

Developing Tangible Mathematics: Design and Validation of Rational Number Manipulatives for Grade 7

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Abstract– This study reports the design and validation of tangible mathematics materials for enhancing grade 7 students' understanding of rational numbers, identified as the most challenging topic in a preceding needs assessment. Using design-based research methodology with iterative refinement, we developed acrylic manipulatives (fraction circles, bars, tiles, and operational pieces) featuring systematic color-coding and movable connections. Validation by five mathematics education experts yielded exceptional ratings for content quality (37.8/40), technical accuracy (16/16), and instructional design (22/24). Teacher feedback confirmed the materials' effectiveness for concept visualization while suggesting physical enhancements. Preliminary testing with 84 students demonstrated significant performance improvements in the experimental group using manipulatives compared to controls. Both quantitative data and classroom observations confirmed enhanced understanding, engagement, and problem-solving abilities with the tangible approach. These findings establish a strong foundation for implementing tangible mathematics in grade 7 classrooms, addressing the critical transition from concrete to abstract mathematical thinking.

Keywords: manipulatives, design-based, understanding, problem-solving, engagement

I. Introduction

Grade 7 represents a pivotal transition in mathematics education as students move from concrete to abstract mathematical thinking (Wilkie & Sullivan, 2017). Despite curricular reforms, challenges with fundamental concepts like rational numbers continue to affect student achievement (OECD, 2023).

This study describes the development phase of a research project examining how tangible mathematics can improve grade 7 students' engagement, understanding, and problem-solving abilities. While manipulatives have proven effective in early mathematics education, their systematic use at the grade 7 level remains underdeveloped, especially for abstract concepts. Our research addresses this gap by creating targeted tangible interventions that connect concrete and abstract understanding of rational numbers, which our needs assessment identified as particularly challenging for students.

The development phase builds directly on findings from the needs assessment phase, which revealed that rational numbers were unanimously identified by teachers as the most challenging topic for Grade 7 learners, with 78.6% of learners rating these operations as "very difficult." The stark contrast between learners' confidence with visual-spatial concepts (mean rating 4.77/5.00) versus rational numbers (1.12/5.00), coupled with all learners (100%) expressing preference for visual aids and hands-on materials, provides clear direction for the development of tangible mathematics interventions.

II. Literature Review

Theoretical Foundations for Tangible Mathematics

Three fundamental learning theories form the foundation for tangible mathematics approaches. Constructivism (Piaget, 1952; Bruner, 1966) explains how learners build mathematical knowledge through physical interaction, with concrete manipulation facilitating the transition to abstract thinking. Cognitive Load Theory (Sweller, 1988) demonstrates how physical representations can externalize mathematical concepts, reducing mental burden and allowing learners to focus on understanding core principles rather than struggling with abstract representations. Embodied Cognition (Lakoff & Núñez, 2000) further supports this approach by establishing that even abstract mathematical concepts are grounded in physical experiences, providing theoretical justification for concrete materials in mathematics education.

Bridging Number and Algebraic Understanding

The theoretical connections between numerical and algebraic understanding provide important foundations for grade 7 mathematics instruction. Kaput et al. (2017) developed a comprehensive theoretical framework that reconceptualizes algebra not as a separate domain but as a strand of mathematical thinking that should develop alongside arithmetic understanding from early education onward. Their theoretical model identifies three primary strands of algebraic reasoning: the study of structures and systems abstracted from computation, the study of functions and relations, and the application of modeling languages for expressing and supporting reasoning about situations.

Carraher et al. (2006) further developed this theoretical integration through their research on how children develop algebraic thinking through generalized arithmetic. Their theoretical approach emphasizes that "arithmetic has an inherently algebraic character when it is conceived in terms of operations on sets rather than simply as computation". This perspective suggests that the arithmetic-to-algebra gap often observed in Grade 7 students results not from developmental constraints but from instructional approaches that emphasize computational procedures over structural understanding.

