

Students' Errors in Solving Word Problems Involving Angles of Elevation and Depression and Performance Level in Grade 9 Trigonometry: Basis for Development of a Teacher-Made Material

Reñel M. Sabanal and Analyn A. Guevara

Saint Mary's University

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Abstract: This study aimed to identify and classify the errors committed by Grade 10 students of a public high school and private high school when solving word problems involving angles of elevation and depression and to determine their performance level in trigonometry. Employing sequential-explanatory and correlational designs along with qualitative and quantitative approaches, the study used Newman's Error Hierarchical Model to analyze the errors committed by the students which included reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors. Data were gathered using worksheet, summative test and interview. Results showed that students' reading errors were very low but very high in terms of comprehension error, transformation error, process skills error and encoding error. The summative test in trigonometry revealed that majority of the students performed poorly, with only a small percentage achieving fairly satisfactory or satisfactory results. Moreover, the findings revealed that there is a significant relationship between the errors committed by the students and their performance score in trigonometry. Based on the findings, a teacher-made material with corrective instructional activities which may be adopted by mathematics teachers in addressing errors in solving word problems in trigonometry was developed.

Keywords: Comprehension error, encoding error, Newman's error hierarchical model, process skills error, transformation error

I. Introduction

Mathematics is a vital medium for real-world application. Learning mathematics teaches us not just how to count but also how to enhance our logical, analytical, and systematic thinking skills and use them in everyday life (Sevgi & Arslan, 2020). Learners continue to study mathematics throughout college, with many specializing in statistics, applied mathematics, or theoretical mathematics.

The Philippine Mathematics Curriculum in the current K to 12 curriculum provides that different branches of mathematics are effectively delivered to students quarterly (Balagtas et al., 2019). One of these branches is trigonometry. Trigonometry covers the study of triangles, the relationships between their sides and angles, the functions of sine and cosine, tangent and cotangent, and secant and cosecant (Walsh et al., 2017). Trigonometry is a critical topic for students because it enables them to be prepared for higher mathematics like calculus (Hidayati, 2020). Its application extends to professional fields like architecture, engineering, cartography, and other advanced fields (Galarza, 2017).

The Third International mathematics and Science Study (TIMSS) 2019 data revealed that Filipino students' poor mathematical performance has placed the country in the 57th rank out of 58 countries worldwide. Another report revealed that students from Cagayan Valley in the Philippines, Region 2, perform poorly in mathematics where the average grade for Grade 6 pupils, Grade 10 and Grade 12 students are respectively 36.66%, 36.91%, and 31.02% based on the National Achievement Test (2019).

One method that can be utilized to evaluate and examine students' errors is Newman's Error Hierarchical Model which originated from a 1970s investigation of mathematical language difficulties which has 5 phases namely reading, comprehension, transformation, process skills, and encoding (Lestari et al., 2018). Reading errors stem from inadequate basic reading skills. Comprehension errors arise from misunderstanding the problem's requirements. Transformation errors occur when students struggle to translate problem information into mathematical models or select suitable problem-solving methods. Process skill errors involve incorrect procedures, leading to procedural, arithmetic, or incomplete solutions. Encoding errors entail correct problem solving but inaccurate or omitted answer presentation.

As of November 2016, information from the Department of Education (DepEd) revealed that the organization lacked 235 million instructional and other learning materials (Umil, 2017) putting educational process in danger. In particular, Maligalig et al. (2011) found that one prevalent concern in the country's current educational system is the lack of learning materials and necessary teaching resources in the mathematics classroom.

In this study, Newman's Error Hierarchical Model was utilized to determine the errors committed by students, the results of which was used to craft a teacher-made material that can efficiently help teachers handle student errors and effectively help students become better at solving word problems involving angles of elevation and depression. This study identifies common errors in math word problem solving, benefiting DepEd, administrators, teachers, students, and researchers.

Theoretical Framework

Newman's Error Hierarchical Model, integrated with a constructivist teaching approach based on Piaget's theory, offers insight into handling student errors and learning new concepts. This framework prioritizes error analysis to understand reasoning behind wrong responses, aiding teachers in identifying areas needing support. Learning is viewed as collaborative, fostering debate and teamwork through error analysis, promoting deeper understanding among students.

Conceptual and Analytical Framework

This study aimed to identify and classify students' committed errors in solving word problems involving angles of elevation and depression based on Newman's Error Hierarchical Model and to investigate their performance level in trigonometry.

Newman's Error Hierarchical Model has been shown to be a reliable model for error analysis and is used by mathematics teacher researchers. As indicated in the study of Nurmeidina and Rafidiyah (2019), trigonometry is a very important subject in mathematics. However, it is alarming to note that many students are underachievers in trigonometry. Figure 2 displays the research paradigm of the study.

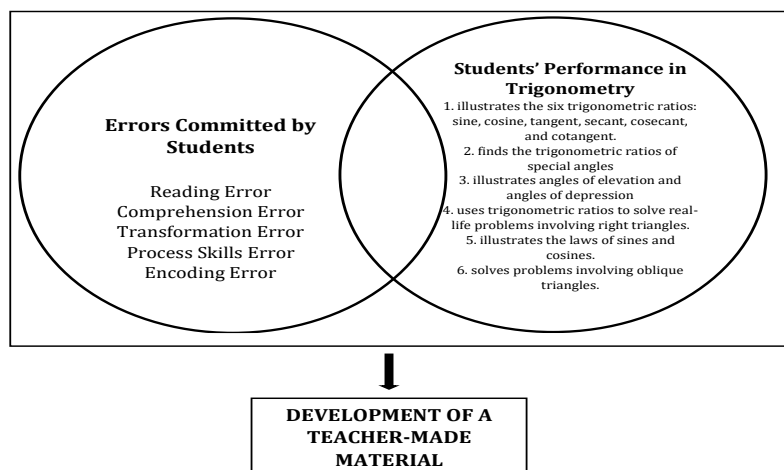


Figure 1. Research Paradigm

The study examines students' errors in trigonometry word problems on angles of elevation and depression. Figure 1 illustrates the relationship between these errors and trigonometry performance. Newman's Error Hierarchical Model guides error analysis, supplemented by triangulated data from worksheets and interviews to develop targeted teaching materials addressing common errors.

Statement of the Objectives

The study aimed to analyze the errors that Grade 10 students at Cagasat High School and Saint John Berchmans High School Incorporated for the SY 2023-2024 commit in solving word problems involving angles of elevation and depression. Specifically, the objectives of the study were the following:

1. Identify and classify the common errors of students in solving word problems involving angles of elevation and depression using Newman's Error Hierarchical Model;
2. Determine the level of performance of students in trigonometry for Grade 10;
3. Describe the relationship between the number of errors committed and performance of Grade 10 students; and
4. Develop teacher-made material based on the identified errors of the students in solving word problems involving angles of elevation and depression and their performance in trigonometry.

Statement of Null Hypothesis

1. There is no significant relationship between the errors committed and the performance of Grade 10 students.

II. Methodology

This study utilized sequential-explanatory and correlational designs along with qualitative and quantitative approaches. Quantitative analysis was employed to calculate the number of errors committed by the students per error indicator and to determine their performance level in trigonometry. The qualitative analysis was carried out by describing each student's errors when solving word problems based on Newman's Hierarchical Error Model. Moreover, it was used to identify the reason behind the errors committed by the students through the use of interviews. Lastly, a correlational study was done to examine the relationship between the errors made by the students when solving word problems and their performance score in trigonometry.

The researcher has selected secondary schools in Cordon, Isabela as the research environment of this study. The town of Cordon, located in Isabela which is in Cagayan Valley, a part of Luzon, has a limited number of secondary schools. Specifically, Cordon comprises five public secondary schools and two private secondary schools. Because of the relatively low count of both public and private secondary schools, the researcher selected one representative for public secondary school and one representative for private secondary school.

