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Scenario of THDE model in Bianchi type VI_0 Universe

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ABSTRACT

In this paper, we study the anisotropic Bianchi type VI_0 metric that consists of Tsallis holographic dark energy (THDE) and dark matter (DM). The hybrid expansion law for the average scale factor is used to calculate some cosmological parameters and achieve exact solutions to Einstein's field equations. These parameters geometrical and physical behaviours demonstrate how the universe eventually becomes homogeneous, flat, and isotropic. Analyzing the equation of state (EoS) parameter also reveals that our model acts like a lambda cold dark matter (Λ) scenario in the late universe.

Keywords: Bianchi-VI_0 type, Hybrid expansion laws, Tsallis holographic dark energy model.

INTRODUCTION

The universe is undergoing an accelerated growth period as indicated by type Ia supernovae [1], CMB radiation [2], and galaxy redshift surveys [3]. This has accelerated the expansion of a new kind of energy known as dark energy (DE) [4]. To investigate the correct nature of DE, numerous researchers have talked about various DE theories. The cosmological constant lambda [5], quintessence [6], tachyon [7], phantom [8], k-essence model [9], and chaplygin gas model [10] are just a few of the models.

Among other DE models, the holographic dark energy (HDE) model holds an important position. The holographic theory, which states that the number of degrees of freedom in a bounded system should be constrained and related to the area of the boundary, is the foundation of the model HDE. The energy density of the HDE is $\rho_{HD} = 3c^2 M_{pl}^2 L^{-2}$ [11] (M_{pl} Planck mass, L infrared (IR) cut-off radius and c numerical constant). Recently, a type of HDE model known as "Tsallis holographic dark energy (THDE)" [12] has been proposed to use Tsallis generalized entropy '$S_\delta = \gamma A^\delta$' where γ is a constant and δ is the parameter (non-additive) to explain the cosmological phenomena.

According to the holographic theory, which posits that a physical system's degrees of freedom should correspond with the boundary area rather than its volume [13] and must adhere to an infrared cut-off, Cohen, Kaplan, and Nelson [14] established an inequality $L^3\Lambda^3 \le S^{\frac{3}{4}}$. By integrating this with $S_\delta = \gamma A^\delta$, they derived $\Lambda^4 \le \left(\gamma(4\pi)^\delta\right)L^{-4+2\delta}$ where Λ^4 represents the vacuum's energy density. The energy density of THDE can be determined using this inequality as $\rho_T = DL^{2\delta-4}$ where D is an unknown variable [15]. When $\delta = 1$, THDE simplifies to the HDE model. By applying the Hubble horizon as the system's IR cut-off, we express the energy density of THDE as $\rho_T = DH^{4-2\delta}$.

Recently several scholars like Santhi and Sobhanbabu [16], Pandey et al. [17], Sadeghi et al. [18], Sharif et al. [19], Sarma [20, 21] explored THDE in different prospect. These works encouraged us to use the hybrid expansion approach to explore Bianchi type VI_0 metric stuffed with DM and THDE.

The following is a breakdown of the paper's structure: Section 2 contains the metric and field equation. Section 3 describes the THDE model's cosmological solutions and physical properties. In Section4, we sum up our findings.

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2. Bianchi type-VI_0 metric and field equations

For the metric functions $a_1(t)$, $a_2(t)$ and $a_3(t)$, the Bianchi type VI_0 metric is defined as

$$ds^2 = dt^2 - {a_1}^2 dx^2 - {a_2}^2 e^{2x} dy^2 - {a_3}^2 e^{-2x} dz^2 \tag{1}$$

We consider that the universe is stuffed with DM and THDE components.

Here the Einstein's field equations are

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\left(T_{\alpha\beta} + \bar{T}_{\alpha\beta}\right) \tag{2}$$

where $T_{\alpha\beta}$ and $\bar{T}_{\alpha\beta}$ are the tensors of matter energy and the THDE energy momentum respectively.

For the matter energy density (ρ_m), the tensor for matter energy momentum ($T_{\alpha\beta}$) is

$$T_{\alpha\beta} = diag[\rho_m, 0,0,0] \tag{3}$$

In addition, the tensor for THDE energy momentum is taken as

$$\begin{aligned}\bar{T}_{\alpha\beta} &= diag[\rho_T, -{p_T}^x, -{p_T}^y, -{p_T}^z] \\ &= diag[1, -{\omega_T}^x, -{\omega_T}^y, -{\omega_T}^z]\rho_T \\ &= diag[1, -\omega_T, -\omega_T, -\omega_T]\rho_T \end{aligned} \tag{4}$$

where ρ_T is the energy density of THDE, pressures and EoS parameters of THDE along *x* axis ${p_T}^x$ and ${\omega_T}^x = \omega_T$; along *y* axis ${p_T}^y$ and ${\omega_T}^y = \omega_T$; along *z* axis ${p_T}^z$ and ${\omega_T}^z = \omega_T$ respectively.