Design Principles for Mathematical Manipulatives

The design of mathematical manipulatives requires careful consideration of several key principles identified through research. Moyer-Packenham and Westenskow (2013) conducted a comprehensive analysis of manipulative design features, finding that effective materials must align closely with mathematical concepts while remaining accessible to learners. Their research emphasizes the importance of clear connections between physical representations and mathematical ideas.

The theoretical foundations for effective manipulative design have been explored by several researchers. Laski et al. (2015) examined what makes mathematics manipulatives effective by drawing lessons from cognitive science and Montessori education. Their analysis identified four key principles for manipulative effectiveness: (1) design that connects directly to mathematical concepts, (2) explicit scaffolding that bridges concrete and abstract representations, (3) consistent use over time, and (4) gradual removal of the physical support.

Comprehensive conceptualizations of manipulatives in mathematics education have been provided by Bartolini and Martignone (2020). Their research establishes a theoretical framework for understanding how manipulatives function in the learning process, distinguishing between their roles as tools for representation, exploration, and concept development. This nuanced understanding of manipulative functions provides crucial guidance for designing materials that support specific learning objectives.

III. Methodology

Research Design

The researchers employed design-based research (DBR) to develop tangible mathematics materials for grade 7 algebra instruction. This methodology was chosen for its effectiveness in connecting theoretical principles with classroom practice while facilitating iterative improvements.

The development process followed four sequential phases:

1. Initial Design: We conceptualized materials based on needs assessment findings, creating preliminary designs with Lightburn software specifically targeting identified challenges in fraction learning.
2. Prototype Development: We constructed initial prototypes from chipboard to test dimensions, proportions, and basic design elements.
3. Expert Evaluation and Refinement: Five mathematics educators assessed the materials using the Department of Education's assessment rubric for manipulatives. We analyzed their feedback and implemented appropriate modifications.
4. Final Production: We manufactured the refined designs using clear acrylic materials, incorporating color coding and labeling systems for the complete manipulative set.

Participants and Subject of the Study

The development phase brings together multiple stakeholder groups to support the development and validation of tangible mathematics materials:

Panel Evaluators. Five mathematics educators providing critical assessment of material design and educational effectiveness.

Teacher Reviewers. Experienced grade 7 mathematics teachers offering practical insights into implementation feasibility.

Learner Testers. 84 grade 7 learners providing essential feedback on material usability and engagement.

For the preliminary testing, the 84 grade 7 learners were divided into control ($n=37$) and experimental ($n=47$) groups based on their intact class sections. The distribution between the groups showed the experimental group containing 56% of total participants and the control group having 44%.

Research Instruments

Panel Evaluation Forms. Incorporated detailed rubrics for content evaluation, technical accuracy assessment, and instructional design review, ensuring comprehensive assessment of material quality.

Teacher Feedback Forms. Focused on material usability assessment, implementation feasibility review, and suggestion documentation, providing practical insights for refinement.

Learner Testing Protocols. Included structured observations of material interaction, understanding check assessments, and engagement evaluation forms, offering direct evidence of material effectiveness from the learner perspective.

Tangible Mathematics (Figure 1). The study developed and utilized specific manipulatives designed to support rational number understanding, including:

- Fraction Circles: Clear acrylic manipulatives with 5-inch diameter, designed for exploring fraction relationships and equivalence
- Fraction Bars: 18.25-inch by 1.5-inch clear acrylic bars for fraction comparisons and operations
- Fraction Tiles: Clear acrylic tiles for rational number operations
- Operational Tiles: 2-inch by 1.5-inch tiles featuring mathematical symbols and operations



Fig. 1. Tangible Mathematics

Ethical Considerations

This study adhered to established educational research ethics guidelines. Written informed consent was secured from all participants, including teachers and panel experts. For learner participants, the researchers obtained both parental/guardian consent and learner assent using age-appropriate forms that clearly explained the research purpose, procedures, and confidentiality measures. All data were anonymized during collection, with participants assigned identification codes to protect their identities. Data storage followed secured protocols in compliance with institutional data protection policies. Participation was voluntary, and participants were informed of their right to withdraw without consequence at any point during the study.

