

Implementation of Queuing Model to Improve Operations in University Libraries

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Abstract: The application of queuing models in university libraries is essential for enhancing service efficiency and improving overall user experience. These models help manage user flow, minimize waiting times, and optimize resource allocation by analyzing key factors such as arrival rates and service durations. Specifically, this study applied the M/M/1 queuing model, which assumes a single-server system with exponentially distributed inter-arrival and service times. The model was used to determine waiting times, queue lengths, and resource utilization in university library facilities. The analysis was conducted using QM (Quality Management) software for Windows. Deterministic numerical results were compared with software-made outputs to validate the model. Additionally, the study included graphical representations of probabilities and cumulative probabilities against the number of users in the system. Various plots were created to analyze utilization factors between the fixed parameters μ_1 and μ_2 under varied arrival rates (λ). A regression equation was developed to examine the relationship between waiting time in the queue and utilization factors. Finally, the correlation between these factors was established to aid decision-making. By analyzing these parameters, library administrators can make informed decisions about staffing and service strategies to better meet user demand. In conclusion, implementing this mathematical queuing model significantly improves operational efficiency, user satisfaction, and the effective use of resources in university libraries.

Keywords: Library Facilities, Mathematical Queuing Model, Resource Allocation

I. Introduction

A library is defined as a collection of printed and non-printed materials, organized and kept for reading, studying and consultations (Itsekor & Nwokeoma, 2017). It can be defined as a building or room that contains a collection of books and other informational resources for reading or studying.

Academic libraries are associated with institutions of higher education, such as universities, and are designed to support the educational, research, and learning endeavors of their respective institutions by offering sufficient resources to achieve the library's goals. Libraries offer a range of services that encompass the collection and provision of access to both printed and non-printed materials. This includes books, journals, CDs, and various other formats of recorded information, all of which are intended to be accessible to users.

University library facilities are essential resources for students and faculty, providing access to a wide range of academic materials and resources. Library facilities are those facilities required to be acquired by the library for effectiveness of library services (Ajegbomogun & Diyaolu, 2018). To ensure efficient and effective utilization of these facilities, it is important to implement mathematical queuing model to manage the flow of users and optimize resource allocation. By applying queuing theory, library administrators can analyze and improve service levels, reduce waiting times, and enhance user satisfaction.

Queuing theory is a branch of applied mathematics that focuses on the study of waiting lines, or queues, and the behaviour of models that involve the processing of entities in a sequential manner. Palvannan & Teow (2012) examined the hospital process model using queuing theory and thoroughly examined the key components and service components of the entire process regarding medical activities.

In the perspective of university library facilities, queuing theory can be used to analyze the flow of users, predict service times, and determine optimal staffing levels. By collecting data on user arrival rates, service times, and queue lengths, administrators can develop mathematical models to simulate and evaluate different queuing scenarios. This information can then be used to make informed decisions about resource allocation, facility design, and service policies.

In addition to improving operational efficiency, mathematical queuing models can also enhance the overall user experience in university library facilities. By minimizing waiting times and ensuring timely access to resources, students and faculty can make better use of their time and achieve their academic goals more effectively. Furthermore, by leveraging queuing theory to optimize service levels, library administrators can create a more welcoming and productive environment for users, ultimately contributing to the overall success of the institution.

In conclusion, the utilization of mathematical queuing models in university library facilities is a valuable tool for optimizing resource allocation, improving operational efficiency, and enhancing user satisfaction. By applying queuing theory to analyze and manage the flow of users, administrators can make data-driven decisions that benefit both the institution and its users. As technology continues to evolve and user expectations change, the application of mathematical queuing models will become increasingly important in ensuring the success of university library facilities.

The aim of this study is to apply a mathematical queuing model to university library operations to enhance service delivery and resource efficiency. The specific objectives are as follows: to determine waiting times, queue lengths, and resource utilization using the M/M/1 queuing model; to solve the M/M/1 model by predicting these performance measures using QM (Quality Management) software for Windows; to compare deterministic analytical results with those generated by QM for Windows; to plot probability and cumulative probability distributions against the number of users in the system; to generate various plots illustrating utilization factors for fixed service rates ($\mu_1 = 10$ and $\mu_2 = 12$) under varying arrival rates ($\lambda = 1 : 1: 9$); to develop a regression equation that relates waiting time in the queue to utilization factors, using a fixed service rate ($\mu = 10$) and varying arrival rates ($\lambda = 1: 1: 9$); and to establish the correlation between queue waiting time and utilization factors for the given service and arrival rates.

II. Literature Review

Most of the scientific researchers have investigated the queueing theory in the following ways. Wang (2023) carried out research on the queueing theory based on M/M/1 queueing model. His work contained some fundamental queueing theory formulae for the M/M/1 queueing model. He simulated M/M/1 queueing model using Rstudio. He did not study the regression between the utilization factors and the awaiting time in queue. Most of mathematical applicable problems have been solved computationally and analytical in [11 – 25].

