

Analytical Approach for Optimal Inventory Policies with Salvage Value and Partial Backlogging: Convexity Insights and Sensitivity Observations

Bhawna Vyas¹, Mahender Poonia², Garima Sharma³

^{1,2} Om Sterling Global University, Hisar, Haryana, India

³ Mody University of Science & Technology, Laxmangarh, Rajasthan, India

DOI: <https://doi.org/10.51583/IJLTEMAS.2025.140500017>

Received: 18 May 2025; Accepted: 22 May 2025; Published: 31 May 2025

Abstract- The paper shows optimization of an inventory model with the considered assumptions that the demand rate is quadratic time-dependent and the deterioration rate is exponential. Further we have included the assumptions of the salvage value and partial backlogging. Taking ideas by the limitations of prior research where constant or linear demand models are taken, the given inventory model considers scenarios in which the customer demand changes non-linearly (quadratically), and the item deterioration has an exponential rate pattern, commonly observed in the industries that deal with the decaying or technology driven items or products. A total expense function is formulated, and an optimal replenishment time interval is calculated to minimize overall inventory expenses. The convexity of the inventory expense function is analytically checked with the help of MAPLE 2025 to make sure that the unique optimal solution of this optimized inventory model exists. Also, a sensitivity analysis is performed using MAPLE to check the influence or impact of key inventory parameters on the optimal solutions, providing important results for decision-making. Further, the effects of the salvage value on the optimization are shown to reflect practical applicability in the modern inventory systems. Numerical examples, supported by graphical analysis and explanation, shows the significant applicability and efficiency of this given inventory model along with the dynamic market conditions.

Keywords- Quadratic time-dependent demand, Exponential deterioration, Salvage value, Partial backlogging, Convexity analysis, Sensitivity analysis

I. Introduction

Inventory simply means by the stock of goods to be consumed for sale, resale or production purpose in the industries or business models. An efficient and accurate inventory system focuses on reducing holding expenses, reducing or preventing the shortages [20], and improving overall customer requirement fulfilment [17], [14]. Many researchers have worked upon various inventory models to align with the practical life inventory problem complexities, for example, taking deteriorating items [19], [20], dynamic demand rate change patterns, salvage value benefit, and the partial backlogging. Earlier work by Dave (1989) [1][2] showed the studies on inventory systems with time-varying demand rates and the stock shortages, giving the framework for more improved models. Next the works such as Ouyang and Cheng (2005) [3] and Soni et al. (2006) [4] took the partial backlogging and credit policies in consideration, giving more realistic inventory system environments. Alamri and Balkhi (2007) [5], and Teng et al. (2007) [6] observed the effects of learning, forgetting, and the pricing with the deteriorating conditions, while Chung and Wee (2008) [7] worked on the inventory models with inspection of planning and stock-dependent demand. The consideration of time-dependent holding costs and variable demand rates became more prevalent in the later studies, such as the studies those by Roy (2008) [8], Tripathy and Mishra (2010) [9], and Begum et al. (2012) [10], further used the concept of quadratic or the cost dependent demand rate along with the decays or deterioration of the inventory items. Further, models by Amutha and Chandrasekaran (2013) [11], and Mishra (2013) [12], combined the concept of quadratic demand rate and the partial backlogging to better represent consumer behaviour in real life dynamic markets. The recent studies by Khare and Sharma (2022) [14] further improved this line of research by analysing time-dependent deterioration and variable costs. Panigrahi et al. (2023) further worked upon the study which shows the uses of RFID (Radio Frequency Identification technology) and the concept of VMI (Vendor Managed Inventory stocks) which brings improvements in the earlier inventory system efficiency and their overall production process [15]. Ogbonna et al. (2024) further explained the deterministic inventory system model with the two-stage stock system along with the assumptions of deteriorating items with the real-life complexities added such as increasing inflation rates and the salvage value [16]. The modern need for sustainable and cost-effective inventory practices requires the combination of salvage value into inventory models, as demonstrated by Khare and Sharma (2024) [19][20], and supported by Prameswari et al. (2024) [18], who showed the circular economy frameworks for more practical inventory systems. The study works upon an EOQ inventory model framework that considers all these aspects, along with variable holding costs and convexity verification, thereby contributing to a more robust and practically applicable inventory optimization model.

Notations

C_1 : It represents the holding cost of the inventory stock per unit and per unit time

C_2 : It represents the shortage stock cost per unit and per unit time

C_3 : It represents the ordering expense of inventory in each complete cycle

C_4 : It represents the Deterioration expense

δ : It represents the salvage value constant

t_1 : It is the time when the shortages of the inventory start

T : It shows the length of a complete ordering cycle

W : It is the maximum level of inventory at time $t = 0$ for complete ordering cycle

S : It shows the maximum amount of shortage that is backlogged for each complete ordering cycle

Q : It shows the order quantity for the given ordering cycle ($Q = W + S$)

$I(t)$: It is the level of inventory at some time t

TIC : It is denoted for the Total inventory cost over the full cycle period $(0, T)$

Assumptions

1. This inventory model considers only one product or item.

2. We have taken the lead time as zero.

3. The rate of demand is quadratic time dependent as:

$$R(t) = a + bt + ct^2 ; a \geq 0, b \geq 0, c \geq 0$$

4. The rate of decay of deterioration at any point of time $t > 0$ follows the Weibull Distribution with two parameters. $\theta(t) = e^{\alpha t}$; $0 < \alpha < 1$;

5. The partial backlogging rate is taken as: $e^{-u(T-t)}$

6. Salvage value constant is δ ; $0 < \delta < 1$

Mathematical formulation

The demand rate is taken to be a quadratic function of time. The inventory items are assumed to have an exponential time-dependent deterioration. Shortages are allowed in our inventory system, and it is assumed that only a part of the customers are willing to wait for backlogged items. A salvage value is considered for leftover or the deteriorated inventory at the end of the complete inventory cycle. Various cost elements are as below: Ordering or Setup cost, Holding cost, Deterioration cost, Shortage and the backlogging cost and the Credit from salvage value. The total inventory cost function is mathematically formulated by summing all relevant costs and deducting salvage value returns. The convexity of the total cost function is mathematically verified using MAPLE 2025. The derived cost function and its first and second derivatives are handled using MAPLE 2025. A detailed numerical example is solved to validate the model optimization. Graphs are drawn to observe the behaviour of the total cost function and decision variables. Practical insights are drawn for inventory managers.

