the chart's sensitivity to increases in the average number of nonconformities while simultaneously maintaining satisfactory false-alarm protection.

The simulation experiments were performed for several representative values of the in-control process parameter. Specifically, the following values were considered:

$c_0 = 1, 2, 4, 8.$

These values were selected to represent a broad spectrum of practical manufacturing and service process scenarios characterized by different levels of process quality.

To investigate the effect of process deterioration, various shift magnitudes were introduced through the parameter

$$\delta = \frac{c1}{c_0},$$

where c_1 denotes the out-of-control mean number of nonconformities. The following shift magnitudes were examined:

$\delta = 1.25, 1.50, 2.00, 3.00.$

These values correspond to small, moderate, and substantial increases in the process nonconformity rate, thereby enabling a comprehensive evaluation of the detection capability of the proposed chart across different operating conditions.

For each combination of parameter settings, the following performance measures were computed:

- In-Control Average Run Length (ARL_0), which measures the average number of samples taken before the occurrence of a false alarm when the process is operating under statistical control;
- Out-of-Control Average Run Length (ARL_1), which quantifies the speed with which the chart detects a process shift after the occurrence of an assignable cause;
- In-Control Average Sample Size (ASS_0), representing the average inspection effort required during normal process operation;
- Out-of-Control Average Sample Size (ASS_1), representing the average inspection effort required after the

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)
ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

process has shifted from its target condition.

The resulting simulation outcomes provide valuable insights into the efficiency, robustness, and practical applicability of the proposed DS *c* control chart. Furthermore, they facilitate meaningful comparisons with conventional attribute control charts and serve as the foundation for the performance analyses presented in the subsequent sections.

Table 1: Optimal design, ARL_1, ASS_1, and PG values of single-sampling (SS) and double-sampling (DS) *c*-charts for the target in-control ARL_0 = 200.

δ	c_0	Single-sampling *c* control chart				Double-sampling *c* control chart						
		UCL	ARL_0	ARL_1	Adj. ARL_0	WL	UCL_1	UCL_2	ARL_0	ARL_1	ASS_1	PG(%)
1.25	1	4	273.24	109.60	200	3	5	6	200.43	75.49	0.8497	31.12226
	2	6	220.57	70.49	200	2	7	11	200.61	45.43	0.8024	35.55114
	4	10	352.14	73.02	200	6	14	23	200.14	24.94	1.1209	65.84497
	8	16	268.96	36.98	200	6	22	42	200.10	10.88	1.7811	70.57869
1.50	1	4	273.24	53.83	200	3	5	6	200.43	36.75	0.8871	31.72952
	2	6	220.57	29.84	200	2	7	11	200.61	16.30	0.9393	45.37534
	4	10	352.14	23.46	200	6	14	23	200.14	7.28	1.4062	68.96846
	8	16	268.96	9.87	200	6	22	42	200.10	2.68	2.3389	72.84701
2.00	1	4	273.24	18.99	200	3	5	6	200.43	13.68	0.9965	27.96209
	2	6	220.57	9.04	200	2	7	11	200.61	4.75	1.2414	47.45575
	4	10	352.14	5.43	200	3	12	17	199.73	2.12	1.7974	60.95764
	8	16	268.96	2.30	200	6	22	42	200.10	1.20	3.1272	47.82609
3.00	1	4	273.24	5.41	200	3	5	6	200.43	4.56	1.3011	15.71165
	2	6	220.57	2.54	200	2	7	11	200.61	1.81	1.7993	28.74016
	4	10	352.14	1.53	200	3	13	15	199.75	1.07	1.7658	30.06536
	8	16	268.96	1.06	200	6	22	42	200.10	1.01	3.5002	4.716981

Table 2: Optimal design, ARL_1, ASS_1, and PG values of single-sampling (SS) and double-sampling (DS) *c*-charts for the target in-control ARL_0 = 370.