Table 1. List of Selected Schools in Cordon, Isabela

Classification	Chosen School
Public	Cagasat High School
Private	Saint John Berchmans High School Incorporated

Moreover, Table 1 provides the location of the two selected schools in Cordon, Isabela. Saint John Berchmans High School Incorporated (SJBHSI), a private institution, is located along National Highway in Brgy. Magsaysay, Cordon, Isabela and founded in 1962. In contrast, Cagasat High School (CHS), a pioneer public secondary school located on Tasani St. in Brgy. Gayong, Cordon, Isabela, was established in 1971.

The respondents of the study were the Grade 10 students who studied trigonometry in their ninth grade enrolled during the school year 2022-2023 in the selected schools in Cordon, Isabela. Through the use of Slovin's Formula, 97 Grade 10 students at Cagasat High School and 128 Grade 10 students at Saint John Berchmans High School Incorporated students were selected with a 0.05 margin of error. Slovin's Formula was applied to determine the appropriate sample size from the stated population of the study.

Table 2 shows the number of Grade 10 students according to their sex and age. The table displays that there are 110 male students (48.89%) and 115 female students (51.11%) selected in this study as respondents. There are more female respondents than male respondents in this study. The table further displays that there are 111 respondents (49.33%) falling under the age of 14 years old, 110 respondents (48.89%) fall under the age of 15 years old, and four respondents (1.78%) fall under the age bracket 16-19 years old.

The study included a total of 225 respondents, drawn from the nine sections of Grade 10 students at Cagasat High School and Saint John Berchmans High School Incorporated. The selection process employed systematic random sampling, specifically choosing every third student present both during the administration of the worksheet and summative test in each class. To ensure confidentiality, each respondent involved in the study was assigned a pseudonym. This measure was taken to protect the privacy of the participants and maintain the ethical standards of research. In the data collection process, all selected participants were interviewed and asked to read the word problems presented on the worksheet. This interactive approach allowed the researcher to directly assess each student's reading ability. For those students demonstrating pattern errors, follow-up questions were administered to gain insights into their thought processes and reasoning.

Table 2. Demographic Profile of Grade 10 Respondents

	Name of Schools		Total	Percent
	CHS	SJBHSI		
Age				
14 years old	80	31	111	49.33
15 years old	15	95	110	48.89
16- 19 years old and above	2	2	4	1.78
Sex				
Male	54	56	110	48.89
Female	43	72	115	51.11

The researcher employed three research instruments namely worksheets on word problems involving angles of elevation and depression, interview guidelines, and a summative test on trigonometry. The three research instruments were validated by three validators with the use of a validation checklist. The researcher utilized five items of word problems involving angles of elevation and depression to evaluate the students' errors in trigonometry. Moreover, the researcher made a 50-item summative test in trigonometry which was used to measure the performance level of students in trigonometry. Lastly, the researcher used semi-structured interviews as a tool to discover the causes of students' committing such errors from the perspective of the students themselves.

The researcher carried out a quantitative and qualitative content analysis on the collected data from the prepared worksheet. Newman's Error Hierarchical Model was utilized to conduct data analysis in identifying the actual errors made by the students

when solving the word problems involving angles of elevation and depression. The errors committed by the students were coded in each domain of the Newman Error Hierarchical Model from each word problem. Furthermore, it was acknowledged that students can commit multiple errors in their submitted worksheets. The interview provided a deeper insight into the students' errors from their perspective. Students' performance in trigonometry was described in frequency, percent and averages, maximum value, minimum value, and standard deviation.

Inferential data analysis was conducted using Statistical Product and Service Solutions (SPSS). Due to the non-normality of the data, Spearman's Rho Correlation Test was selected as the appropriate statistical tool. To investigate these relationships, the study specifically focused on relating the number of errors per error indicator committed by the students with their performance scores in trigonometry.

In the process of developing teacher-made material, the focus was directed toward the learning competency associated with the application involving right triangles. The teacher-made material includes corrective instruction activities, strategically recommended to rectify the errors observed in the students' problem-solving skills. Within the teacher-made material, details are provided outlining the types of errors commonly observed, proposing corrective instruction activities, and describing the procedures for implementing these activities.

The researcher conducted the study not for personal gain, but to improve mathematics instruction and learning. There was no conflict of interest in the conduct of the study. The researcher ensured that the instruments that were ready for usage, coding, and analysis before the actual study. The profiles of the respondents and the data collected were treated with utmost confidentiality. The collected tests and worksheets were stored in a secure location. Specifically, the answered worksheets and tests were placed in expanding envelopes. The study's vulnerable populations (those under the age of 18) were not exploited and the researcher considered their needs as well as any difficulties that might arise in their capacity to provide complete informed consent. The involvement of the respondents in the study carried some risks. The respondents may be inconvenienced or at risk due to the time taken away from them. The existence of an informed consent form in this study served as a proactive measure to address and mitigate potential risks.

III. Results and Discussion

Section 1. Common Errors of Grade 10 Students in Solving Word Problems Involving Angles of Elevation and Depression

Table 3 provides a detailed breakdown of student errors organized by specific indicators according to Newman's Error Hierarchical Model. Each error is thoroughly examined and discussed in succeeding subsections for clarity.

Table 3. Summary of Error Codes

Error Codes	Item 1		Item 2		Item 3		Item 4		Item 5		μ (%)
	n	%	n	%	n	%	n	%	n	%	
R1	30	13.3	23	10.2	2	0.9	24	10.7	1	0.4	35.5
R2	9	4.0	1	0.4	43	19.1	5	2.2	29	12.9	7.72
C1	15	6.7	45	20.0	62	27.6	52	23.1	101	44.9	24.6
C2	121	53.8	165	73.3	134	59.6	167	74.2	105	46.7	61.53
T1	47	20.9	88	39.1	118	52.4	103	45.8	158	70.2	45.68
T2	88	39.1	121	53.8	86	38.2	117	52.0	55	24.4	41.5
P1	51	22.7	92	40.9	131	58.2	105	46.7	163	72.4	48.18
P2	113	50.2	123	54.7	76	33.8	120	53.3	56	24.9	43.38
E1	84	37.3	125	55.6	152	67.6	134	59.6	173	76.9	59.4
E2	94	41.8	94	41.8	57	25.3	91	40.4	48	21.3	34.12

Reading Errors

Failure to read symbols or signs that prevent students from continuing the calculation process is called a reading error. There are reading errors in the different word problems committed by the students. Specifically, 39 students (17.3%) encountered reading errors with the first word problem. The second word problem posed reading error challenges for 24 students (10.7%). The third word problem presented reading errors for 45 students (20%). Moving on to the fourth word problem, 29 students (12.9%) struggled with reading errors. Lastly, the fifth word problem perplexed 30 students (13.3%) who struggled with reading. On average, the reading error rate across these word problems is at 14.84%. The types of students' reading error in this domain were divided into two, namely: (R1) the student cannot find the meaning of the mathematical symbols, and (R2) the student cannot read the questions correctly. R1 indicates that students struggle to understand the interpretation of mathematical symbols

contained in the word problem while R2 implies that the student is having difficulty in reading the words contained in the word problems.

Researcher: Can you read the word problem #1?

Axel: (Reads the 1st word problem but he read the symbol ° as percent)

Researcher: What do you call this symbol again? (points at ° symbol)

Axel: Percent Sir.

(The researcher explains that the symbol ° should be read as degree/s)

Researcher: Can you read the word problem #3?

Belle: (Reads the 1st word problem but he read the word “leans” as lens)

Researcher: Can you read the first statement of word problem #3 again?

Belle: (Reads the 1st statement and he read again the word “leans” as lens)

(The researcher explains that the pronunciation of the word “leans” is /lins/)

The majority of students demonstrated proficiency in overcoming reading errors when reading mathematical symbols and words contained in the word problems involving angles of elevation and depression. However, a small number of students, including Axel and Belle, encountered difficulties in reading the problems. Axel acknowledged misinterpreting the mathematical symbol, reading the ° as a percent due to his prior knowledge. Some students chose to skip reading the ° symbol altogether, while others mistakenly identified it as a degree Celsius symbol. Approximately 35.5% of the respondents exhibited an indicator of reading error R1.