Using (1), (3), (4) in (2), we get the following equations

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{1}{a_1^2} = -\omega_T \rho_T \tag{5}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{1}{a_1^2} = -\omega_T \rho_T \tag{6}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{1}{a_1^2} = -\omega_T \rho_T \tag{7}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{1}{a_1^2} = \rho_m + \rho_T \tag{8}$$

$$\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2} = 0 \tag{9}$$

For the metric (1) scale factor, spatial volume, the mean generalised Hubble's parameters are expressed as

$$a = (a_1 a_2 a_3)^{\frac{1}{3}} \tag{10}$$

$$V = a^3 = a_1 a_2 a_3 \tag{11}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_x + H_y + H_z\right) \tag{12}$$

where $H_x = \frac{\dot{a}_1}{a_1}$, $H_y = \frac{\dot{a}_2}{a_2}$, $H_z = \frac{\dot{a}_3}{a_3}$ are the Hubble's parameters along three axes respectively.

Moreover, the parameter for deceleration (q), growth scalar(θ), shear scalar(σ^2), average anisotropy parameter(A_m) are described as

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$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{13}$$

$$\theta = 3H = \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{1}{2}\left(\frac{\dot{a}_1^2}{a_1^2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{\dot{a}_3^2}{a_3^2}\right) - \frac{\theta^2}{6} \tag{15}$$

$$A_m = \frac{1}{3}\sum_{\alpha=1}^{3}\left(\frac{\Delta H_\alpha}{H}\right)^2 \tag{16}$$

where $\Delta H_\alpha = H_\alpha - H\ (\alpha = x, y, z)$.

3. Cosmological solutions and physical properties of THDE model

From (9), we obtain

$$a_3 = m a_2\ (m = constant) \tag{17}$$

Using (17), the equations (5-8) now can be written as

$$2\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{1}{a_1^2} = -\omega_T\rho_T \tag{18}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{1}{a_1^2} = -\omega_T\rho_T \tag{19}$$

$$2\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} - \frac{1}{a_1^2} = \rho_m + \rho_T \tag{20}$$

Solving equations (18) and (19), after using equation (11) we get

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = \frac{c_1}{V} exp\left(-\int \frac{\frac{2}{a_1^2}}{\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1}}\ dt\right) \tag{21}$$

where c_1 is constant.

We use the condition

$$\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1} = \frac{2}{a_1^2} \tag{22}$$

which is presented by K. S. Adhav [22] to solve equation (21).

Using (22) in (21), we get

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = \frac{c_1}{V} e^{-t} \tag{23}$$

Also conservation law of energy $T^{\alpha\beta}_{;\beta} = 0$ gives the continuity equation as

$$\dot{\rho}_m + 3H\rho_m + \dot{\rho}_T + 3H(\rho_T + p_T) = 0 \tag{24}$$

Since the two fluids under consideration are non-mixing, the continuity equation (24) can be used differently for matter and THDE, therefore the continuity equation for matter becomes

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{25}$$

and the continuity equation for THDE is

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$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \tag{26}$$

The barotropic EoS is

$$\omega_T = \frac{p_T}{\rho_T} \tag{27}$$

To find the parameters a_1, a_2, ρ_m, ρ_T , p_T we consider two additional conditions which are given below.

(i) Density of THDE as

$$\rho_T = DH^{4-2\delta} \tag{28}$$

(ii) The scale factor 'a' as

$$a = t^i e^{jt} \tag{29}$$

where $i \geq 0$ and j≥ 0 are constants[23].

From (29) and (11), we obtain V of this model as

$$V = a^3 = t^{3i} e^{3jt} \tag{30}$$

Again Eq. (23) gives

$$\frac{a_1}{a_2} = c_2 exp\left[c_1 \int e^{-t(1+3j)} t^{-3i} dt\right] \tag{31}$$