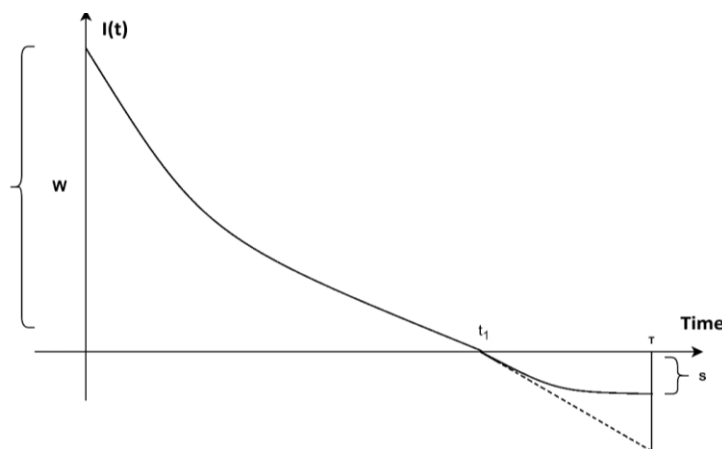


Figure 1: Inventory level $I(t)$ Vs time

Based on upon Mathematical formulations, the inventory model is formulated into two main differential equations as given below:

$$\frac{d}{dt}I_1(t) = -(e^{\alpha t}) - ct^2 - bt - a ; 0 < t < t_1 \quad [1]$$

$$\frac{d}{dt} I_2(t) = -(a + b t + c t^2) e^{-u(T-t)} ; t_1 < t < T \quad [2]$$

With the boundary conditions (obtained from the given mathematical modelling) are given as:

$$I_1(t_1) = 0$$

$$I_1(0) = W$$

$$I_2(t_1) = 0$$

$$I_2(T) = -S$$

Solving both the equations with the help of given boundary conditions, we get:

$$I_1 = -\frac{\alpha^3 t_1^4}{24} + \frac{(-\alpha^2 + 2c)t_1^3}{6} + \frac{(b-\alpha)t_1^2}{2} + (a-1)t_1 + \frac{\alpha^3 t^4}{24} + \frac{(\alpha^2 - 2c)t^3}{6} + \frac{(-b+\alpha)t^2}{2} + (1-a)t \quad [3]$$

$$I_2 = \frac{1}{120} (((12c t_1^5 + 15(-2Tc + b)t_1^4 + 20(T^2c - 2Tb + a)t_1^3 + 30T(Tb - 2a)t_1^2 + 60T^2at_1 - 60t((\frac{1}{3}c t^2 + \frac{1}{2}bt + a)T^2 - t(\frac{1}{2}c t^2 + \frac{2}{3}bt + a)T + \frac{t^2(\frac{3}{5}c t^2 + \frac{3}{4}bt + a))}{3}))u^2) + \frac{1}{12} ((3c t_1^4 + 4(-Tc + b)t_1^3 + 6(-Tb + a)t_1^2 - 12Tat_1 + 2(2t^3c + 3t^2b + 6at)T - 3c t^4 - 4b t^3 - 6a t^2)u) - \frac{t^3c}{3} + \frac{c t_1^3}{3} - \frac{t^2b}{2} + \frac{t_1^2b}{2} - at + t_1a \quad [4]$$

Maximum level of inventory W at time $t = 0$

$$W = -\frac{\alpha^3 t_1^4}{24} + \frac{(-\alpha^2 + 2c)t_1^3}{6} + \frac{(b-\alpha)t_1^2}{2} + (a-1)t_1 \quad [5]$$

Peak level of Shortage S is:

$$S = -\frac{1}{120} (((12c t_1^5 + 15(-2Tc + b)t_1^4 + 20(T^2c - 2Tb + a)t_1^3 + 30T(Tb - 2a)t_1^2 + 60T^2at_1 - 60T((\frac{1}{3}T^2c + \frac{1}{2}Tb + a)T^2 - T^3(\frac{1}{2}T^2c + \frac{2}{3}Tb + a) + \frac{T^2}{3}(\frac{3}{5}T^2c + \frac{3}{4}Tb + a)))u^2) - \frac{1}{12} (((4(-Tc + b)T^3 + 6(-Tb + a)T^2 - 12Tat_1 + 2(2T^3c + 3T^2b + 6Ta)T - 4b T^3 - 6T^2a)u) + \frac{T^2b}{2} - \frac{t_1^2b}{2} + Ta - t_1a \quad [6]$$

$$Q = W + S$$

$$Q = -\frac{\alpha^3 t_1^4}{24} + \frac{(-\alpha^2 + 2c)t_1^3}{6} + \frac{(b-\alpha)t_1^2}{2} + (a-1)t_1 - \frac{u^2}{120} (12c t_1^5 + 15(-2Tc + b)t_1^4 + 20(T^2c - 2Tb + a)t_1^3 + 30T(Tb - 2a)t_1^2 + 60T^2at_1 - 60T((\frac{1}{3}T^2c + \frac{1}{2}Tb + a)T^2 - T^3(\frac{1}{2}T^2c + \frac{2}{3}Tb + a) + \frac{T^2}{3}(\frac{3}{5}T^2c + \frac{3}{4}Tb + a))) - \frac{1}{12} (((4(-Tc + b)T^3 + 6(-Tb + a)T^2 - 12Tat_1 + 2(2T^3c + 3T^2b + 6Ta)T - 4b T^3 - 6T^2a)u) + \frac{T^2b}{2} - \frac{t_1^2b}{2} + Ta - t_1a \quad [7]$$

Holding Cost HC: $C_1 \int_0^{t_1} I_1 dt$

$$HC = C_1 (-\frac{\alpha^3 t_1^5}{30} + \frac{(\frac{\alpha^2 - c}{6})t_1^4}{4} + \frac{(\frac{-b + \alpha}{2})t_1^3}{3} + \frac{(1-a)t_1^2}{2} + \frac{(-\alpha^2 + 2c)t_1^4}{6} + \frac{(b-\alpha)t_1^3}{2} + (a-1)t_1^2) \quad [8]$$