Meanwhile, R2 exemplifies specific instances of misreading like “leans” being read as “lens” in the 3rd word problem, “attached” as “attacked” in the 1st word problem, “lighthouse” as “lithows” in the 4th word problem, and “diver” as “driver” in the 5th word problem. However, only 7.72% of respondents displayed the R2 reading error indicator.

Among the reading errors observed in students, it was noted that they often bypass unfamiliar words, leading to potential misunderstandings. Additionally, some students tended to incorrectly articulate mathematical symbols, which could potentially impact the accuracy of their responses. As Lubis et al. (2021) have found, students face challenges in identifying the given information within the word problem which hinders their ability to solve the word problem.

Comprehension Errors

Comprehension errors refer to situations where students can read the questions, yet struggle to grasp the underlying meaning, resulting in their responses not aligning with the intended inquiry (Lestari et al., 2018). Comprehension errors were experienced by 136 students in the 1st word problem (60.4%), 210 students in the 2nd word problem (93.3%), 196 students in the 3rd word problem (87.1%), 219 students in the 4th word problem (97.3%), and 206 students in the 5th word problem (91.6%) with an average of 85.94%. In this study, comprehension error arises when students cannot illustrate the given problem (C1) and when students do not understand the problem (C2).

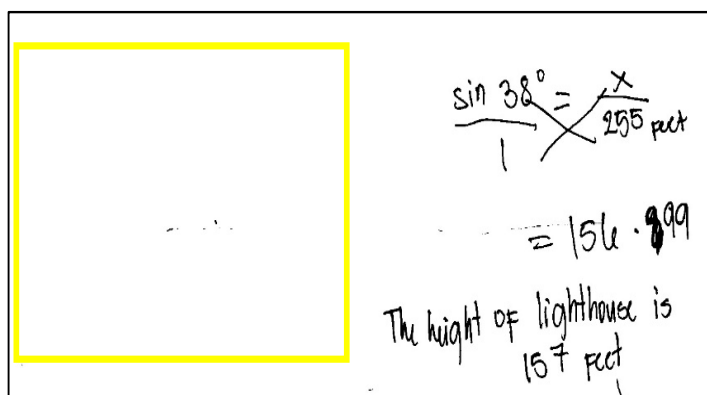


Figure 2. Clara's Answer to the Fourth Word Problem with C1 Error

Figure 2 shows Clara's solution to the fourth word problem. Clara appears to be experiencing a comprehension error in indicator C1, which is shown by a yellow box. Clara supplied a solution, but she did not provide an illustration. The lack of an illustration may limit a comprehensive grasp of her answer, particularly in the step of determining the appropriate trigonometric ratio. This error may result in an incorrect answer and conclusion.

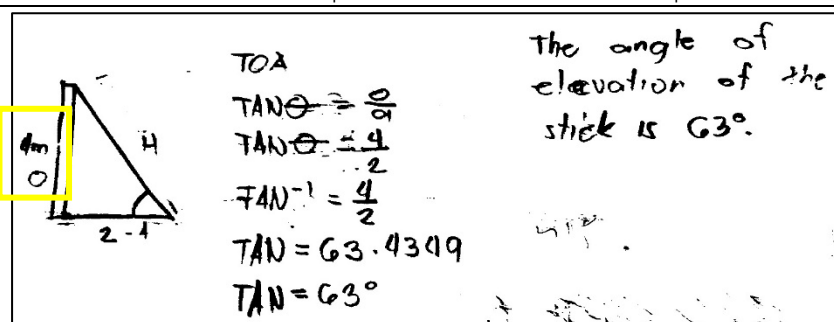


Figure 3. David's Answer to the Third Word Problem with C2 Error

Figure 3 displays David's response to the third word problem. It appears that David is encountering a comprehension error in indicator C2, which is highlighted with a yellow box. David misplaced one of the provided pieces of information, leading to an incorrect illustration. David mistakenly recorded "4" as the height of the wall, when it should have been the length of the ladder. This misstep led to an inaccurate illustration, directly impacting the trigonometric ratio. Consequently, this error in the trigonometric ratio can lead to an incorrect answer and an inaccurate conclusion.

Some students also have difficulty representing angles of depression in their illustrations, incorrect placement of the provided angle, and misplacing an object to the triangle. Overall, comprehension errors were experienced by students while solving word problems, with a percentage of 24.6% and 61.53% for C1 and C2, respectively. These errors can potentially lead to a misinterpretation or misunderstanding of the solution. Visual representations often play a crucial role in conveying complex concepts accurately. As such, encouraging the students to include an illustration alongside their solution would greatly contribute to ensuring the correctness and clarity of their answers (Prasetyaningrum et al., 2022).

Transformation Errors

Transformation errors result from a lack of familiarity with the appropriate formula to employ in problem-solving (Oktafia et al., 2020). Among the respondents, 135 students (60%) encountered transformation errors in the 1st word problem. In the 2nd word problem, 209 students (92.9%) faced similar challenges. This trend continued with 204 students (90.7%) experiencing transformation difficulties in the 3rd word problem, 220 students (97.8%) in the 4th word problem, and 213 students (94.7%) in the 5th word problem. On average, this results in an overall transformation rate of 87.22%. In this study, transformation errors particularly refer to instances where students struggle with selecting the correct mathematical model (T1) and the failure to write down the mathematical model completely and accurately (T2).

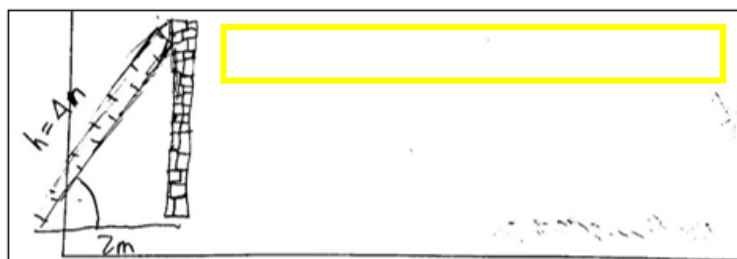


Figure 4. Hector's Answer to the Third Word Problem with T1 Error

Figure 4 illustrates Hector's response to the third word problem. It indicates that Hector encountered a transformation error in indicator T1, marked by a yellow box. While Hector can comprehend and illustrate the problem, he faces difficulty in selecting the appropriate trigonometric ratios for the given scenario. This error in selecting the appropriate trigonometric ratio led to the absence of an answer and a lack of a conclusion. This indicates a specific area of challenge for Hector in applying trigonometric concepts to problem-solving.

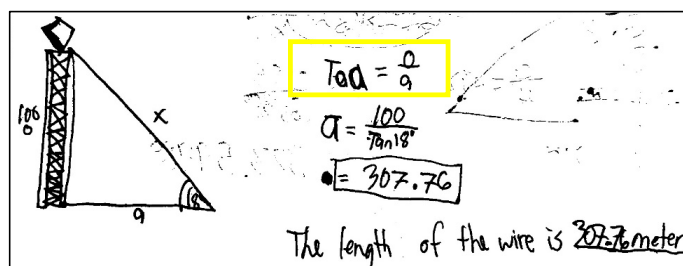


Figure 5. Ian's Answer to the First Word Problem with T2 Error

Figure 5 showcases Ian's response to the first word problem. Ian faced a transformation error in indicator T2, marked by a yellow box. Despite providing an accurate illustration of the word problem, Ian made an error in writing the appropriate trigonometric ratio. He wrote the trigonometric ratio tangent instead of sine when attempting to express the trigonometric ratio. Ian explained that he selected and wrote tangent because the adjacent side of the reference angle was not mentioned in the word problem.