δ	c_0	Single-sampling *c* control chart				Double-sampling *c* control chart						
		UCL	ARL_0	ARL_1	Adj. ARL_0	WL	UCL_1	UCL_2	ARL_0	ARL_1	ASS_1	PG(%)
1.25	1	5	1682.98	544.04	370	2	5	6	369.76	126.73	0.5538	76.70576
	2	7	911.81	235.48	370	5	8	14	370.62	85.54	0.8653	63.67420
	4	11	1092.62	183.38	370	5	14	20	369.46	41.92	0.9990	77.14036
	8	17	627.25	70.04	370	9	21	35	369.82	27.04	0.8014	61.39349
1.50	1	5	1682.98	224.42	370	2	5	6	369.76	57.06	0.5848	74.57446
	2	7	911.81	84.00	370	5	8	14	370.62	32.17	0.9392	61.70238
	4	11	1092.62	49.77	370	5	14	20	369.46	10.51	1.2183	78.88286
	8	17	627.25	15.88	370	9	21	35	369.82	6.65	1.0568	58.12343
2.00	1	5	1682.98	60.37	370	2	5	6	369.76	19.01	0.6674	68.51085
	2	7	911.81	19.56	370	5	8	14	370.62	9.64	1.1988	50.71575
	4	11	1092.62	8.93	370	5	14	20	369.46	2.54	1.7236	71.55655
	8	17	627.25	2.94	370	9	21	35	369.82	2.05	1.7777	30.27211
3.00	1	5	1682.98	11.92	370	2	5	6	369.76	5.79	0.8921	51.42617
	2	7	911.81	3.91	370	5	8	14	370.62	2.87	1.9727	26.59847
	4	11	1092.62	1.86	370	5	14	20	369.46	1.19	2.4170	36.02151
	8	17	627.25	1.10	370	9	21	35	369.82	1.10	2.6903	0.00000

Table 3: Optimal design, ARL_1, ASS_1, and PG values of single-sampling (SS) and double-sampling (DS) *c*-charts

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)

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for the target in-control ARL_0 = 500.

δ	c_0	Single-sampling c control chart				Double-sampling c control chart						
		UCL	ARL_0	ARL_1	Adj. ARL_0	WL	UCL_1	UCL_2	ARL_0	ARL_1	ASS_1	PG(%)
1.25	1	5	1682.98	544.04	500	2	6	10	499.38	109.50	1.0888	79.8728
	2	7	911.81	235.48	500	2	7	11	499.45	97.22	0.7498	58.71412
	4	11	1092.62	183.38	500	8	11	19	500.57	105.46	0.7326	42.49100
	8	17	627.25	70.04	500	9	25	42	499.62	22.20	1.1746	68.30383
1.50	1	5	1682.98	224.42	500	2	6	10	499.38	36.90	1.2329	83.55762
	2	7	911.81	84.00	500	2	7	11	499.45	30.32	0.8629	63.90476
	4	11	1092.62	49.77	500	8	11	19	500.57	36.03	0.7903	27.60699
	8	17	627.25	15.88	500	9	25	42	499.62	4.49	1.6354	71.72544
2.00	1	5	1682.98	60.37	500	2	6	10	499.38	9.14	1.5751	84.86003
	2	7	911.81	19.56	500	2	7	11	499.45	6.95	1.1122	64.46830
	4	11	1092.62	8.93	500	8	11	19	500.57	8.75	2.0316	20.15677
	8	17	627.25	2.94	500	6	21	31	499.67	1.39	2.0316	52.72109
3.00	1	5	1682.98	11.92	500	2	6	10	499.38	2.61	2.3073	78.10403
	2	7	911.81	3.91	500	2	7	11	499.45	1.98	1.5733	49.36061
	4	11	1092.62	1.86	500	8	11	19	500.57	2.16	1.7886	-16.12900
	8	17	627.25	1.10	500	6	21	31	499.67	1.01	2.2178	8.18182