Shortage Cost SC: $-C_2 \int_{t_1}^T I_2 dt$

$$SC = -C_2 (-\frac{c u^2 (T^6 - t_1^6)}{60} + \frac{1}{5} ((\frac{(30Tc - 15b)u^2}{120} - \frac{cu}{4})(T^5 - t_1^5)) + \frac{1}{4} ((\frac{(-20T^2c + 40Tb - 20a)u^2}{120} + \frac{(4Tc - 4b)u}{12} - \frac{c}{3})(T^4 - t_1^4)) + \frac{1}{3} ((\frac{(-30T^2b + 60Ta)u^2}{120} + \frac{(6Tb - 6a)u}{12} - \frac{b}{2})(T^3 - t_1^3)) + \frac{1}{2} ((-\frac{1}{2}T^2a u^2 + Tau - a)(T^2 - t_1^2)) + \frac{1}{120} ((12c t_1^5 + 15(-2Tc + b)t_1^4 + 20(T^2c - 2Tb + a)t_1^3 + 30T(Tb - 2a)t_1^2 + 60T^2at_1)u^2(T - t_1)) + \frac{1}{12} ((3c t_1^4 + 4(-Tc + b)t_1^3 + 6(-Tb + a)t_1^2 - 12Tat_1)u(T - t_1)) + \frac{c t_1^3 (T - t_1)}{3} + \frac{t_1^2 b (T - t_1)}{2} + t_1 a (T - t_1)) \quad [9]$$

Deterioration cost DC: $C_4 (W - \int_0^{t_1} (a + bt + ct^2) dt)$

$$DC = C_4 (-\frac{\alpha^3 t_1^4}{24} + \frac{(-\alpha^2 + 2c)t_1^3}{6} + \frac{(b-\alpha)t_1^2}{2} + (a-1)t_1 - \frac{t_1^2b}{2} - t_1a) \quad [10]$$

Ordering cost OC = C_3 [11]

Salvage value SV: $\delta(DC)$ [12]

Now calculation the total inventory cost $TIC = \frac{1}{T} (HC + DC + SC + OC - SV)$ [13]

Now to obtain the optimal solutions of Total Inventory cost and Optimal order quantity, we must satisfy the necessary conditions given below:

$$\frac{\partial TIC(t_1, T)}{\partial t_1} = 0, \quad \frac{\partial TIC(t_1, T)}{\partial T} = 0 \quad [14]$$

$$\text{And, } \frac{\partial^2 TIC(t_1, T)}{\partial T^2} > 0 \quad [15]$$

Numerical Example

Solving these Equations using mathematical software MAPLE 2025 for the give numerical example to obtain optimal solutions:

Solving equations (14) and (15) using maple 2025 for the values of parameters:

$$a = 20, b = 3, c = 1, u = 0.09, C_1 = 20, C_2 = 4, C_3 = 200, C_4 = 50, C_5 = 40, \alpha = 0.01, \delta = 0.099$$

We get the results:

Optimal values of T and t_1 (Solving Using Maple 2025):

$$T = 2.225986192 \text{ Years}, t_1 = 0.4149400799 \text{ Years}$$

Optimal Order Quantity:

$$Q_1 = 43.01727250 \text{ Units per Cycle}$$

Optimal Value of Total Inventory cost:

$$OTIC = \text{Rs. } 161.7874720$$

Verification of Convexity of the graphs

Graphs were drawn between TIC and T to check convexity of the graph to ensure that the given inventory system has the global optimal solution.

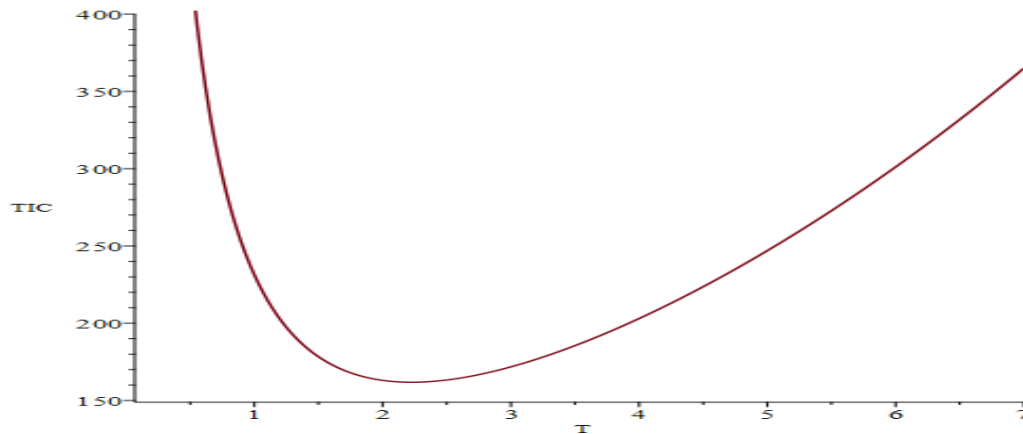


Figure 2: Graph between total inventory cost TIC and T

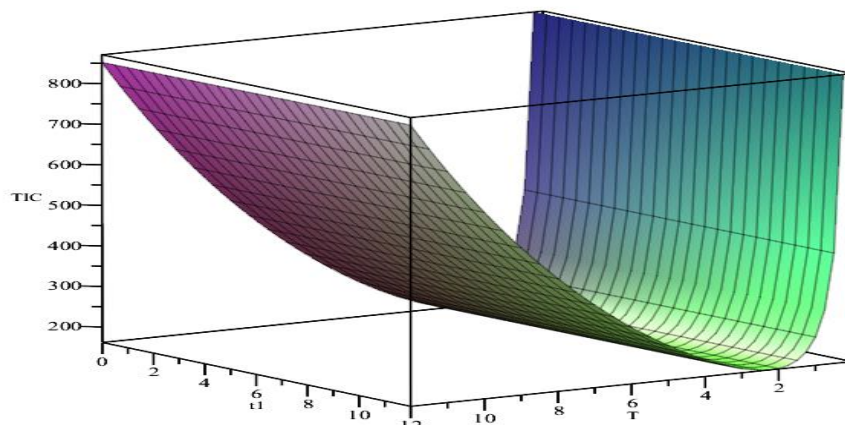


Figure 3: 3D - representation of total inventory cost TIC Vs time

Sensitivity analysis on the Inventory parameters:

Detailed Sensitivity analysis was performed on the key inventory parameters $a, b, c, u, C_1, C_2, C_3, C_4, \alpha, \delta$ on Total Inventory Cost TIC , Complete cycle time T , Depletion time t_1 and optimal order quantity Q_1 .