Development Ethics

The study maintains ethical standards throughout the development process. The development process adheres to ethical material development practices, ensuring content appropriateness and cultural sensitivity. Learner testing environments are carefully monitored to ensure participant safety and comfort. All feedback and testing data are handled confidentially, with careful attention to protecting participant privacy while maintaining the integrity of the development process.

Data Collection and Analysis

Data collection focused on gathering detailed feedback for material development and refinement through panel validation, teacher review, and learner testing. The analysis employed both quantitative and qualitative methods:

- Panel Evaluation Analysis: Examination of content validation scores, synthesis of panel recommendations, and documentation of refinement needs.
- Teacher Feedback Analysis: Examination of usability ratings and thematic analysis of suggestions.
- Learner Testing Analysis: Analysis of interaction patterns, assessment of material effectiveness through observational data and direct feedback, and statistical comparison of control and experimental group performance.

IV. Results and Discussion

Development of Tangible Mathematics

The developed manipulatives, "Tangible Mathematics," consist of two main component types: fraction-based manipulatives and operational tiles, each designed with specific features to support mathematical learning. All components were manufactured using clear acrylic material, chosen for its durability and visual clarity.

The design incorporated several key features to enhance educational effectiveness:

- A systematic color-coding system across all fraction pieces
- Movable connections enabling learners to align and manipulate pieces during problem-solving
- Clear fraction markings applied using acrylic spray paint
- Precise laser-cutting techniques ensuring accurate mathematical representations

Evaluation of the Manipulatives: Tangible Mathematics

The tangible mathematics materials achieved exceptional evaluation scores across all assessment categories. As shown in Table 1, Content quality (Factor A) received 37.8 out of 40 points (94.5%), with all ten indicators rated "Very Satisfactory" (3.26-4.00 on a 4-point scale). This demonstrates that the materials effectively support curriculum objectives while engaging student interest through appropriate design features.

Table 1 Evaluation In Terms Of Factor A (Content)

Factor A. Content	Mean Rating	Descriptive Interpretation
1. Content reinforces, enriches, and/or leads to the mastery of certain learning competencies for the level and subject it was intended.	3.8	Very Satisfactory
2. Material has the potential to arouse interest of the target users.	4	Very Satisfactory
3. Facts are accurate.	4	Very Satisfactory
4. Information provided is up – to – date.	4	Very Satisfactory
5. Visuals are relevant to the text.	3.6	Very Satisfactory
6. Visuals are suitable to the age level and interests of the target user.	3.6	Very Satisfactory
7. Visuals are clear and adequately convey the message of the subject or topic.	3.8	Very Satisfactory
8. Typographic layout/design facilitates understanding of concepts presented.	3.8	Very Satisfactory
9. Size of the material is appropriate for use in school.	3.6	Very Satisfactory
10. Material is easy to use and durable.	3.6	Very Satisfactory
Total Points	37.8	Passed

*Note: Score must be at least 30 points out of a maximum 40 points to pass this criterion.
Very Satisfactory, 3.26 – 4.00; Satisfactory, 2.56 – 3.25; Poor, 1.76 – 2.50; and Not Satisfactory, 1.00 – 1.75.*

For Factor B, which focused on identifying potential errors, the materials received a perfect score of 16 out of 16 points, signifying the complete absence of conceptual, factual, grammatical/typographical, and other technical errors, as shown in Table 2.

Table 2 Evaluation In Terms Of Factor B (Other Findings)

Indicators	Mean Rating	Descriptive Interpretation
1. Conceptual errors	4.00	Not Present
2. Factual errors	4.00	Not Present
3. Grammatical and/or typographical errors	4.00	Not Present
4. Other errors (i.e., computational errors, obsolete information, errors in the visuals, etc.)	4.00	Not Present
Total	16	Passed

*Note: Score must be at least 16 points out of a maximum 16 points to pass this criterion.
Not Present, 3.26 – 4.00; Present but with very minor and must be fixed, 2.56 – 3.25; Present and requires major redevelopment, 1.76 – 2.50; and Poor, 1.00 – 1.75.*

The evaluation of Factor C shown in Table 3, examining both instructional and technical design aspects, achieved a total score of 22 out of 24 points, which substantially exceeds the minimum passing requirement of 18 points. The results demonstrate strong performance across both design categories, with most indicators falling within the "Very Satisfactory" range.