Students with an error of T1 experiencing comprehension errors while solving word problems were at 45.68%, whereas in T2, this percentage was lower at 41.5%. Understanding and correctly applying mathematical equations is crucial in various areas of mathematics, especially in geometry, trigonometry, algebra, and calculus (Star et al., 2015). Encountering transformation errors in this context indicates that the students may need further practice and guidance in this area. In many cases too, students demonstrate an understanding of the problem statement, but struggle with determining the correct sequence of steps required to arrive at the solution accurately (Yuliana et al., 2021). This could involve revisiting the definitions of trigonometric ratios and practicing their application in different types of problems.

Process Skills Errors

Process skills errors were experienced by 164 students in the 1st word problem (72.9%), 215 students in the 2nd word problem (95.6%), 207 students in the 3rd word problem (92%), 225 students in the 4th word problem (100%), and 219 students in the 5th word problem (98.2%) with an average of 91.56%. This shows that process skills errors are quite high. In this study, process skills errors mean that students have no solution (P1) and student does not correctly apply the steps in detail and systematically (P2).

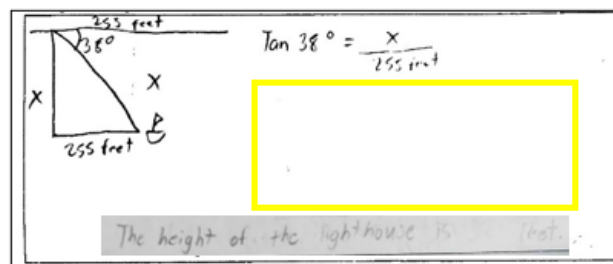


Figure 6. Madison's Answer to the Fourth Word Problem with P1 Error

Figure 6 presents Madison's response to the fourth word problem. Madison encountered a process skills error in indicator P1, highlighted by a yellow box. She provided a precise illustration of the given word problem and correctly supplied a trigonometric ratio to solve for the unknown. However, despite initially offering an accurate illustration and providing an appropriate trigonometric ratio, she lacked a solution and answer. Additionally, Madison attempted to write the conclusion of the word problem, but since she did not have an obtained answer, her conclusion was incomplete, also indicating an encoding error.

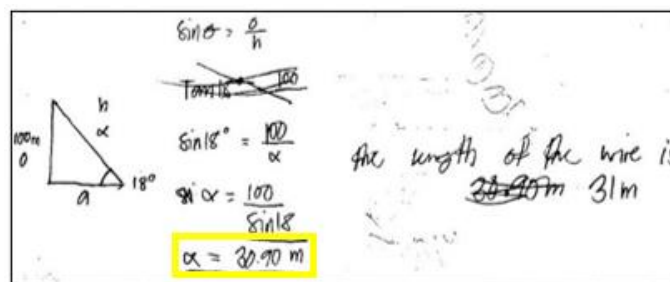


Figure 7. Nelson's Answer to the First Word Problem with P2 Error

Figure 7 reveals Nelson's response to the first word problem. Nelson encountered a process skills error in indicator P2, indicated by a yellow box. Nelson committed a process skill error in providing the correct answer despite providing a precise illustration of the word problem and using the correct trigonometric ratio. Nelson has exhibited difficulties in using a scientific calculator to execute his calculations accurately. According to Nelson's interview, he mentioned multiplying 100 and sin 18 instead of obtaining their quotient.

Siskawati et al. (2021) also found that students exhibit weaknesses in their grasp of process skills, particularly in comprehending the mathematical equation presented on the word problem. In this study, process skills errors were experienced by students while solving word problems, with a percentage of 48.18% and 43.38% for P1 and P2, respectively. Students fail to develop systematic steps in problem-solving in a consistent, detailed, and accurate manner. In relation to this, it is crucial to provide students with clear instructions and ample practice opportunities (Juneidi et al., 2015).

Encoding Errors

An encoding error arises when a student successfully arrives at a solution but encounters difficulty in articulating the concluding statement of the answer. Encoding errors were experienced by 178 students in the 1st word problem (79.1%), 219 students in the

2nd word problem (97.3%), 209 students in the 3rd word problem (92.9%), 225 students in the 4th word problem (100%), and 221 students in the 5th word problem (98.2%) with an average of 93.5%. In this study, encoding errors refer to students not writing the conclusion (E1) and students not able to answer the problem with the right conclusion (E2).

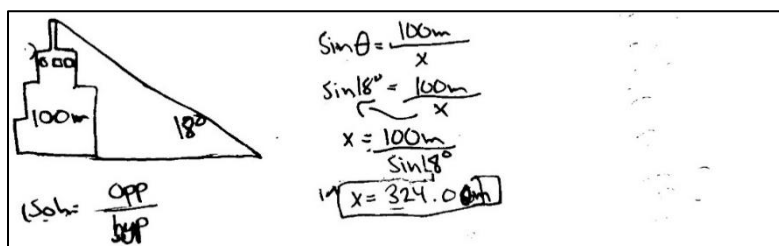


Figure 8. Pamela's Answer to the First Word Problem with E1 Error

Figure 8 displays Pamela's response to the first word problem. Pamela encountered an encoding error in indicator E1. She provided an accurate illustration of the problem and correctly applied the appropriate trigonometric ratio to determine the unknown, resulting in a precise and correct result despite this encoding error. However, Pamela did not provide a conclusion. By including a conclusive statement, Pamela could have provided a more holistic and well-rounded response, ensuring that the recipient of her work gains a thorough understanding of the problem's resolution.

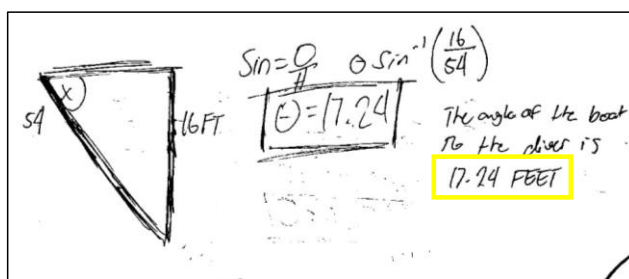


Figure 9. Ralph's Answer to the Fifth Word Problem with E2 Error

Figure 9 presents Ralph's answer to the fifth word problem. Ralph encountered an encoding error in indicator E2, represented by the yellow box. Ralph has effectively provided an appropriate illustration of the problem, utilized the correct trigonometric ratios, and arrived at an accurate solution. However, there was a slight oversight in providing the correct unit of measurement for the solution. Ralph included feet in his conclusion when the accurate unit of measurement should have been degrees, as the unknown involved an angle. This emphasizes the significance of careful attention to detail, particularly in specifying units, which is integral to conveying precise mathematical solutions.

Other students experienced encoding errors, which included the use of improper grammar or an incomplete thought in their conclusions and the application of incorrect units of measurement. Overall, students encountered encoding errors while solving word problems, with an average of 59.4% for E1 and 34.12% for E2. These errors may have occurred because of the student's lack of prioritization, diminished interest, and a certain level of indifference toward formulating an accurate concluding statement.

Section 2. Student's Performance Level in Trigonometry

Table 4. Mean Percent Scores and Performance Level in Each Learning Competency in Trigonometry

Learning Competencies	Mean Percent Score	Performance Level
1. Illustrates the six trigonometric ratios: sine, cosine, tangent, secant, cosecant, and cotangent.	42.88	Poor Performance
2. Finds the trigonometric ratios of special angles	32.20	Poor Performance
3. Illustrates angles of elevation and angles of depression	46.92	Poor Performance
4. Uses trigonometric ratios to solve real-life problems involving right triangles.	34.27	Poor Performance
5. Illustrates the laws of sines and cosines.	19.64	Poor Performance
6. Solves problems involving oblique triangles.	32.91	Poor Performance
Overall	34.80	Poor Performance

Table 4 displays the mean percent scores and performance level in each learning competency in trigonometry of Grade 10 students. The analysis of student performance in the first learning competency, reveals a poor performance among the students with a mean percent score of 42.88%. The table also indicates that students exhibited a poor performance in the second learning competency with a mean percent score of 32.20%. Table 4 further indicates that students exhibited a poor performance in the third learning competency as reflected in a mean percent score of 46.92%. Moreover, the table reveals that majority of students demonstrated a poor performance in the fourth learning competency as evidenced by a mean percent score of 34.27%. The table also indicates that students demonstrated a poor performance in the fifth learning competency with a mean percent score of 19.64%. Lastly, the table reveals that majority of students demonstrated a poor performance in the sixth learning competency having an average percent score of 32.92%. The overall mean percent score of 34.80% suggests that the students, on average, performed poorly in trigonometry across various learning competencies. This could indicate a need for improvement in teaching methods, student engagement, or the overall learning environment.