Effect of the Demand rate Constant a

Table I						
S N	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	a	+20%	2.1001	0.3855	47.8044	175.0908
2		+10%	2.1602	0.3994	45.4583	168.5309
3		-10%	2.2952	0.4321	40.4765	154.8474
4		-20%	2.3922	0.4515	37.8152	147.6962

An increase in the demand constant a gives shorter cycle length T and depletion times. The total inventory cost (TIC) increases due to more usual replenishments. The optimal order quantity also increases as the demand increases. When the demand rate constant a is reduced, the demand rate decreases, which increases the cycle duration and reduces overall cost due to fewer orders needed.

Overall Effect on the Market of the demand constant a :

Markets with higher demand rate need faster inventory turnover which leads to higher operating costs. Efficient logistic models and dynamic order situations become crucial in such inventory settings to prevent frequent shortages and service delays.

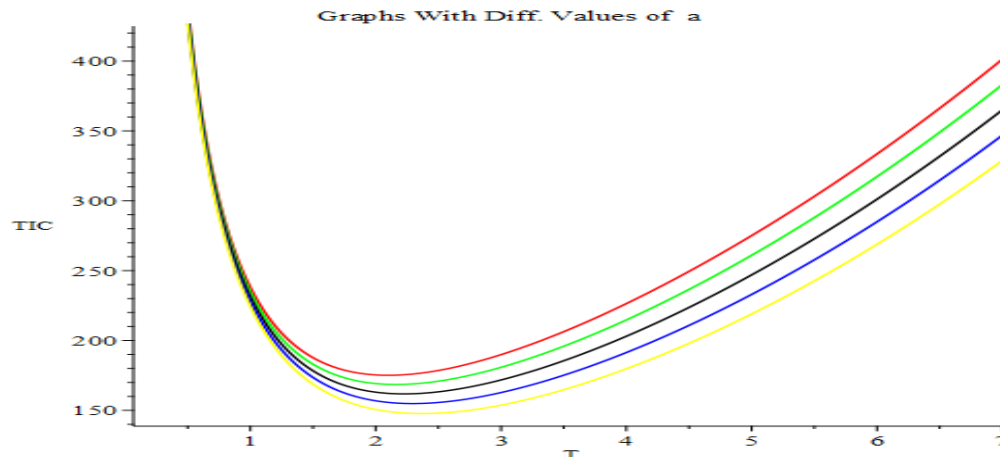


Figure 4: Graph between total inventory cost TIC and T

Effect of the Demand rate constant cost b

Table II						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	b	+20%	2.1896	0.4095	43.8037	163.5316
2		+10%	2.2075	0.4122	43.4167	162.6663
3		-10%	2.2451	0.4177	42.6043	160.8943
4		-20%	2.2650	0.4206	42.1770	159.9862

As the linear rate constant b of the demand rate increases, there is a little decrement in order quantity and an increment in TIC . Higher value of b necessitates marginally shorter cycle times. However, the change in values is relatively small which shows that b is moderately sensitive.

Overall Effect on the Market of the demand rate constant b :

Markets in which the demand grows steadily over time (e.g., in urban expansion or the tech adoption phases) need gradually adaptive inventory policies. By ignoring such linear trends, the system may not cause immediate losses but can accumulate inefficiencies over longer periods.

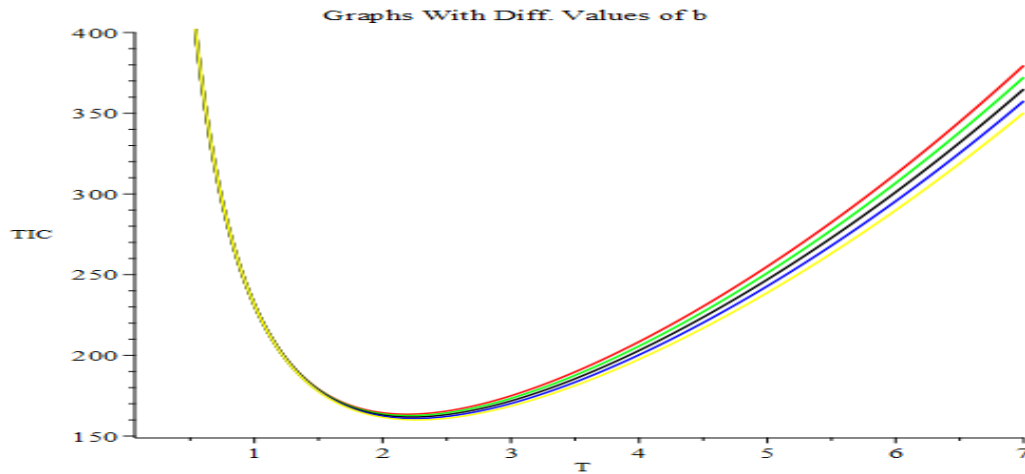


Figure 5: Graph between total inventory cost TIC and T
Effect of the Demand rate constant cost c

Table III						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	c	+20%	2.2035	0.4091	42.6138	162.5511
2		+10%	2.2145	0.4120	42.8124	162.1721
3		-10%	2.2377	0.4179	43.2287	161.3968
4		-20%	2.2499	0.4210	43.4473	160.9999

Changes in the demand rate constant c , which is the quadratic growth rate constant of demand, have a subtle effect. As c increases, TIC increases lightly and Q_1 decreases. This shows the sensitivity is low in the short time periods but could grow with long planning periods or seasonal trends.

Overall Effect on the Market:

The systems which are prone to accelerating demand (like consumer electronics or seasonal products), even small values of c could disrupt the overall plans. Proper forecasting and flexibility in orders help maintain cost-effectiveness.