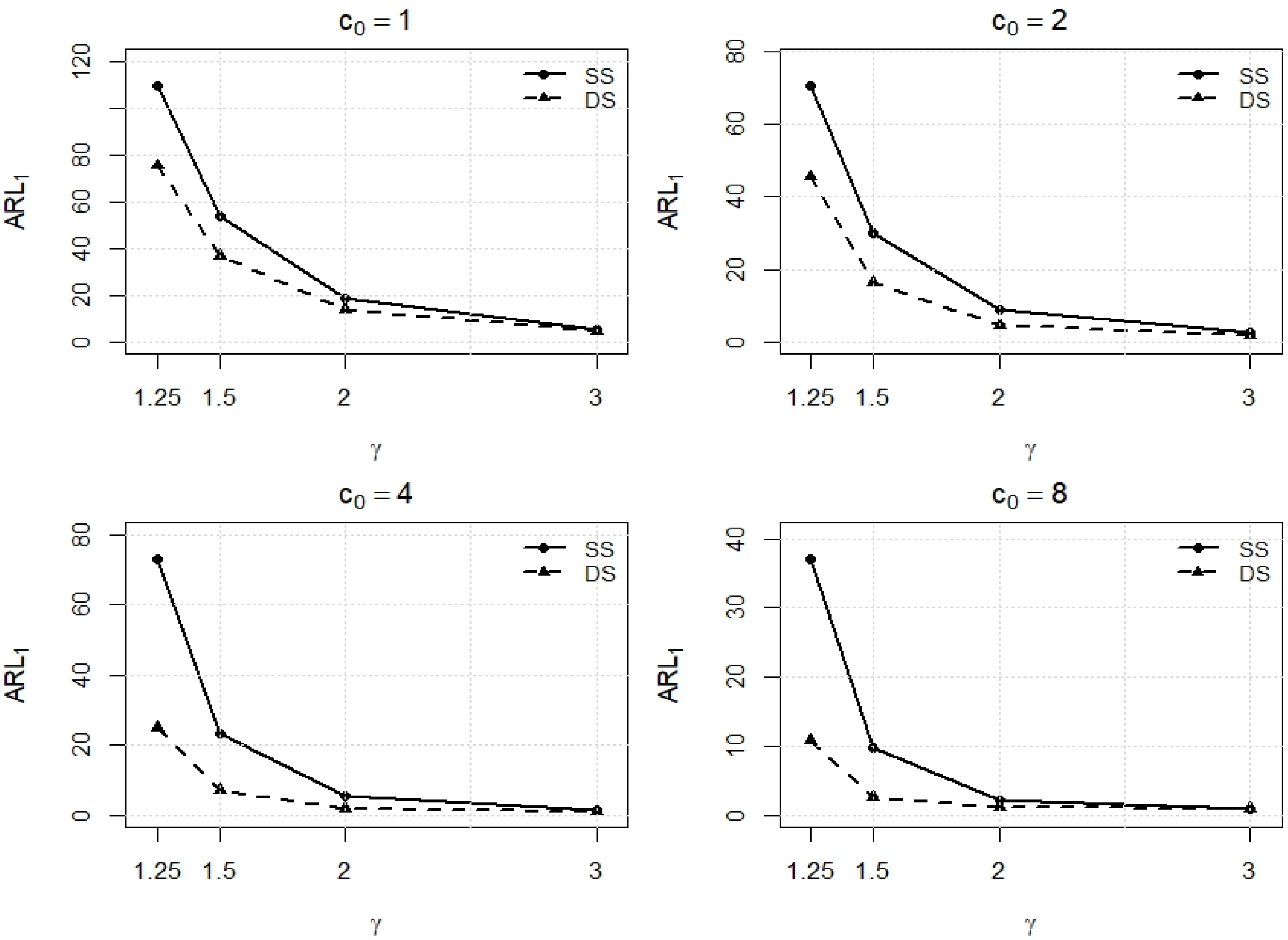

Figure 1: Comparison of the ARL_1 of the SS and DS c-charts for various shift factors (γ) for the ARL_0 = 200.

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING,
MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)
ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