Table 3 Evaluation In Terms Of Factor C (Additional Requirements For Manipulative)

Indicators	Mean Rating	Descriptive Interpretation
Instructional Design	3.4	Very Satisfactory
1. Adequate support material is provided.		
2. Activities are summarized, extension activities are provided.	4.0	Very Satisfactory
3. Suggested activities support innovative pedagogy.	3.6	Very Satisfactory
Technical Design		
4. Manipulative is safe to use.	4.0	Satisfactory
5. Size and composition of manipulative is appropriate for intended audience.	3.2	Very Satisfactory
6. Suggested manual tasks within the activities are compatible with the motor skills of the intended users.	3.8	
Total	22	Passed

*Note: Score must be at least 18 points out of a maximum 24 points to pass this criterion.
Very Satisfactory, 3.26 – 4.00; Satisfactory, 2.56 – 3.25; Poor, 1.76 – 2.50; and Not Satisfactory, 1.00 – 1.75.*

Overall, the evaluation results indicate that the tangible mathematics manipulatives not only met but substantially exceeded the minimum requirements prescribed by the Department of Education for manipulatives across all evaluation areas, as shown in Figure 2.

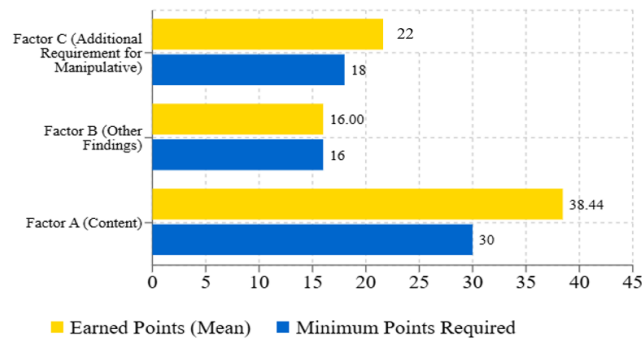


Fig. 2. Summary of Ratings for the Areas of Evaluation

Teacher Feedback Analysis

The analysis of teacher feedback shown in Table 4 revealed five major themes: Physical Design Features, Instructional Applications, Learning Impact, Extended Applications, and Areas for Enhancement.

Table 4 Thematic Analysis of Teachers' Feedback with Supporting Quotes

Theme Category	Key Elements	Supporting Quotes
Physical Design Features	- Precise sizing - Attractive colors - Tangible/detachable components - Easy to manipulate	"The size is very precise" "The color is attractive to students" "It can easily be manipulated and colorful" "Color combinations and the tangible/detachable manipulatives"
Instructional Applications	- Preparatory/motivational activities - Group learning activities - Fraction operations teaching - Concept visualization support - Recreational learning tool	"It can be used as preparatory activity or motivation before the lesson proper" "Can be used best in group activities" "the manipulatives can be a recreational tool as motivation in the lesson"
Learning Impact	- Enhanced concept understanding - Improved attitude toward fractions - Concrete learning experiences - Learner engagement - Concept visualization	"help the students visualize fractions" "This will help students not to hate fractions" "Concrete learning among learners" "education value to help students appreciate and understand more in fractions"
Extended Applications	- Measurement concepts - Algebraic expressions - Different fraction forms - Improper fractions - Dissimilar fractions	"Measurement" "Can be used in Rational Algebraic Expressions" "teaching improper or dissimilar fractions"
Areas for Enhancement	- Durability concerns - Size adjustments needed - Suggestions for larger dimensions - Prevention of piece loss	"Best if students can manipulate other fraction forms" "Durability" "size of the materials and its durability" "Bigger sizes and more colorful" "the size so it won't get lost easily"

Note: Themes derived from qualitative analysis of teachers' open-ended responses

Teachers identified several strong features of the manipulatives, including precise sizing, attractive color scheme, and tangible/detachable nature. They emphasized how the manipulatives helped learners visualize fractions and facilitated learner engagement through their interactive design.