Section 3. Relationship Between the Committed Errors and the Performance of Grade 10 Students in Trigonometry

Table 5. Correlation of the Number of Committed Errors of the Respondents to their Performance Score in Trigonometry

Error Domains	Error Indicators	Correlation Coefficient	Significance Level
Reading Error	R1	-0.059	0.380
	R2	-0.261**	<0.001
Comprehension Error	C1	-0.258**	<0.001
	C2	0.072	0.284
Transformation Error	T1	-0.355**	<0.001
	T2	0.213**	0.001
Process Skills Error	P1	-0.337**	<0.001
	P2	0.234**	<0.001
Encoding Error	E1	-0.386**	<0.001
	E2	0.308**	<0.001

** Significant at 0.01 level of significance

A study found a significant negative correlation ($r = -0.261, p < 0.001$) between R1, indicating reading comprehension issues, and Grade 10 trigonometry performance. More reading errors were linked to poorer trigonometry scores, suggesting reading difficulties may hinder math performance in this group.

Moreover, the study revealed a significant relationship between comprehension errors on C1, and the performance score of Grade 10 students in trigonometry, $r(225) = -0.258, p < 0.001$. This indicates that fewer student illustrations correlate with decreased trigonometry scores, suggesting a potential impact on performance.

The study found significant relationships between transformation error indicators, T1 and T2, and trigonometry performance among students ($p < 0.001$ for T1, $p = 0.001$ for T2). T1 errors showed a weak negative correlation ($r = -0.355$), indicating lower trigonometry scores with more T1 errors. Conversely, T2 errors exhibited a weak positive correlation ($r = 0.216$), suggesting slight improvement in trigonometry scores with more T2 errors. This underscores the role of effort and engagement in learning.

Table 5 reveals significant relationships between process skills errors, P1 and P2, and trigonometry performance ($p < 0.001$ for P1, $p < 0.001$ for P2). P1 errors show a weak negative correlation ($r = -0.337$), indicating lower performance scores with more P1 errors. Conversely, P2 errors display a weak positive correlation ($r = 0.234$), suggesting slight improvement in performance scores with more P2 errors. This underscores the notion that even imperfect effort and engagement can foster better understanding and performance.

Furthermore, encoding errors, both E1 and E2, significantly relate to trigonometry performance ($p < 0.001$ for E1, $p < 0.001$ for E2). E1 exhibits a weak negative correlation ($r = -0.386$), indicating higher E1 errors coincide with lower trigonometry scores. Conversely, E2 shows a weak positive correlation ($r = 0.308$), suggesting slight performance score improvement with more E2 errors. In essence, more E1 errors are linked to lower performance scores, while more E2 errors are associated with minor performance score increases in trigonometry. These errors were statistically significant at the 0.01 level.

Section 4. Development of a Teacher-made Material Based on the Identified Errors of the Students in Solving Word Problems Involving Angles of Elevation and Depression and their Performance Level in Trigonometry.

To lessen the committed errors and to enhance students' performance in the aforementioned learning competency, providing corrective instruction activities through teacher-made material tailored to students' specific error categories is considered an effective approach. The researcher developed a teacher-made material that aims to serve as a comprehensive guide for teachers, enabling the students to improve their performance in learning trigonometry specifically when solving word problems involving angles of elevation and depression.

These corrective instruction activities are intended to assist students in reducing errors, as outlined in Newman's Error Hierarchical Model, particularly when solving word problems involving angles of elevation and depression. Furthermore, these activities aim to foster the development of students' higher-order thinking skills, improved conceptual understanding, enhanced problem-solving skills, and individualized learning.

Table 6. Corrective Instruction Activities in the Teacher-Made Material

Activity No.	Activity	Types of Errors	Targeted Learning Competencies
1	Trigonometric Tales: Heights, Distances, and Angles!	Reading and Comprehension Errors	Illustrates angles of elevation and angles of depression.
2	SOHCAHTOA Spectacular: Unraveling Math Mysteries!	Transformation Errors	Illustrates the six trigonometric ratios: sine, cosine, tangent, secant, cosecant, and cotangent
3	Trig-Tacular Calculator Challenge!	Process Skills Errors	- Uses trigonometric ratios to solve real-life problems involving right triangles. - Illustrates the six trigonometric ratios: sine, cosine, tangent, secant, cosecant, and cotangent
4	Enigma Escapade: Mastering Conclusion Statements!	Encoding Errors	Uses trigonometric ratios to solve real-life problems involving right triangles.

Table 6 shows an overview of the corrective instruction activities integrated into the teacher-made material. This instructional material comprises four distinct activities, each specifically designed with two objectives in mind: firstly, to reduce the occurrence of the targeted error type committed by the students, and secondly, to enhance their performance in the specified learning competency in trigonometry. The incorporation of these activities into the curriculum is expected to not only supplement students' grasp of the subject matter but also contribute significantly to the development of their mathematical proficiency specifically in solving word problems involving angles of elevation and depression (Bayos, 2020).

In the first activity, titled "Trigonometric Tales: Heights, Distances, and Angles!", students engage in reading and comprehension exercises to rectify errors associated with understanding word problems. The collaborative nature of the task, where students work with partners, aims to enhance their ability to read and comprehend trigonometric scenarios. The goal of this activity is to illustrate angles of elevation and angles of depression. Figure 10 shows some sample items from the first activity.

1. Kite Soaring High: A Mathematical Flight

1. An airplane is flying really high, about 6000 meters up in the sky. When the people inside the plane look down, they see the town of Cordón, but not straight down – it's at a bit of an angle, like 14° . Now, they want to figure out how far away the town of Cordón is from the airplane.

- Identify the mode of transportation mentioned in the problem: _____
- Determine the height above the ground, known as the altitude, of the transportation: _____
- Illustrate the transportation in the provided box on the next page. Extend a line to represent the altitude and label it with the given altitude value.
- Specify where the people inside the plane are looking outside: _____
- State the provided angle in the problem: _____
- Classify whether the given angle represents an angle of elevation or depression: _____
- Sketch a line from the passengers to the observed location. Write the given angle in the box below. (Note: The reference angle should start from the horizontal line to the line of sight)
- Complete the visual representation by forming a right triangle in the figure, connecting the endpoint of the line of sight to the base of the transportation.
- Identify the unknown in the provided word problem: _____
- Write the variable "x" as the unknown in the designated box: The distance between the airplane and the municipal hall.

3. Ladder Logic: Scaling Heights with Trigonometry

A student in their room wanting to put a plant on a high wall that's 13 feet tall. They have a 14-foot ladder but need to figure out the right way to position it to reach the top of the wall. The challenge is to find the angle at which the ladder should be set up.

- Draw a wall and write its given length in the box below.
- Illustrate a ladder leaning against the wall and write its length in the box below.
- Determine the unknown in the word problem.
- On your illustration, label the unknown with "x."
- Finish your drawing by forming a right triangle.

Figure 10. Sample Items from the First Activity

Figure 10 displays the first, third, and fifth items from the first activity in the teacher-made material. The items in this activity are designed to assess and enhance students' reading and comprehension skills related to angles of elevation and angles of depression. Specifically, the first and second items provide a complete guided statement to assist students in illustrating the given word problem. The third and fourth word problems include a guided statement that is intentionally left incomplete, challenging students to illustrate the problem even with partial guidance. The fifth item, without a guided statement, aims to evaluate students' comprehension by requiring them to independently illustrate the given problem. Additionally, students are required to read the word problem and the guided question, providing a test of their reading skills. Overall, these activities are crafted to contribute to the improvement of students' performance in the learning competency of illustrating angles of elevation and angles of depression.