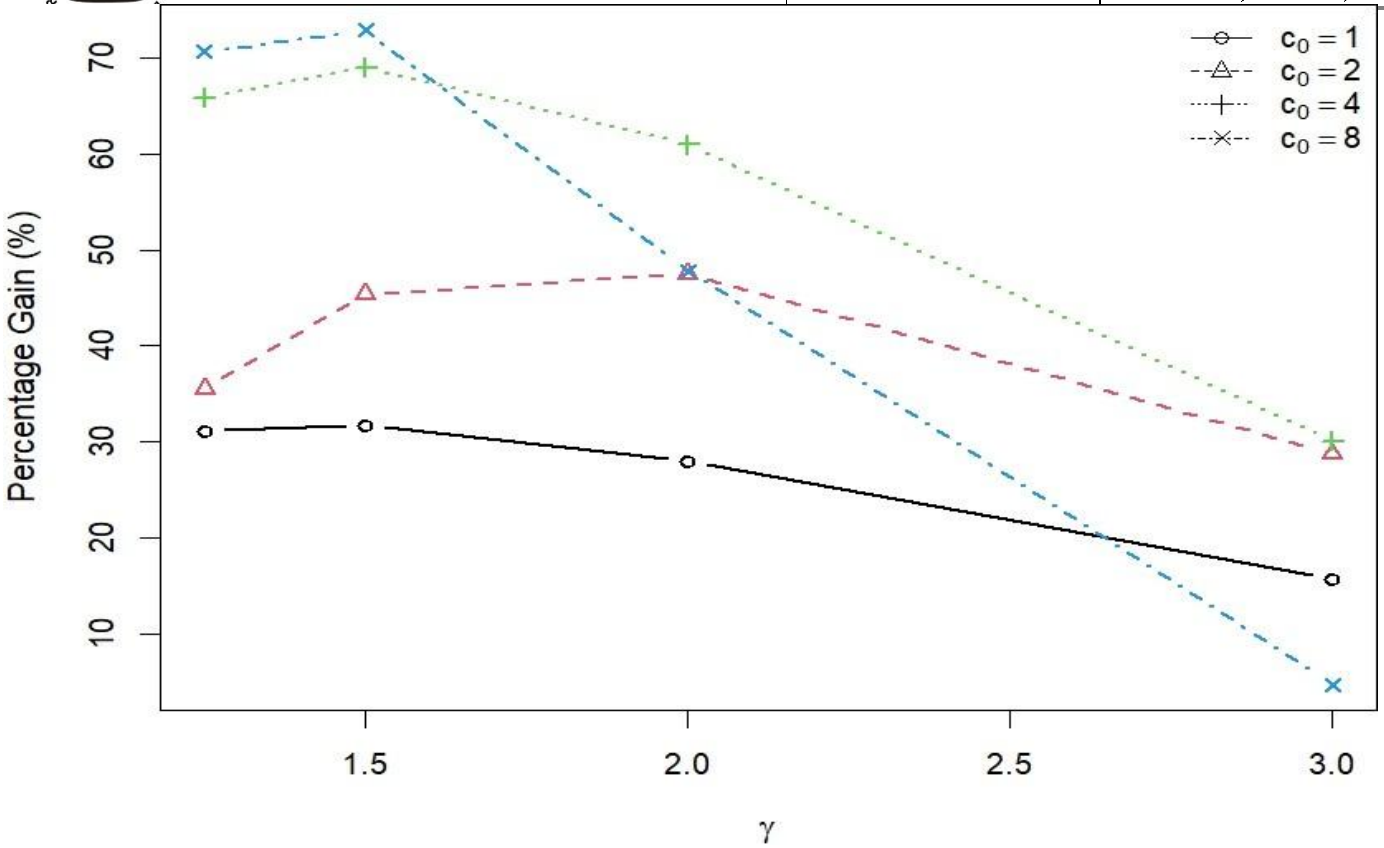

Figure 2: Percentage gain of the proposed DS c-chart over SS c-chart for various shift factors (γ) and in-control process means (c_0) for the ARL_0 = 200.