For instructional applications, teachers identified diverse ways to integrate the manipulatives into their teaching practice, particularly for preparatory activities, motivation, and group work. They noted significant positive effects on learner learning, especially in helping learners better understand mathematical concepts and reducing negative attitudes toward fractions.

Teachers also suggested potential improvements, primarily focused on physical aspects such as durability and size, with recommendations for larger dimensions to enhance usability in classroom settings.

Learner Testing Phase

The preliminary testing with 84 grade 7 learners showed promising results. Initially, as shown in Table 5, both the control and experimental groups started at relatively similar levels, with no significant difference at pretest ($U = 692$, $z = -1.60$, $p > 0.05$). Following the intervention, both groups showed significant improvement, but the experimental group demonstrated significantly better performance in the posttest ($U = 456$, $z = -3.73$, $p < 0.001$).

Table 5 Between-Group Comparisons (Mann-Whitney U Test)

Comparison	U-value	z-score	p-value	Interpretation
Pretest	692	-1.60	$p > 0.05$	Not significant; groups started at similar levels
Posttest	456	-3.73	$p < 0.001$	Highly significant difference between groups

Note: Mann-Whitney U test was selected due to non-normal distribution of test scores as determined by Shapiro-Wilk test ($p < 0.05$). Effect size $r = 0.41$ indicates a medium to large effect according to Cohen's criteria. Groups were equivalent at pretest ($p > 0.05$), confirming that posttest differences can be attributed to the intervention rather than pre-existing differences between groups.

The experimental group achieved a higher posttest mean score of 22.23 (SD = 4.02) compared to the control group's mean of 19.35 (SD = 2.54), with a larger mean gain of 16.72 points versus 14.65 points for the control group, as shown in Figure 3.

The graph in Figure 1 displays mean test scores for both groups, illustrating the significantly greater improvement observed in the experimental group using tangible mathematics materials ($n=47$) compared to the control group ($n=37$). Error bars represent standard deviations.

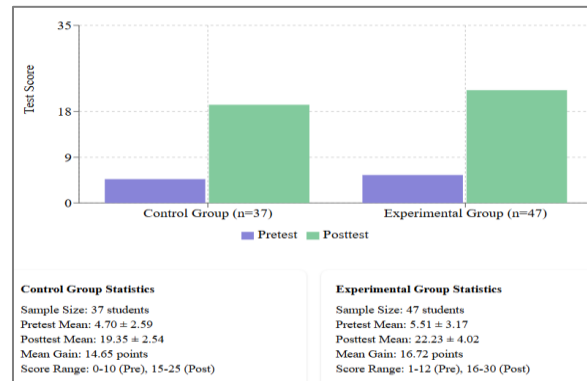


Fig. 3. Before and After Results Comparison

The learner feedback, as shown in Table 6, was overwhelmingly positive, with five major themes emerging: Enhanced Learning Experience, Engagement and Enjoyment, Learning Support, Visual Appeal, and Suggestions for Improvement. Learners particularly emphasized improved understanding and ease of learning, active participation, and the fun learning environment created by the manipulatives.

Table 6 Thematic Analysis Of Learner Feedback

Enhanced Learning Experience	Understanding and Comprehension	"I understand the lesson" "It helps most in understanding fractions in math" "It's easy to understand" "I like the math manipulatives because we can understand the solution"
	Ease of Learning	"It explains more than just teachers talking about it" "It's easy to use" "It's easy to solve and answer the questions" "I can easily learn fast" "The manipulatives are easy to use"
Engagement and Enjoyment	Active Participation	"I like to participate in class when we use these math manipulatives" "I feel excited when I get to work with these math manipulatives" "I enjoy working with my classmates when we use these manipulatives"
	Fun Learning Environment	"Combination of playing and learning math" "It feels like I am building something and turn back to I was a kid" "Cool and useful" "Manipulatives are cool"
Learning Support	Instructional Aid	"It helps us to learn math" "We can learn more about math" "It helps me learn in math" "It helps a lot when teachers discuss" "It helps me to know better in math"
	Educational Value	"The math manipulatives improve our knowledge of solving" "They are amazing and satisfying. They are useful for learning" "This can help us learn many lessons" "It can improve the lesson more"
Visual Appeal	Aesthetic Elements	"The color scheme is quite amazing" "These math manipulatives make learning interesting"
Suggestions for Improvement	Physical Modifications	"It will be cooler if it has magnet" "Bigger sizes"