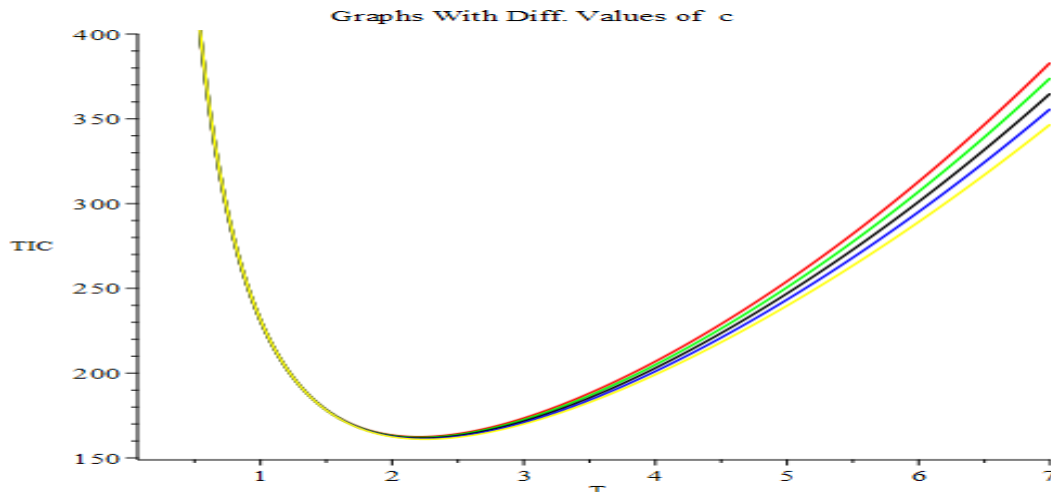


Figure 6: Graph between total inventory cost TIC and T

Effect of the Backlogging constant cost u

Table IV						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	U	+20%	2.2604	0.4106	41.4440	160.4516
2		+10%	2.2430	0.4128	42.2677	161.1226
3		-10%	2.2093	0.4170	43.6992	162.4460
4		-20%	2.1931	0.4191	44.3196	163.0983

As customer tolerance for backlogging increases u , the total cost decreases and the system allows smaller order sizes and longer cycles. Reducing the backlogging constant u leads to large value of the order quantities and cost because of the need to meet demand more rapidly.

Overall Effect on the Market of the Backlogging constant u :

In the markets where customers are ready to wait (e.g. customized or high-value goods), firms benefit from flexible ordering costs. In highly competitive markets where alternatives are already available, a low value of the Backlogging constant u demands greater inventory responsiveness.

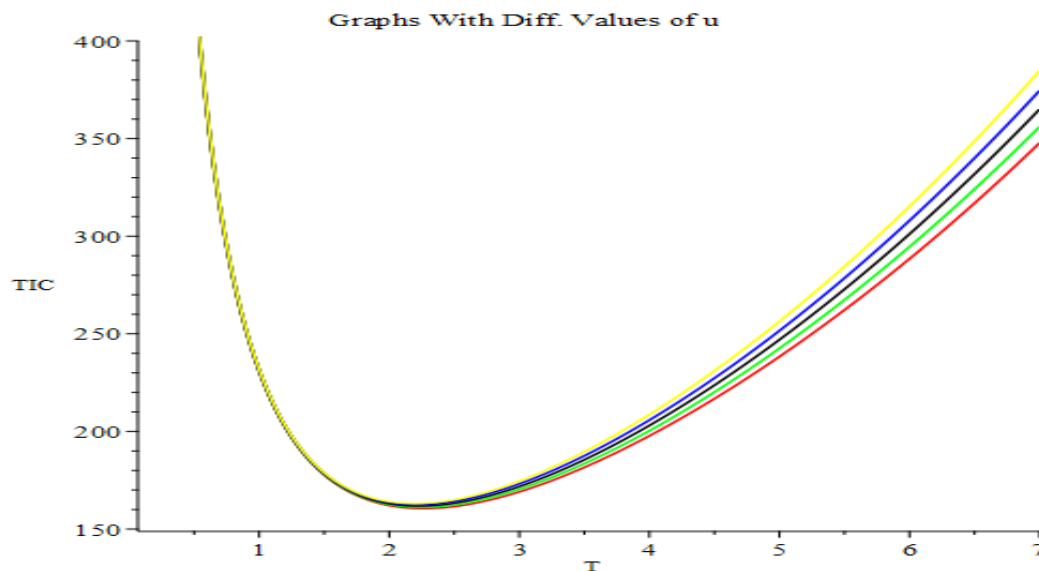


Figure 7: Graph between total inventory cost TIC and T

Effect of the Holding cost C_1

Table V						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	C_1	+20%	2.2169	0.3569	42.7097	164.4340
2		+10%	2.2211	0.3838	42.8485	163.2111
3		-10%	2.2318	0.4531	43.2182	160.1104
4		-20%	2.2388	0.4944	43.4613	158.1081

As holding cost C_1 increases, the inventory system is supposed to contain fewer goods for a shorter time, decreasing t_1 and mildly reducing Q_1 . TIC increases with higher holding costs, emphasizing the penalty for maintaining large inventories.

Overall Effect on the Market of the holding cost C_1 :

In markets where the warehouse rents or utility costs are increasing with time, firms must take up by tightening their inventory policies. Efficient storage and just – in – time delivery systems become more interesting to minimize excessive carrying costs.

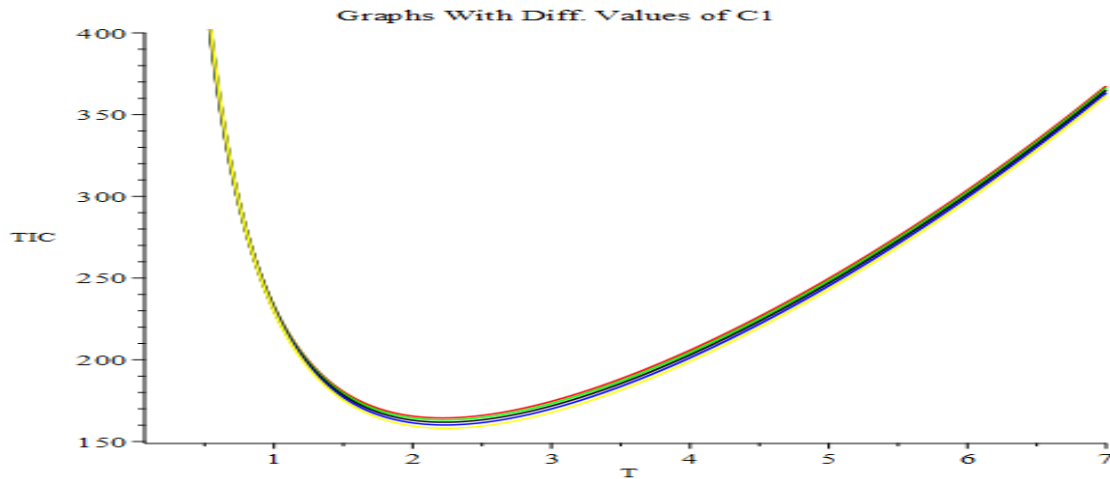


Figure 8: Graph between total inventory cost TIC and T
Effect of the Shortage cost C_2

Table VI						
S N	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	C_2	+20%	2.0667	0.4423	40.4180	173.7916
2		+10%	2.1408	0.4291	41.6464	168.0020
3		-10%	2.3249	0.3995	44.5522	155.0841
4		-20%	2.4416	0.3827	46.2728	147.8108

When shortage cost C_1 rises, the system tends to order more frequently, maintain higher safety stocks, and hence incur greater TIC to avoid expensive stockouts. Conversely, if the shortage costs decrease, longer cycles or large value of T and smaller safety stocks are recommended.