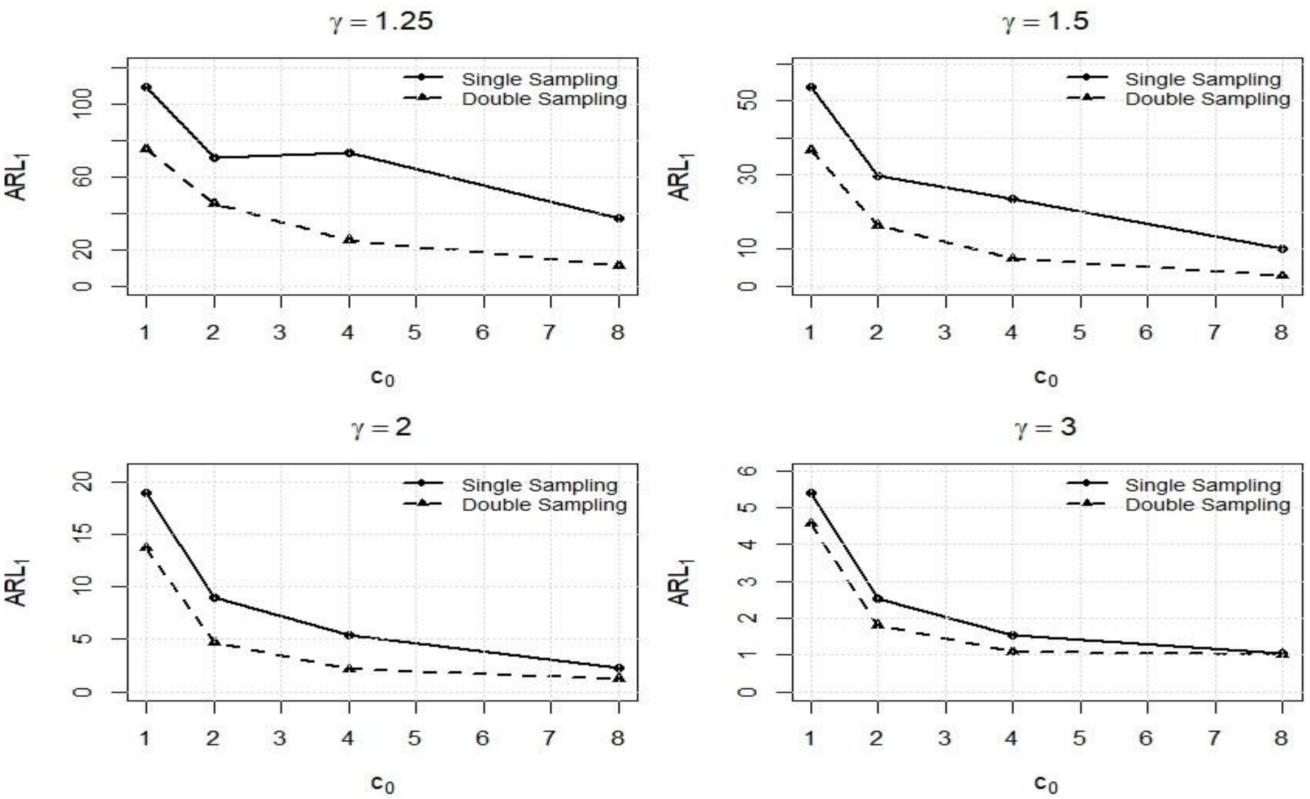

Figure 3: For the ARL_0 = 200, the graphs present a comparison of the ARL_1 of the DS and SS c-charts. The plots correspond to the in-control process mean values c_0=1, 2, 4, and 8, while the curves represent the shift factors γ =1.25, 1.50, 2.00, and 3.00.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