Eq. (17), (30) and (31) yield

$$a_1 = c_2^{\frac{2}{3}} m^{-\frac{1}{3}} t^i e^{jt} \times exp\left[\frac{2c_1}{3} \int e^{-t(1+3j)} t^{-3i} dt\right] \tag{32}$$

$$a_2 = c_2^{-\frac{1}{3}} m^{-\frac{1}{3}} t^i e^{jt} \times exp\left[\frac{-c_1}{3} \int e^{-t(1+3j)} t^{-3i} dt\right] \tag{33}$$

$$a_3 = c_2^{-\frac{1}{3}} m^{\frac{2}{3}} t^i e^{jt} \times exp\left[\frac{-c_1}{3} \int e^{-t(1+3j)} t^{-3i} dt\right] \tag{34}$$

The directional Hubble's parameter, deceleration parameter, average anisotropy parameter, expansion scalar, shear scalar become

$$H_x = \frac{\dot{a}_1}{a_1} = \frac{i}{t} + j + \frac{2c_1}{3} e^{-t(1+3j)} t^{-3i} \tag{35}$$

$$H_y = H_Z = \frac{\dot{a}_2}{a_2} = \frac{\dot{a}_3}{a_3} = \frac{i}{t} + j - \frac{c_1}{3} e^{-t(1+3j)} t^{-3i} \tag{36}$$

$$H = \frac{i}{t} + j \tag{37}$$

$$q = -1 + (i + jt)^{-2} i \tag{38}$$

$$A_m = \frac{2}{9} {c_1}^2 t^{2-6i} e^{-2t(1+3j)} (i + jt)^{-2} \tag{39}$$

$$\theta = 3\left(\frac{i}{t} + j\right) \tag{40}$$

$$\sigma^2 = \frac{1}{3} {c_1}^2 e^{-2t(1+3j)} t^{-6i} \tag{41}$$

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From Eqs. (30), (37), (40) and (41), it is found that for $t = 0$, V (spatial volume) is zero and H, θ, σ are diverges and H, θ , σ are approaches to zero and $V \to \infty$ when $t \to \infty$. As a result, the universe starts with null volume and evolves at a rate of infinity.

From Eq. (38), we notice that $q < 0$ for $i(1-i) < 2ijt + j^2t^2$ and $q = -1$ for $i = 0$. It is observed that for $t = 0$, $q = \frac{1}{i} - 1$. In this case, $q < 0$ and $q > 0$ according to $i > 1$or < 1 , respectively. It can be seen from (39), that $A_m \to constant$ when $t \to \infty$, indicating that universe's evolution is anisotropic.

From Eq. (27), we get THDE EoS parameter as

$$\omega_T = -1 - \frac{2(\delta-2)i}{3(i+jt)^2} \tag{42}$$

It shows that $\omega_T \to -1$ when $t \to \infty$. As a result, it acts like ΛCDM model. The astrophysical data, particularly SNe Ia data [24], the three years WMAP data [25], the SDSS data [26], all show that the ΛCDM is the standard model for describing the universe's evolution.

Now, by using equations (18), (32), (33), (34) in (25), (26) we get

$$\rho_T = \frac{D(i+jt)^4\left(j+\frac{i}{t}\right)^{-2\delta}}{t^4} \tag{43}$$

$$\rho_m = c_3 t^{-3i} e^{-3jt} \ \ (c_3 = constant) \tag{44}$$

From these equations, we observe that energy densities of THDE, matter reduce with cosmic time. For this model, THDE density parameter Ω_T and matter density parameter Ω_m are expressed as

$$\Omega_T = \frac{\rho_T}{3H^2} = \frac{1}{3} D \left(\frac{i}{t} + j\right)^{2(1-\delta)} \tag{45}$$

and

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{1}{3} c_3 \left(\frac{i}{t} + j\right)^{-2} t^{-3i} e^{-3jt} \tag{46}$$

The total energy density (Ω) parameter is

$$\Omega = \Omega_T + \Omega_m$$

$$= \frac{1}{3}\left(\frac{i}{t} + j\right)^{-2} \left[c_3 t^{-3i} e^{-3jt} + D\left(\frac{i}{t} + j\right)^{4-2\delta}\right] \tag{47}$$

The Ω (*total energy density*) approaches to 1, as shown by (47).

CONCLUSIONS

Here, using the hybrid expansion rule in the framework of general relativity, we investigated the THDE in a Here, Bianchi type VI_0 universe. We looked at a number of cosmological factors in our model to explain the accelerated evolution of the universe. The q (deceleration parameter) is seen to represent universe's accelerated phase under certain condition, which is in strong agreement with observations.

It is observed that the energy densities of matter and THDE decrease with time (t). Furthermore, the total density tends to 1 as the universe gets older. Later on, our model becomes flat, isotropic, and spatially uniform. Additionally, THDE's EoS parameter(ω_T) tends to -1 in the future. Consequently, our model and the Λ CDM model are identical.

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