Overall Effect on the Market of the shortage cost C_2 :

Industries where customer loyalty is highly sensitive to the service levels (like pharmaceuticals or luxury goods) face higher penalties for shortages, which makes it crucial to make have robust inventory cushions. Lower shortage costs might apply to commodities with abundant substitutes.

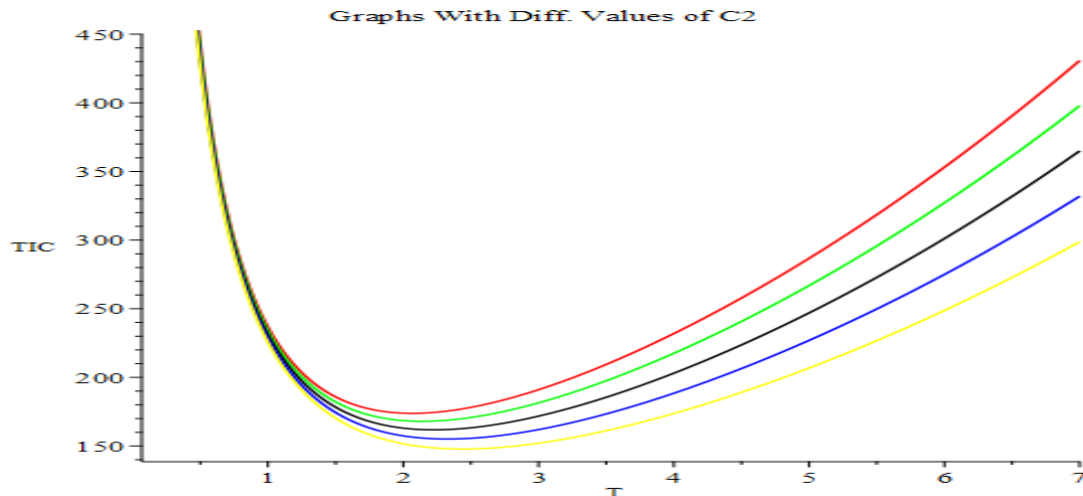


Figure 9: Graph between total inventory cost TIC and T

Effect of the Ordering cost C_3

Table VII						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	C_3	+20%	2.4158	0.4371	46.0628	179.0180
2		+10%	2.3236	0.4264	44.6141	170.5788
3		-10%	2.12200	0.4024	41.2545	152.5884
4		-20%	2.0106	0.3888	39.3034	142.9101

An increment in the ordering cost C_3 leads to larger, less frequent orders (higher T , larger Q_1), inflating TIC as well. Lower ordering costs favour smaller, more frequent orders and cost reductions.

Overall Effect on the Market of the ordering cost C_3 :

Businesses facing rising administrative, transportation, or customs fees per order must optimize by bulk purchasing. Conversely, lower ordering costs enable leaner, faster inventory practices supporting to the supply chains.

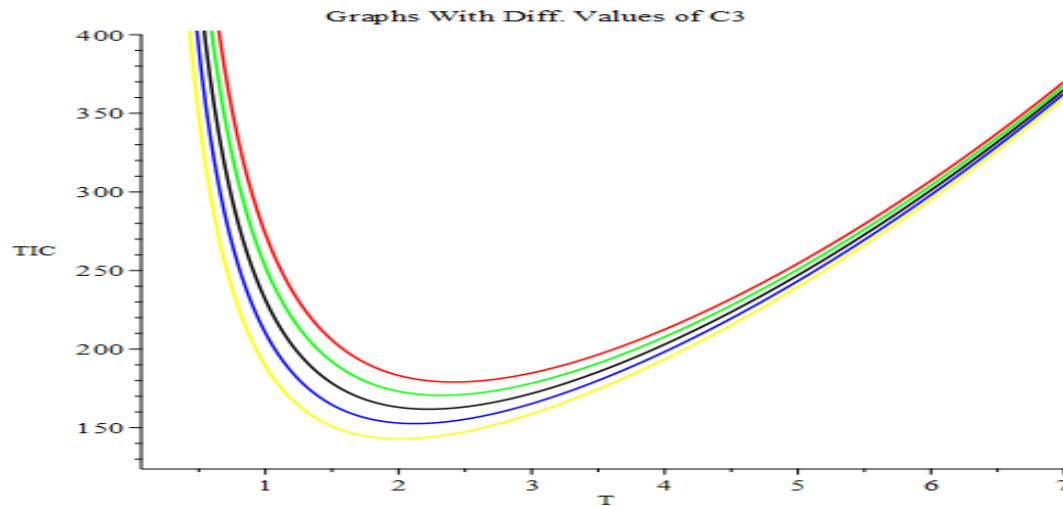


Figure 10: Graph between total inventory cost TIC and T

Effect of the Deterioration cost C_4

Table VIII						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	C_4	+20%	2.2177	0.4282	42.9215	160.1753
2		+10%	2.2219	0.4216	42.9714	160.9877
3		-10%	2.2297	0.4080	43.0590	162.5743
4		-20%	2.2333	0.4009	43.0967	163.3480

Increasing deterioration cost C_4 leads to little decrement in cycle time T and quantity, which shows faster turnover is favoured to minimize losses from spoilage or damages. However, the overall sensitivity is low.

Overall Effect on the Market of the deterioration cost C_4 :

Sectors which deal with decaying or perishable goods (like food, medicines, or batteries) must tighten cycle times as damage penalties increase. Efficient stock rotation and faster delivery cycles become vital to protect margins.