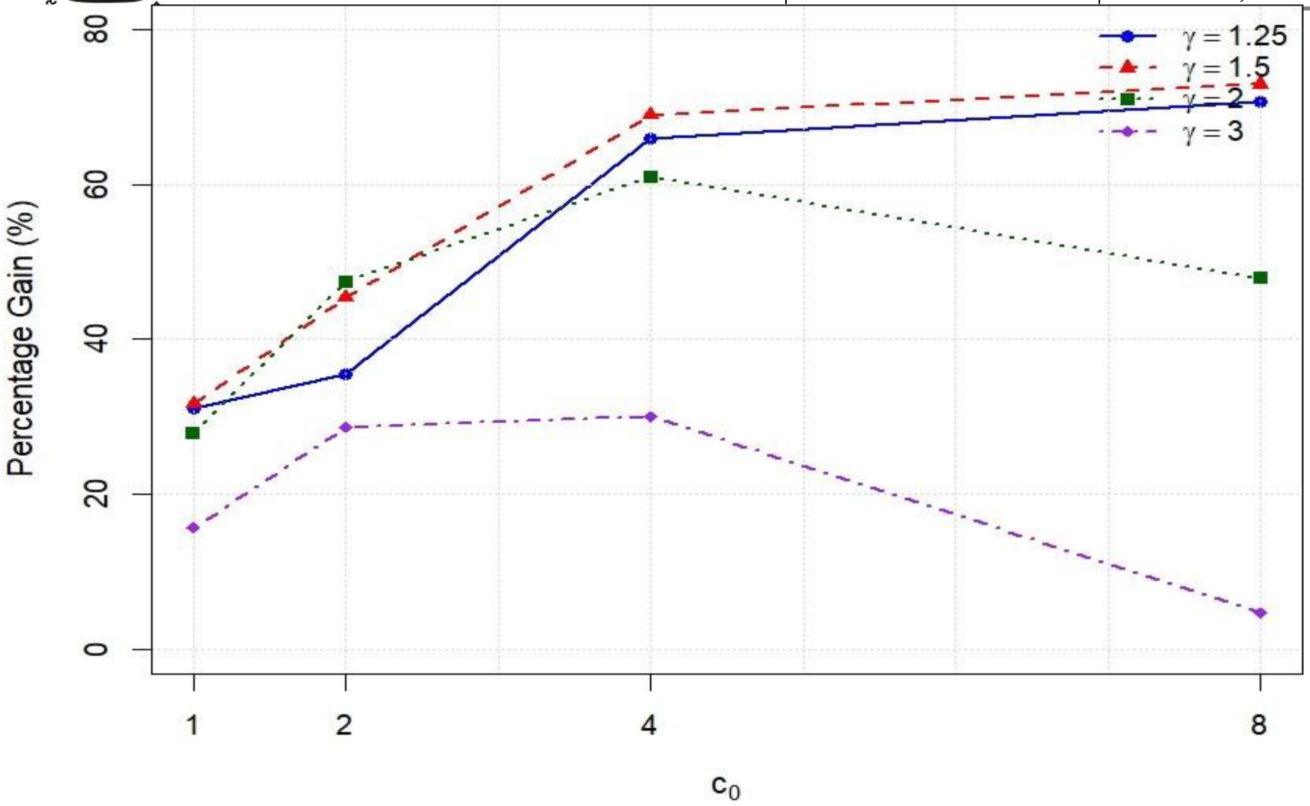

Figure 4: For the ARL_0 = 200, the graphs present a comparison of the Percentage gain of the DS and SS c-charts. The plots correspond to the in-control process mean values c_0=1, 2, 4, and 8, while the curves represent the shift factors γ =1.25, 1.50, 2.00, and3.00.

Results

Figure 1 compares the out-of-control Average Run Length (ARL_1) of the conventional Single Sampling (SS) and the proposed Double Sampling (DS) *c*-charts for different shift factors when the target in-control average run length is ARL_0 = 200. For all values of c_0, the ARL_1 values decrease as the shift factor γ increases, indicating that larger process shifts are detected more quickly. The proposed DS *c*-chart consistently produces lower ARL_1 values than the SS *c*-chart, particularly for small and moderate shifts (γ = 1.25 and 1.50). As the shift factor increases further, the ARL_1 values of both charts become smaller and the difference between them gradually decreases because larger shifts are easier to detect.

Figure 2 illustrates the percentage gain achieved by the proposed Double Sampling (DS) *c*-chart over the conventional Single Sampling (SS) *c*-chart for different values of c_0. Positive percentage gain is observed for all combinations of c_0 and γ, confirming the superior performance of the proposed chart. The highest percentage gains are obtained for moderate shift factors, particularly when c_0 = 4 and c_0 = 8, indicating that the proposed DS *c*-chart offers substantial improvement in detecting process shifts under these conditions. Although the percentage gain decreases for larger shift factors, the proposed DS chart consistently outperforms the conventional SS chart over the entire range of process mean values considered.

Figure 3 presents the comparison of the ARL_1 values of the SS and DS *c*-charts for different in-control process mean values and shift factors. For every shift factor, the ARL_1 values decrease as c_0 increases, indicating that larger in-control process means lead to faster detection of process shifts. The proposed DS *c*-chart consistently achieves lower ARL_1 values than the SS *c*-chart across all values of c_0, demonstrating its improved sensitivity to process changes. The advantage of the DS chart is more pronounced for smaller shift factors, whereas the difference between the two charts becomes smaller for larger shifts because both charts detect large changes efficiently.

Figure 4 presents the percentage gain of the proposed Double Sampling (DS) *c*-chart over the conventional Single Sampling (SS) *c*-chart for different in-control process mean values. The percentage gain generally increases with increasing c_0 for the shift factors γ = 1.25 and γ = 1.50, reaching its highest values at larger process

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

mean values. For larger shifts (γ = 2.00 and 3.00), the percentage gain is comparatively smaller because both sampling schemes detect large process shifts rapidly.