Moving on to the second activity, "SOHCAHTOA Spectacular: Unraveling Math Mysteries!", the focus is on correcting transformation errors related to providing accurate trigonometric ratios. The learning competency targeted in this activity is the illustration of the six trigonometric ratios: sine, cosine, tangent, secant, cosecant, and cotangent. Guided questions will assist the students in overcoming challenges and ensure the correct application of trigonometric concepts. Figure 11 shows some sample items from the second activity.

1. As the school gathered for the national anthem, a teacher stood proudly 25 feet away from the school flagpole. With the *Lupang Hinirang* hymn about to begin, the teacher looked up at the Philippine flag, which was flying at a 43° angle. What is the distance of the teacher to the Philippine flag?

3. Under the afternoon sun, a student took on the challenge of measuring the height of a flagpole. Recording a shadow length of 34 feet cast by the flagpole, the student considered the 38° angle formed by the sun's rays with the ground. How tall is the flagpole?

a) With respect to the angle, what part of the triangle is x ? Opposite, adjacent, or hypotenuse?

b) With respect to the angle, what part of the triangle is 25 ft? Opposite, adjacent, or hypotenuse?

c) Consider the relationships you identified in a and b. Which trigonometric ratio (sine, cosine, tangent) relates the known sides to the angle?

d) Substitute the given information into your chosen trigonometric ratio in the box.

Figure 11. Sample Items from the Second Activity

Figure 11 displays the first and third items from the second activity in the teacher-made material. This activity aims to assess students' ability to transform given word problems and illustrations into trigonometric ratios, with a focus on the trigonometric ratios: sine, cosine, tangent, secant, cosecant, and cotangent. In the first and second items, a complete question guide is provided to assist students in identifying the sides of the triangle, determining the unknown, and selecting the appropriate trigonometric ratio applicable to the problem. The third and fourth word problems also include a question guide intentionally left incomplete, challenging students to provide a trigonometric ratio even with partial guidance. The fifth item, without a question guide, tests students' critical thinking skills in determining the appropriate trigonometric ratio for the given problem. This activity is designed to enhance students' performance in the learning competency of illustrating the six trigonometric ratios, fostering their understanding of the relationships between angles and sides in a triangle.

The third activity, titled "Trig-Tacular Calculator Challenge!", addresses transformation errors associated with performing operations in trigonometric equations and utilizing scientific calculators. By requiring the students to execute operations using calculators, this activity enhances their practical application of trigonometric concepts. The learning competencies addressed include using trigonometric ratios to solve real-life problems involving right triangles, as well as illustrating the six trigonometric ratios. Figure 12 shows some sample items from the third activity.

1. $\sin 28^\circ = \frac{x}{20}$

$(\sin 28^\circ = \frac{x}{20}) 20$

$20 \sin 28^\circ = x$

$9.38 = x$
or $x = 9$

Multiply both sides of the equation by 20

Simplify the equation

To solve for $20 \sin 28$, your calculator must be in degree mode. Then press the buttons **2** **0** **sin** **2** **8** **=**

4. $\cos 13^\circ = \frac{x}{7}$

Multiply both sides of the equation by 7

Simplify the equation

Perform the expression on the calculator

Figure 12. Sample Items from the Third Activity

Figure 12 features the first and fourth items from the third activity in the teacher-made material. This activity is designed to evaluate and enhance students' procedural skills in solving for the value of the unknown using a digital scientific calculator. The focus is on applying trigonometric ratios to solve real-life problems involving right triangles and demonstrating proficiency in illustrating the six trigonometric ratios. In the first and second items, a complete step-by-step procedure is provided to guide students in calculating the unknown. The fourth and fifth word problems feature an incomplete procedure, challenging students to execute the equation and solve for the unknown with partial guidance. The third and sixth items lack a provided procedure, testing students' ability to independently apply their procedural skills in solving for the unknown using the calculator. This activity aims to contribute to the improvement of students' performance in the learning competency involving the application of trigonometric ratios to solve real-life problems associated with right triangles.

Last with the fourth activity, "Enigma Escapade: Mastering Conclusion Statements!", the focus is on rectifying encoding errors related to writing conclusion statements. The students are encouraged to create conclusive statements based on word problems and the given answer, addressing challenges in drawing conclusions. The learning competency targeted in this activity is the application of trigonometric ratios to solve real-life problems involving right triangles. Figure 13 shows some sample items from the fourth activity.

1. An airplane is flying really high, about 6000 meters up in the sky. When the people inside the plane look down, they see the town of Cordon, but not straight down – it's at a bit of an angle, like 14°. Now, they want to figure out how far away the town of Cordon is from the airplane. Answer: 24,801
Conclusion: The _____ from the airplane to the town of Cordon is _____.
2. Picture yourself in the school playground, flying a colorful kite held by a 65-meter string. Look up at the kite and notice that it's at a slant, about 70° from the ground. Now, let's figure out how high your kite is above your head. Answer: 61.08
Conclusion: The altitude of the _____ above your head is _____.

Figure 13. Sample Items from the Fourth Activity

Figure 13 shows the first and second items from the fourth activity in the teacher-made material. The objective of this activity is to assess and improve students' ability to articulate conclusions based on the given word problem and its corresponding answer. The emphasis is on enhancing their performance in the learning competency of applying trigonometric ratios to solve real-life problems involving right triangles. In the first to fifth items, a conclusion is partially provided with two blanks, requiring students to fill in the missing information. This challenges them to connect the word problem with the solution and accurately articulate the appropriate conclusion. The sixth to tenth word problems lack a pre-written conclusion, demanding that students generate a complete and accurate conclusion based on the given word problem and its solution. This activity aims to further develop students' proficiency in drawing meaningful conclusions from trigonometric problem-solving scenarios, fostering a deeper understanding of the application of trigonometric ratios in real-life situations involving right triangles.

Together, these activities form a comprehensive set designed to address specific challenges, and enhance learning competencies in trigonometry.

IV. Conclusions and Recommendations

The identified errors, categorized under Newman's Error Hierarchical Model, mainly include reading with most being comprehension, transformation, process skills, and encoding errors. This highlights the areas that require targeted corrective instruction activities. The analysis of the students' performance level in the summative test in trigonometry revealed a range of outcomes, with a proportion falling into the fair to poor performance categories. The correlational analysis established connections between specific error types in word problems and students' overall performance in trigonometry. Reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors displayed significant correlations with students' overall performance in trigonometry. The corrective instruction activities in the teacher-made material are based on the committed errors made by the students in solving word problems involving angles of elevation and depression and students' performance in trigonometry. All the error domains are addressed in the material since all of the errors were considered restrictions.

The following are the suggestions based on the study's findings and analysis of the data obtained:

1. The researcher recommends that teachers conduct a thorough error analysis to identify the most prevalent types of errors experienced by students when solving word problems.
2. Given that the most prevalent errors committed by the students were comprehension, transformation, process skills, and encoding errors, it is recommended that future researchers conduct a similar study across different grade levels. Additionally, a study focused on evaluating the effectiveness of corrective instruction activities could yield valuable insights into improving performance in trigonometry. School administrators are also encouraged to implement programs aimed at enhancing trigonometry instruction across all educational levels from basic education units to higher levels.
3. The researcher also recommends that teachers prioritize familiarizing students with various mathematical word problems. This approach is essential for enhancing students' problem-solving abilities, ultimately contributing to their overall proficiency in the subject.
4. To address the underlying causes of errors committed by the students, it is advised that teachers employ suitable instructional procedures. Additionally, teachers may find it beneficial to integrate the developed teacher-made material from this study. The utilization of the teacher-made material is recommended not only in SJBHSI and CHS but also in other schools whose students face similar challenges in solving word problems related to angles of elevation and depression. This inclusive approach can effectively address common error patterns and contribute to improved performance in trigonometry.
5. The researcher recommends for the teacher-made material to undergo quality assurance by the Learning Resource Management Division (LRDMC) before it is utilized. The researcher also recommends the enhancement of the material for the utilization in Department of Education.