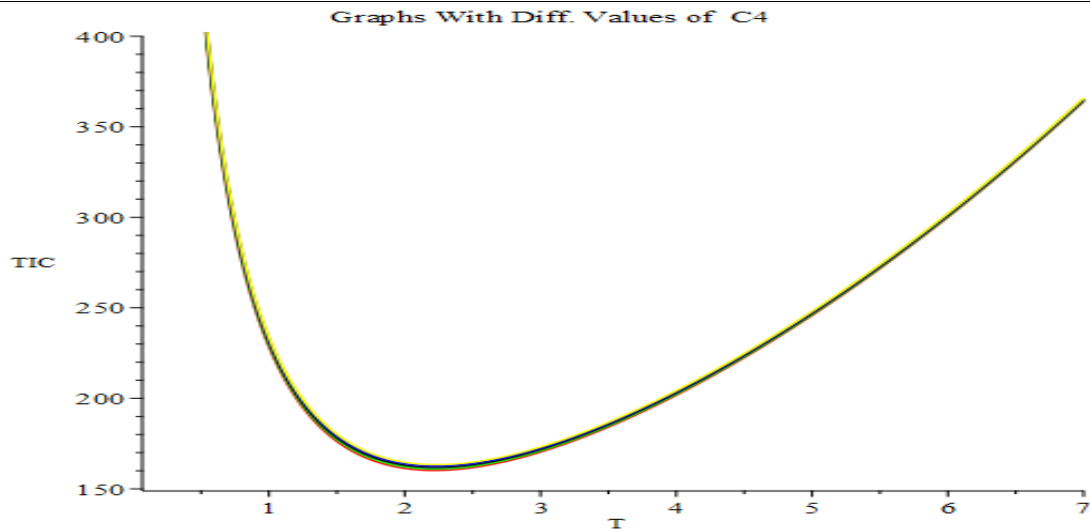


Figure 11: Graph between total inventory cost TIC and T
Effect of the Deterioration rate constant α

Table IX						
SN	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	α	+20%	2.2260	0.4150	43.0176	161.7835
2		+10%	2.2259	0.4149	43.0174	161.7855
3		-10%	2.2259	0.4148	43.0171	161.7894
4		-20%	2.2259	0.4148	43.0169	161.7913

The deterioration rate constant α influences how rapidly items degrade over time. As α increases, goods deteriorate faster, prompting little smaller inventory holdings and adjustments in replenishment timing. However, changes in TIC are mild, suggesting robustness of the model to moderate variations in α .

Overall Effect on the Market of the Deterioration rate constant α :

Industries that deal with decaying or tech-sensitive goods (e.g., food inventory systems, pharmaceuticals and the electronics) must focus on the deterioration or decaying rates of the products. Higher α values demand tighter inventory control and rapid turnover to reduce losses from spoilage or damages.

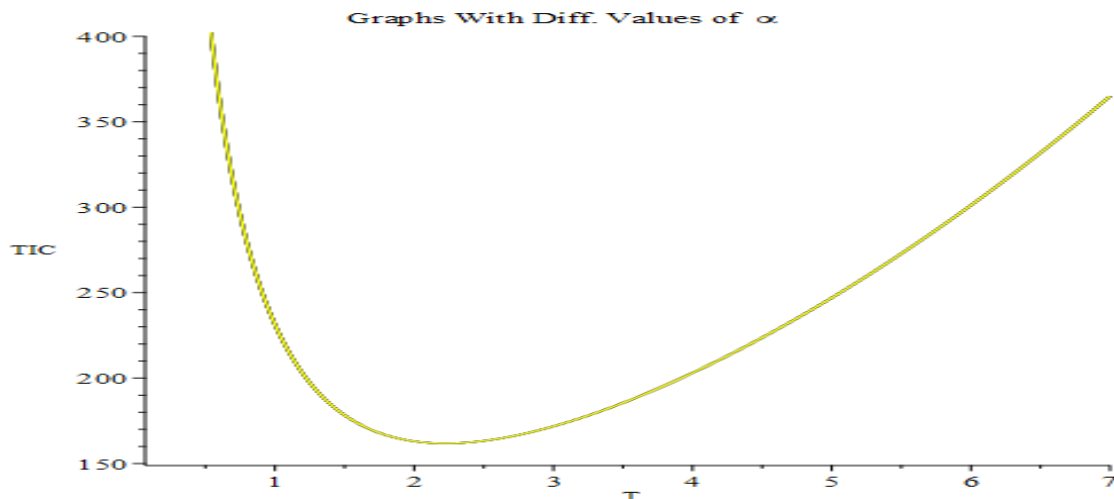


Figure 12: Graph between total inventory cost TIC and T

Table X						
S N	Parameter	% Change	Value of			
			T	t_1	Q_1	TIC
1	δ	+20%	2.2268	0.4313	43.0268	161.9614
2		+10%	2.2264	0.4141	43.0220	161.8745
3		-10%	2.2255	0.4156	43.0124	161.7002
4		-20%	2.2251	0.4164	43.0075	161.6128

δ represents the sensitivity of holding cost to time, which shows inflationary effects or cost escalation over the storage period. An increase in δ influences earlier consumption and shorter holding, slightly adjusting the replenishment time and lowering inventory levels. However, its effect on TIC remains modest in the tested range.

Overall Effect on the Market of the salvage value constant δ :

In inflation-prone market systems or in the sectors where storage becomes increasingly expensive over time, δ becomes critical. For example, warehousing costs may escalate in urban environments, or energy consumption for refrigeration may rise over time. Monitoring deterioration rate constant δ helps businesses pre-emptively manage time-sensitive holding costs.