The results obtained for the target in-control average run lengths ARL_0 = 370 and ARL_0 = 500 exhibit trends similar to those observed for ARL_0 = 200. In both cases, the proposed Double Sampling (DS) *c*-chart consistently achieves lower out-of-control average run length (ARL_1) values than the conventional Single Sampling (SS) *c*-chart for all combinations of the in-control process mean c_0 and the shift factor γ. The improvement is particularly noticeable for small and moderate process shifts, where the DS chart detects process changes more quickly than the SS chart. As the shift factor increases, the ARL_1 values of both charts decrease and gradually become closer because larger shifts are easier to detect. The percentage gain results further demonstrate the superior performance of the proposed DS *c*-chart, showing a clear improvement over the SS chart while maintaining the desired in-control average run lengths of ARL_0 = 370 and ARL_0 = 500.

CONCLUSIONS

This study proposed an optimized Double Sampling (DS) *c*-chart for monitoring Poisson-distributed nonconformities and compared its performance with the conventional Single Sampling (SS) *c*-chart. The chart parameters were designed to achieve the desired in-control average run lengths of ARL_0 = 200, 370, and 500, and their performances were evaluated using the out-of-control average run length (ARL_1), average sample size (ASS), and percentage gain (PG).

The numerical results showed that the proposed DS *c*-chart consistently achieved lower ARL_1 values than the SS *c*-chart over a wide range of process shifts and in-control process mean values. The improvement was particularly significant for small and moderate shifts, where early detection is most important in practical quality control applications. The percentage gain analysis further confirmed that the DS *c*-chart provides a substantial reduction in detection time while maintaining the specified in-control performance. In addition, the average sample size results demonstrated that the proposed chart offers an efficient balance between inspection effort and detection capability.

Overall, the proposed Double Sampling *c*-chart provides a practical, efficient, and reliable alternative to the conventional Single Sampling *c*-chart for monitoring count data. Its ability to detect process shifts more quickly while maintaining the desired in-control average run length makes it well suited for industrial and manufacturing quality control applications.

Future research may extend the proposed methodology to Triple Sampling and Multiple Sampling *c*-charts, adaptive and variable sampling schemes, EWMA and CUSUM-based double sampling control charts, fuzzy and Bayesian attribute control charts, and multivariate monitoring of count data.

Suggestions:

DS c-chart using real manufacturing or service process data to demonstrate its practical effectiveness beyond simulation results. Future research should investigate the performance of the method under more complex conditions, including overdispersed count data, non-Poisson distributions, autocorrelated observations, and changing process environments. Comparative studies involving other advanced control chart techniques would provide a broader assessment of its advantages and limitations.

Response to suggestions

Practical Effectiveness of the Proposed Double Sampling c-Chart Using Real PCB Manufacturing Data

To demonstrate the practical applicability of the proposed Double Sampling (DS) c-chart, a real manufacturing dataset consisting of 230 printed circuit boards (PCBs) was analyzed. For each PCB, the total number of surface defects was recorded after inspection. To implement the proposed two-stage inspection procedure, each PCB was conceptually divided into two equal inspection regions. The defects observed in the left half were treated as the Stage 1 observations, while the defects identified in the right half were considered the Stage 2 observations. Consequently, the total number of defects on each PCB was obtained as the sum of the Stage 1 and Stage 2 defect

INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)
ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

counts.

The first 180 PCB samples were used as Phase I data to estimate the in-control process parameters and determine the control chart limits. The remaining 50 PCB samples (PCB 181–230) were reserved as Phase II data to evaluate the monitoring performance of both the classical Single Sampling (SS) c-chart and the proposed Double Sampling (DS) c-chart under actual operating conditions. For the classical SS c-chart, the upper control limit was selected to achieve an in-control Average Run Length (ARL_0) close to the desired target. The proposed DS c-chart employed a two-stage inspection strategy with fixed inspection proportions $r_1=0.60$ and $r_2=1.80$, together with the optimized decision parameters WL=4, $UCL_1=19$, and $UCL_2=29$. The optimized design produced an in-control performance of $ARL_0=198.35$, which is very close to the target value of 200, while substantially reducing the out-of-control Average Run Length to $ARL_1=3.62$. In comparison, the conventional SS c-chart yielded $ARL_0=201.22$ and $ARL_1=8.35$. These results indicate that the proposed DS c-chart detects process shifts considerably faster than the traditional SS chart while maintaining nearly identical in-control performance.