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APPENDIX A

Summative Test in Trigonometry

Name:	Score:
Grade and Section:	Date:

Direction: Choose the correct answer from the given options and then write the letter of your answer on the line provided before the number. You may use your digital scientific calculator to answer the items that requires solving. Good Luck!

_____1. With respect to the given angle, what is the ratio of the opposite side to the hypotenuse?

- a. sine b. cosine c. secant d. tangent

_____2. Which of the following is true about six trigonometric ratios?

- a. secant is the reciprocal of sine c. secant is the reciprocal of cosecant
b. cotangent is the reciprocal of cosine d. cosecant is the reciprocal of sine

_____3. Which of the following is a secondary trigonometric ratio?

- a. tangent b. cosine c. secant d. sine

_____4. What is the abbreviation for cosecant?

- a. cos b. csc c. sin d. cot

_____5. Determine the correct formula for the secant ratio of $\angle M$.

- a. $\sec M = \frac{\text{length of side adjacent to } \angle M}{\text{length of the hypotenuse}}$ c. $\sec M = \frac{\text{length of the hypotenuse}}{\text{length of side adjacent to } \angle M}$
b. $\sec M = \frac{\text{length of side opposite to } \angle M}{\text{length of side adjacent to } \angle M}$ d. $\sec M = \frac{\text{length of side adjacent to } \angle M}{\text{length of side opposite to } \angle M}$

For items 6-8, refer to the figure at the right

_____6. What particular trigonometric ratios can be used to solve for x ?

- a. sine b. cosine c. tangent d. secant

_____7. Which of the following statements is correct?

- a. $\sin 60^\circ = \frac{8}{x}$ c. $\tan 60^\circ = \frac{8}{x}$
b. $\cos 60^\circ = \frac{x}{8}$ d. $\sec 60^\circ = \frac{x}{8}$

_____8. What is the length of x in $\triangle ABC$?

- a. 4.61 units b. 5.61 c. 6.61 d. 7.61

_____9. Given $\triangle SOC$ where $\angle C$ is a right angle, which side is opposite to $\angle S$?

- a. $\overline{SC}$ b. $\overline{SO}$ c. $\overline{OC}$ d. none of the above

_____10. Which of the following statements is true about a 45° - 45° - 90° special triangle?

- a. The length of the shorter leg is $\sqrt{3}$ times the length of the longer leg.
b. The length of the hypotenuse is twice the length of the shorter leg.
c. Both legs of the triangle are equal in length.
d. All of the above.

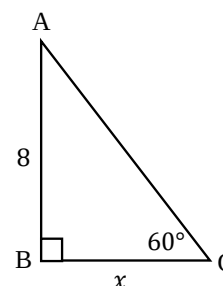
_____11. What is the exact value of $\csc 30^\circ$?

- a. 1 b. 2 c. 3 d. 4

_____12. The value of $\sin 30^\circ$ is _____ the value of $\cos 60^\circ$.

- a. less than b. greater than c. equal to d. not equal to

_____13. Which among the following is the value of $\sec 30^\circ$?



a. $\frac{2\sqrt{3}}{3}$

b. $-\frac{2\sqrt{3}}{3}$

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

14. Evaluate the trigonometric expression $(\cos 60^\circ)(\tan 45^\circ)$.

a. $\frac{1}{2}$

b. $\frac{3}{2}$

c. 2

d. 1

15. Determine the value of $\sin^2 30^\circ + \cos^2 45^\circ$.

a. 2

b. $\frac{3}{2}$

c. $\frac{5}{2}$

d. $\frac{3}{4}$

For items 16-18, refer to the figure at the right.

16. Identify the angle illustrated in the figure.

a. angle of elevation

b. angle of depression

c. line of sight

d. none of the above

17. What is the name of the figure formed in the figure.

a. $\angle LMN$

b. $\angle KLM$

c. $\angle LNM$

d. $\angle LKM$

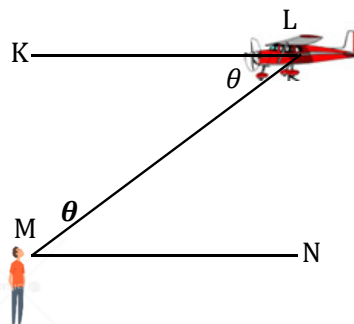
18. What is the line of sight in the figure?

a. $\overline{ML}$

b. $\overline{KL}$

c. $\overline{MN}$

d. $\overline{MN}$



19. This is the angle between the horizontal line of sight and the line of sight up to an object.

a. angle of elevation

b. angle of depression

c. line of sight

d. none of the above

20. How can you find the cosecant on a digital scientific calculator?

a. Press the shift button then the sine button.

b. Press the 1, then division operation button and then sine button.

c. Press the inverse button for cosecant.

d. It is impossible to determine the cosecant on a digital scientific calculator.

For items 21-22, refer to the figure at the right.

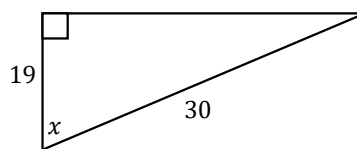
21. Which of the following trigonometric ratios can be used to solve for x?

a. $\cos x = \frac{19}{30}$

c. $\csc x = \frac{19}{30}$

b. $\sin x = \frac{19}{30}$

d. $\tan x = \frac{19}{30}$



22. What is the measurement of x?

a. 29.08°

c. 39.3°

b. 32.35°

d. 50.70°

For items 23-24, refer to the figure at the right.

23. What trigonometric ratio can be used to solve for x?

a. cosine

c. cosecant

b. sine

d. tangent

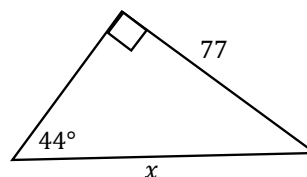
24. What is the length of x?

a. 23.44 units

c. 75.76 units

b. 48.01 units

d. 83.99 units



25. What do you call the angle formed by the horizontal line of sight and the line of sight when the object observed is below the eye?

a. Horizontal line of sight

c. Angle of elevation

- b. Angle of depression d. All of the above

26. What do you call the angle formed by the horizontal line of sight and the line of sight when the object observed is above the eye?

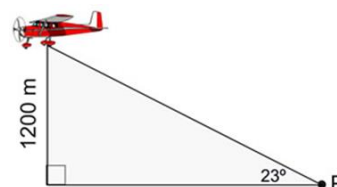
- a. Horizontal line of sight c. Angle of elevation
b. Angle of depression d. All of the above

27. Nick is watching on the ground looking up at a concert. What angle can you measure?

- a. Angle of elevation c. Horizontal line of sight
b. Angle of depression d. Line of sight

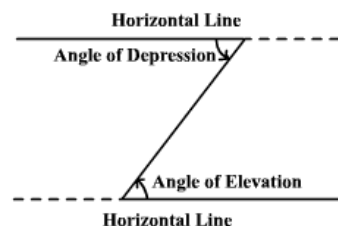
28. What is NOT true about the illustration at the right?

- a. The angle of elevation from point P to the plane is 23° .
b. The angle of depression from the plane to point P is 23° .
c. The altitude of the plane is 1300 meters.
d. The distance from point P to the plane is 3071 meters.



29. Given the figure below, which statements shows the correct relation of the angle of depression and the angle of elevation?

- a. angle of depression is greater than angle of elevation
b. angle of elevation is greater than angle of depression
c. angle of elevation is not equal to angle of depression
d. angle of elevation is equal to angle of depression



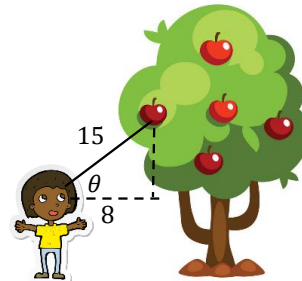
For items 30-32, refer to the figure at the right.