Classroom observations in Table 7 confirmed these findings, revealing high levels of learner engagement, effective implementation of the manipulatives, and successful progression from concrete to abstract understanding, as shown.

Table 7 Thematic Analysis Of Classroom Observations

Major Theme	Sub-themes	Key Observations
Physical and Pedagogical Preparedness	- Classroom Organization - Instructional Planning	- Well-organized and accessible materials - Appropriate arrangement for group work - Strategic placement of visual tools - Clear learning objectives - Structured progression of concepts - Integration of manipulatives with curriculum
Effective Teaching Strategies	- Direct Instruction - Conceptual Development	- Clear communication of objectives - Proper demonstration techniques - Guided practice opportunities - Explicit connections between concrete and abstract concepts - Progressive movement from manipulatives to symbolic representation - Multiple strategy approaches
Learner Engagement and Participation	- Active Learning - Collaborative Learning	- Consistent learner enthusiasm - High levels of participation - Initiative in asking questions - Effective group work - Peer interactions - Shared problem-solving experiences
Learning Process Development	- Problem-Solving Skills - Conceptual Understanding	- Active use of manipulatives - Development of multiple strategies - Application of concepts - Clear explanations using materials - Successful symbolic representation - Connection between concrete and abstract concepts
Assessment and Learning Outcomes	- Continuous Assessment - Learning Effectiveness	- Regular formative assessment - Consistent feedback - Monitoring of progress - Enhanced learner learning - Successful concept integration - Demonstrated understanding

Theoretical Implications

The effectiveness of the tangible mathematics materials aligns with and extends our understanding of key learning theories. The significant improvement in student performance supports Constructivist principles (Piaget, 1952; Bruner, 1966) by demonstrating how physical manipulation facilitates the construction of mathematical knowledge. Students' enthusiastic engagement and reported improved understanding confirm that concrete experiences provide effective scaffolding for abstract mathematical concepts.

Consistent with Cognitive Load Theory (Sweller, 1988), our observations revealed that students using the manipulatives demonstrated greater capacity to focus on conceptual relationships rather than procedural mechanics. When students physically manipulated fraction representations, they appeared to externalize some of the cognitive processing demands, allowing more mental resources for higher-order thinking. This was particularly evident when students progressed from manipulating physical pieces to working with symbolic representations.

The findings also support Embodied Cognition theory (Lakoff & Núñez, 2000), as students frequently referenced their physical experiences with the manipulatives when explaining abstract fraction concepts. Their use of gestures mirroring previous physical manipulations, even when working with symbolic representations, suggests that the embodied experiences facilitated their conceptual understanding.

These theoretical connections provide a robust foundation for further development of tangible approaches in mathematics education, particularly at the critical transition point between concrete and abstract mathematical thinking.

Practical Implementation Recommendations

Based on teacher feedback and classroom observations, we offer the following recommendations for effectively implementing tangible mathematics materials in grade 7 classrooms:

- Sequential Implementation:** Begin with exploratory activities that familiarize students with the manipulatives before introducing formal operations. Teacher feedback indicated that students gained more from the manipulatives when given time to explore relationships without immediate performance pressure.
- Explicit Connections:** Deliberately guide students in connecting physical manipulations to symbolic representations. Effective implementation involved teachers explicitly verbalizing these connections and encouraging students to explain concepts using both physical and symbolic references.
- Group Dynamics:** Organize students in small groups (3-4 students) when using manipulatives to encourage collaborative problem-solving and verbalization of mathematical thinking. Observation data showed higher engagement and more mathematical discourse in appropriately sized groups.
- Material Management:** Develop classroom routines for distributing, using, and collecting manipulatives to maximize instructional time. Teachers noted that established procedures reduced transition time and materials management issues.
- Assessment Integration:** Incorporate manipulative-based assessments alongside traditional evaluations to provide multiple ways for students to demonstrate understanding. This approach acknowledges diverse learning preferences while maintaining rigorous assessment standards.