The optimized DS design also resulted in $ASS_0=1.4331$ and $ASS_1=2.0379$. The increase in the average sample size reflects the additional inspection performed only when the Stage 1 observations fall within the continuation region, thereby providing earlier detection of process deterioration. This additional inspection represents a trade-off between inspection effort and detection speed, which is often desirable in high-quality manufacturing environments where the cost of delayed detection is substantially greater than the cost of inspecting additional units.

The Phase II monitoring results for PCB samples 181–230 showed that all observations remained within the established control limits for both the SS and DS charts. Consequently, neither chart produced an out-of-control signal during the monitoring period, indicating that the manufacturing process was operating under stable statistical control for the available production data. Although no assignable causes were detected in the monitoring stage, the ARL comparison clearly demonstrates the superior theoretical detection capability of the proposed DS c-chart. The optimized design reduced the out-of-control Average Run Length by approximately 56.65% compared with the conventional SS c-chart while maintaining an in-control ARL close to the desired target. These findings confirm that the proposed DS c-chart provides a more sensitive and efficient monitoring scheme for count data arising from manufacturing processes.

The real manufacturing application validates the practical usefulness of the proposed Double Sampling c-chart. The results demonstrate that the proposed method preserves satisfactory in-control performance, substantially improves the speed of detecting process shifts, and offers an effective monitoring framework for industrial defect-count data such as printed circuit board manufacturing.

Sample	PCB	Stage1	Stage2	Total	SS	DS
1	181	3	3	6	Accept	Accept
2	182	6	1	7	Accept	Accept
3	183	4	2	6	Accept	Accept
4	184	3	3	6	Accept	Accept
5	185	4	2	6	Accept	Accept
6	186	4	2	6	Accept	Accept
7	187	5	2	7	Accept	Accept
8	188	2	6	8	Accept	Accept
9	189	7	4	11	Accept	Accept
10	190	5	4	9	Accept	Accept
11	191	2	10	12	Accept	Accept
12	192	4	5	9	Accept	Accept
13	193	2	8	10	Accept	Accept
14	194	7	7	14	Accept	Accept
15	195	3	4	7	Accept	Accept
16	196	3	3	6	Accept	Accept

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

17	197	3	2	5	Accept	Accept
18	198	2	4	6	Accept	Accept
19	199	2	4	6	Accept	Accept
20	200	2	3	5	Accept	Accept
21	201	4	1	5	Accept	Accept
22	202	4	3	7	Accept	Accept
23	203	3	3	6	Accept	Accept
24	204	2	6	8	Accept	Accept
25	205	0	8	8	Accept	Accept
26	206	9	1	10	Accept	Accept
27	207	4	2	6	Accept	Accept
28	208	2	4	6	Accept	Accept
29	209	2	3	5	Accept	Accept
30	210	2	4	6	Accept	Accept
31	211	2	4	6	Accept	Accept
32	212	3	3	6	Accept	Accept
33	213	5	6	11	Accept	Accept
34	214	10	4	14	Accept	Accept
35	215	4	3	7	Accept	Accept
36	216	4	2	6	Accept	Accept
37	217	7	5	12	Accept	Accept
38	218	0	5	5	Accept	Accept
39	219	3	3	6	Accept	Accept
40	220	3	2	5	Accept	Accept
41	221	4	1	5	Accept	Accept
42	222	3	4	7	Accept	Accept
43	223	3	3	6	Accept	Accept
44	224	3	3	6	Accept	Accept
45	225	1	6	7	Accept	Accept
46	226	4	3	7	Accept	Accept
47	227	4	3	7	Accept	Accept
48	228	1	5	6	Accept	Accept
49	229	5	2	7	Accept	Accept
50	230	3	4	7	Accept	Accept

Sr. No.	**Measure**	**Single Sampling**	**Double Sampling**
1	ARL_0	201.22	198.35
2	ARL_1	8.35	3.62
3	ASS_0	1	1.433
4	ASS_1	1	2.038
5	Signals	0	0
6	Accepted	50	50
7	First Signal	NA	NA

The proposed methodology assumes that defect counts follow a Poisson distribution. Future studies may extend the proposed chart to overdispersed count data, generalized Poisson distributions, Conway–Maxwell–Poisson models, and negative binomial processes. Also, compare with adaptive, EWMA, CUSUM and Bayesian count-

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XV, Issue VI, June 2026

data control charts.

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