30. The unknown is an example of _____.

- a. Horizontal line of sight c. Angle of elevation
b. Angle of depression d. All of the above

31. Which of the following is the most appropriate trigonometric ratio that relates to the given and unknown?

- a. $\sin \theta = \frac{15}{8}$ c. $\cos \theta = \frac{8}{15}$
b. $\tan 15 = \frac{\theta}{15}$ d. Insufficient data



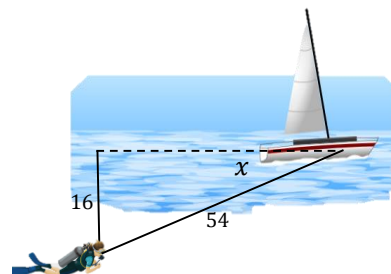
32. What is the measurement of θ ?

- a. 55.77° c. 57.77°
b. 56.77° d. 58.77°

For questions 33 – 35, use the figure at the right.

33. What can you infer from the illustration?

- a. The angle of depression from the boat to the diver is 16° .
b. The angle of elevation from the diver to the boat is 54° .
c. The distance between the boat and the diver is 54 units.
d. The distance between the diver and the surface of water is the unknown.



34. What trigonometric ratio can be used to solve for the unknown?

- a. sine b. cosinec. tangent d. secant

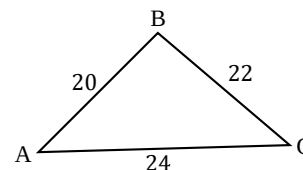
35. What is the measurement of the unknown?

- a. 17.24° b. 21.54° c. 28.9° d. 31.16°

For items 36-39, refer to the given problem below.

The angle of elevation of the top of the flagpole is 42° from a point 18 ft away from base.

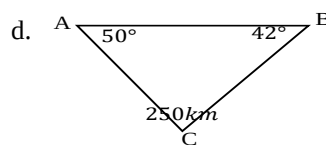
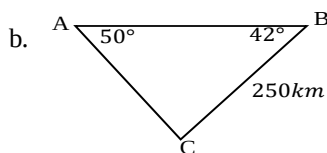
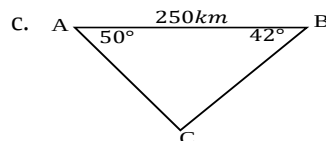
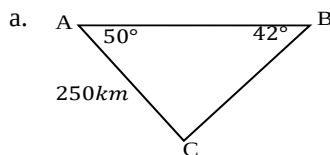
- ____ 36. What trigonometric ratio will be used to find the height of flagpole?
a. sine b. cosine c. tangent d. cosecant
- ____ 37. What is the height of flagpole?
a. 16.21 ft b. 3.06 ft c. 8.37 ft d. 14.2 ft
- ____ 38. What trigonometric ratio will be used to find the distance of the top of the flagpole from the point?
a. sine b. cosine c. tangent d. cosecant
- ____ 39. Calculate the distance of the top of the flagpole from the point?
a. 4.6 b. 15.79 c. 9.98 d. 24.22
- ____ 40. Which of the following best describes oblique triangle?
a. All angles are acute c. One angle is right
b. One angle is obtuse d. It has no right angle
- ____ 41. What case can be solved by the law of cosines?
a. Two sides and an angle opposite one of them are known
b. Two angles and non-included side are known
c. Three angles are known
d. Three side are known
- ____ 42. Which of the following cases can be solved by the law of sines?
a. ASA b. SSS c. SAS d. All of the above
- ____ 43. Which of the following is an equation of law of cosine.
a. $c^2 = a^2 + b^2$ c. $b^2 = a^2 + c^2 - 2bc \cos A$
b. $\frac{\sin A}{a} = \frac{\sin B}{b}$ d. $\csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite leg}}$
- ____ 44. Referring to the figure at the right, which angle can you say for certain is the largest even before solving the triangle?



For items 45-50, refer to the given problem below.

Towers A and B are located 250 km apart. A guard in tower A spots a fire at C and sees $\angle A$ to be 50° . Another guard in tower B finds $\angle B$ to be 42° . How far are the towers from the fire?

- ____ 45. Which among the figures illustrates the problem?



46. Which of the following cases can best described the figure?
a. ASA b. SAS c. SSS d. AAS
47. What is the measurement of $\angle C$?
a. 78° b. 88° c. 98° d. 108°
48. Which of the following can be used to solve for unknown?
a. Law of Sine b. Law of Cosine c. Law of Tangent d. All of the above
49. How far is Tower A from the fire?
a. 167.38 km b. 175.25 km c. 191.63 km d. 201.5 km
50. How far is Tower B from the fire?
a. 167.38 km b. 175.25 km c. 191.63 km d. 201.5 km

APPENDIX B

Worksheet on Angles of Elevation and Depression

Dear Student,

Please consider each word problem carefully. Your answer in this worksheet will be a great help to for identifying the students' errors when solving word problems involving angles of elevation and depression. This will measure your problem solving skill in angles of elevation and depression. The data to to be gathered will be used in my thesis entitled "Students' Errors in Solving Word Problems Involving Angles of Elevation and Depression and Performance Level in Grade 9 Trigonometry: Basis for Development of a Teacher-Made Material".

Rest assured that your responses in this worksheet will be kept confidential.

Thank you and God Bless!

-RESEARCHER-

Name:	Score:
Grade and Section:	Date:

Directions: Understand the following problems. Draw first the appropriate illustration for each item. Write a trigonometric equation which can be used to solve the problem. Rewrite the equation until it is calculator-ready, and then solve. Show your complete solution in the given box below each item. Good Luck! □

1. A wire is attached to the top of a 100-meter tower. If the angle of elevation to the top of the tower is 18° , what is the length of the wire?
2. The angle of depression of a car from the top of a 125-ft tower is 29° . How far is the car from the tower's base?
3. A 4-meter stick leans against a wall and the base of the stick is 2 meters from the base of the wall. What is the angle of elevation of the stick?
4. The angle of depression of a boat from the top of a lighthouse is 38° . What is the height of the lighthouse given that the distance from the boat to the foot of the lighthouse is 255 feet?
5. A diver swims at a depth of 16 feet below sea level. If the direct distance of the diver to a boat is 54 feet, what is the angle of depression of the boat to the diver?

APPENDIX C

List of Interview Guidelines

Adapted from Rohmah & Sutiarto (2017)

The researcher will use semi-structured interviews as a tool in this study to discover the causes of students' committing such errors from the perspective of the students themselves. Interview questions consist of structured questions and unstructured questions (Mahmud & Yunus, 2018). This form of the interview has been classified as an in-depth interview, which is more adaptable in its application than structured interviews. Newman's Error Hierarchy Model (1977), which sought to identify the categories of errors made by the students on this subject, was used to conduct the interviews. There will be also follow-up questions which will be based on the student's responses on the worksheet.

Problem-Solving Aspect Of Newman	Interview Questions
Reading Errors	1) Can you read this problem?
	2) What information do you get after reading this problem?
	3) What are the mathematical symbols contained in this problem?
Comprehension Errors	1) Can you identify the known and the given in this problem?
	2) Can you identify where is the error you did?
	3) Is there a difficulty in determining what is known and unknown in this problem?
Transformation Errors	1) What mathematical equation did you apply when answering this problem?
	2) Can you explain to me how you arrived at this mathematical equation in this problem?
	3) What should be the mathematical equation for this problem?
Process Skills Errors	1) Can you explain how did you solve this problem?
	2) Why didn't you finish solving this problem?
	3) Do you have any difficulty in performing the calculation process in the solution part?
Encoding Errors	1) What conclusion did you draw from this problem?
	2) What should be the unit required to this problem?
	3) Do you have any difficulty in determining the final answer of this problem?