These recommendations reflect both the quantitative findings and qualitative insights gathered throughout the study and provide practical guidance for educators seeking to implement tangible mathematics approaches in their classrooms.

Limitations and Future Directions

While this study demonstrates promising results for tangible mathematics materials, several limitations should be acknowledged. The sample size of 84 grade 7 learners, though sufficient for preliminary testing, limits broad generalizability. The distribution between experimental ($n=47$) and control ($n=37$) groups was not perfectly balanced, which may have influenced comparative results.

The relatively short implementation period (approximately one month) may not have captured potential long-term effects of the manipulatives on mathematical understanding. Additionally, the study was conducted in Philippine setting, and cultural or educational system factors may influence how these materials perform in different contexts.

The research design, while including both quantitative and qualitative components, could be strengthened in future studies through more extensive classroom observations and longitudinal assessment of learning outcomes. Future research should address these limitations by:

1. Extending implementation to larger and more diverse student populations
2. Conducting longitudinal studies to assess retention of concepts learned through tangible approaches
3. Exploring the effectiveness of these materials across different educational contexts and teaching styles
4. Investigating potential variations in effectiveness based on student learning preferences and prior achievement levels.

Generalizability of Findings

While our results show promising outcomes for the tangible mathematics approach, we must consider the context-specific factors that may influence generalizability. The study was conducted within Philippines, which has particular characteristics including curriculum structure, teaching approaches, and cultural context. The significant improvement demonstrated in learner performance suggests potential transferability to similar educational environments.

However, implementation in substantially different contexts may require adaptations to accommodate varying:

1. Educational systems and curriculum structures: Different educational systems may sequence mathematical concepts differently or emphasize different aspects of rational number understanding.
2. Teacher preparation and pedagogical approaches: Effectiveness depends partly on teacher familiarity with manipulative-based instruction and their ability to connect concrete experiences with abstract concepts.
3. Resource availability: The materials developed for this study used specific manufacturing techniques and materials that may not be universally available.
4. Student populations: While our sample included students from diverse achievement levels, broader application across different socioeconomic, cultural, or linguistic contexts would benefit from additional validation.

Future research could strengthen generalizability by implementing these materials across multiple school types, geographical regions, and student populations. A multi-site replication study would provide valuable insights into which aspects of the approach remain effective across contexts and which may require adaptation.

V. Conclusion

The development phase of this study has successfully created and validated tangible mathematics materials specifically designed to address the identified needs of grade 7 learners in understanding rational numbers. The systematic design-based research approach resulted in manipulatives that were highly rated by expert evaluators, enthusiastically received by teachers, and demonstrably effective in preliminary learner testing.

The panel evaluation demonstrated exceptional quality and precision, with the materials exceeding minimum requirements across all evaluation areas. The content evaluation yielded an impressive score of 37.8 out of 40 points, the technical accuracy assessment achieved a perfect score of 16 out of 16, and the instructional and technical design evaluation scored 22 out of 24 points.

Teacher feedback highlighted the materials' potential for enhancing learner understanding through visual and hands-on learning experiences, while also providing practical suggestions for effective classroom use. Particularly significant was teachers' recognition of the manipulatives' potential to improve learners' attitudes toward mathematics, especially concerning fractions.

Preliminary testing with 84 grade 7 learners provided strong evidence of the materials' effectiveness, with the experimental group showing significantly better performance than the control group. Both quantitative assessment data and qualitative classroom observations confirmed that the tangible materials successfully enhanced learner understanding, engagement, and problem-solving abilities in rational number operations.

These findings provide a strong foundation for the subsequent implementation phase of this research, suggesting that the developed tangible mathematics materials have significant potential to address the persistent challenges in grade 7 mathematics education, particularly in the critical area of rational numbers.

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