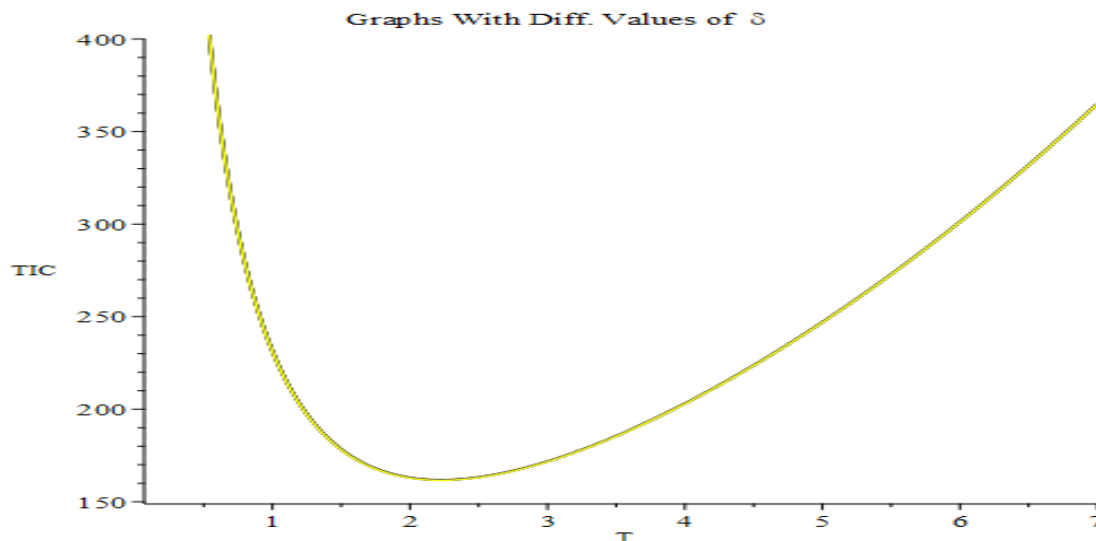


Figure 13: Graph between total inventory cost TIC and T

II. Conclusion

In this study, we have optimized a complicated but realistic inventory model considering the quadratic time-dependent demand, exponentially increasing deterioration rate, salvage value, and partial backlogged stock. By the analysis of convexity verification and mathematical optimization using Maple 2025, we obtained optimal solutions for the key inventory decision variables: complete cycle time T , depletion time t_1 , and order quantity Q_1 : that minimize the total inventory cost. Sensitivity Analysis was performed on all key inventory parameters such as the demand constants a, b, c , the backlogging cost constant u , holding cost C_1 , shortage cost C_2 , ordering cost C_3 , deterioration cost C_4 , deterioration rate α , and salvage value rate δ . The demand rate constant a showed the highest sensitivity effect on the total inventory expense (TIC), order quantity, and the inventory cycle timing. A little change in the demand rate constant a significantly changed the overall inventory decisions. Ordering cost C_3 also showed an important effect on the overall inventory system, where increased ordering costs showed the higher changes TIC and delayed inventory cycle timing. Shortage cost C_2 and deterioration rate α notably affected the whole inventory system performance, it is affecting the holding period and the ordering decision. The convexity verification of the total cost function shows that the obtained solution is globally optimal, providing a strong confidence in the applicability of the model in dynamic or less accurate and deteriorative product environments.

Future scope

This given inventory model can be further studied by taking consideration of price dependent demand rate, inflation effects on the system, or multi-echelon inventory stock systems. Future research could also work to explore fuzzy with the given uncertain

environments, sustainability related constraints, and dynamic expense strategies. Combining the AI tools for real-time data analysis and predictive approach may further improve the system decision-making in complex and fast-changing inventory models.

References

1. Dave, U. (1989). On a heuristic inventory replenishment rule for items with a linearly increasing demand incorporating shortages. *Journal of the Operational Research Society*, 40, 827–830.
2. Dave, U. (1989c). A comment on “An inventory replenishment policy for linearly increasing demand considering shortages – an optimal solution” by Murdeshwar, T.M. *Journal of the Operational Research Society*, 40, 412–416.
3. Ouyang, W.U., and Cheng. (2005). An inventory model for deteriorating items with exponential declining demand and partial backlogging. *Yugoslav Journal of Operations Research*, 15(2), 277–288.
4. Hardik Soni, Nita H. Shah, and Bhavin J. Shah. (2006). An EOQ model for two level credit policy order random input. *Revista Investigacion Operacional*, 27(3), 227–238.
5. Alamri, A.A., and Balkhi, Z.T. (2007). The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. *International Journal of Production Economics*, 107, 125–138.
6. Teng, J.T., Ouyang, L.Y., and Chang, C.T. (2007). A comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items. *International Journal of Production Economics*, 105, 190–203.
7. Chung, C.J., and Wee, H.M. (2008). An integrated production-inventory deteriorating model for pricing policy considering imperfect production, inspection planning and warranty-period- and stock-level-dependent demand. *International Journal of Systems Science*, 39(8), 823–837.
8. Roy, A. (2008). An inventory model for deteriorating items with price dependent demand and time varying holding cost. *Advanced Modeling and Optimization*, 10, 25–37.
9. Tripathy, C.K., and Mishra, U. (2010). An inventory model for Weibull deteriorating items with price dependent demand and time-varying holding cost. *Applied Mathematical Sciences*, 4(44), 2171–2179.
10. Begum, R., Sahu, S.K., and Sahoo, R.R. (2012). An inventory model for deteriorating items with quadratic demand and partial backlogging. *British Journal of Applied Science & Technology*, 2(2), 112–131.
11. Amutha, R., and Chandrasekaran, E. (2013). An EOQ model for deteriorating items with quadratic demand and time dependent holding cost. *International Journal of Emerging Science and Engineering*, 1(5).
12. Mishra, V.K. (2013). Deteriorating inventory model for quadratically time varying demand with partial backlogging. *Working Papers on Operations Management*, 4(2), 16–28.
13. Li, K., Li, D., and Wu, D. (2022). Carbon transaction-based location-routing-inventory optimization for cold chain logistics. *Alexandria Engineering Journal*, 61(10), 7979–7986.
14. Khare, G., & Sharma, G. (2022). Effects on Inventory Model in Time Deteriorate Rate with Variable Cost. *International Journal of Intelligent Systems and Applications in Engineering*, 10(3s), 45–50.
15. Panigrahi, R.R., Jena, D., Sahoo, A., and Nayak, M.R. (2023). Inventory management practice and performances of manufacturing firms: An empirical study of RFID and VMI. In *Emerging Trends in Decision Sciences and Business Operations* (pp. 46–68). Routledge India.
16. Ogbonna, U. U., Oladugba, A. V., & Ezra, N. P. (2024). Deterministic Two-Storage Inventory Model for Deteriorating Items with Salvage Value Under Inflation. *Journal of the Indian Society for Probability and Statistics*, 25(2), 789-806.
17. Patra, S. K., Paikray, S. K., & Dutta, H. (2024). An inventory model under power pattern demand having trade credit facility and preservation technology investment with completely backlogged shortages. *Journal of Industrial and Management Optimization*, 20(8), 2652-2679.
18. Prameswari, B.G., Rahman, A., Muharam, H., and Tjahjana, R.H. (2024). Product inventory optimization with EOQ approach in the context of circular economy. *Research Horizon*, 4(4), 389–398.
19. Khare, G., & Sharma, G. (2024, March). Model for inventory having variable ordering and holding costs with salvage value. In *American Institute of Physics Conference Series* (Vol. 3025, No. 1, p. 030005).
20. Khare, G., & Sharma, G. (2024). An Inventory Model with Fluctuate Ordering and Holding Cost with Salvage Value for Time Sensitive Demand and Partial Backlogging. *Commun. Appl. Nonlinear Anal*, 31(1), 177–186.