Yue et al. (2012) analyzed M/M/1 queueing system with impatient customers and working vacations. They found out that during a working vacation, customers were served at a slower than usual service rate and are likely to become impatient.

Lyua et al. (2021) applied a queueing model to determine library allocation based on the quality of the reader services. They determined the measurement of library resource allocation through the birth and death process of the number of readers. They did not examine the correlation between the utilization factors and the awaiting time in queue.

Concept of Mathematical Queuing Model

The figure 1 below shows the queuing model of library facilities where the readers are the users/students.

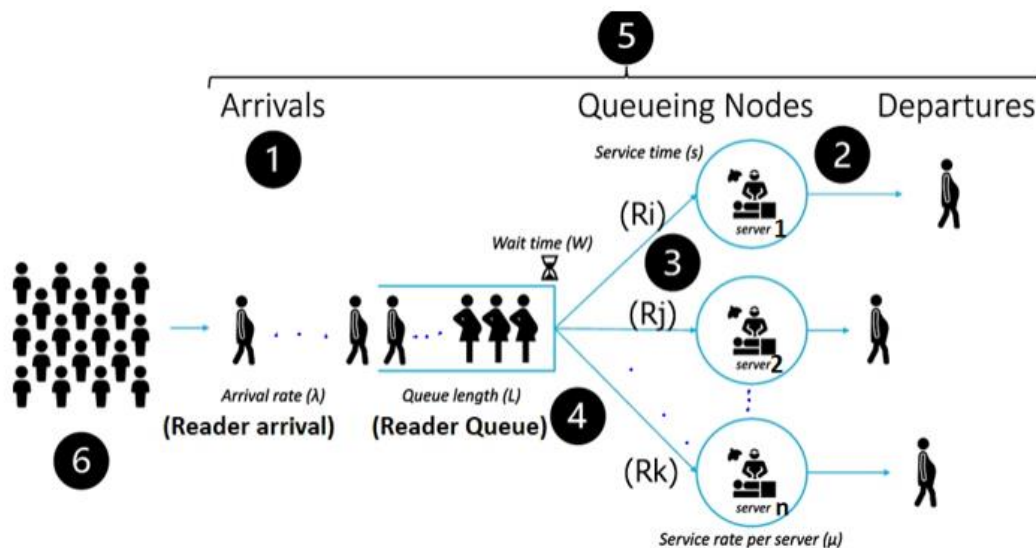


Figure 1: Queuing Model of Library Facilities

III. Methodology of Mathematical Queuing Model

A model is a simplified representation of a system or phenomenon that is used to study or analyse its behaviour while a theorem is a statement that has been proven to be true using mathematical reasoning or logic.

In the M/M/1 model, the "M" stands for Markovian, meaning that arrivals and service times are exponentially distributed. The "1" indicates that there is only one server (in this case, library resources or staff) available to serve the arriving users (students). The exponential distribution is used to model the time until an event occurs. It assumes that the expected future waiting time is independent of the past waiting time. For example, if you expect to wait for 5 minutes in a queue, the expected waiting time is still 5 minutes even if you have already waited for 3 minutes.

This model is used in the case where students arrive randomly and are served on a first-come, first-served basis; it assumes that users arrive independently. This would accurately represent the situation in a university library, where students arrive at different times and need access to library resources or assistance from staff. Following Yue et al. (2012), the M/M/1 model is a queuing process in which users arrive at one server and wait in a queue (if necessary) until the server is available.

By using the M/M/1 queueing model, university libraries can better understand and predict wait times, queue lengths, and resource utilization. This can help in optimizing staff scheduling, managing library resources effectively, and improving overall user experience for students.

The parameter formulation of the effectiveness of M/M/1 queueing system are:

Input:

Arrival rate (number of students/unit time) = λ

Service rate (number of students/ unit time) = μ

Output:

U = Utilization factor = percentage of the time that all servers are busy,

$$(\rho) = \frac{\lambda}{\mu} < 1 \text{ (for stability condition, Tonui et al. (2014))}$$

Probability that there is no student in the system, $P_0 = 1 - \rho$

Probability that there are n users in the system, $P_n = \rho^n P_0$; $P_{n+1} = \rho P_n$

Average time a student spends in the system (in waiting line and being served),

$$W = \frac{1}{\mu - \lambda} \text{ (Wang, 2023).}$$

Following Little (1961) identified in Tonui et al. (2014) that:

Average time a student spends in waiting line waiting for service,

$$W_q = W - \frac{1}{\mu} = \frac{1}{\mu - \lambda} - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\lambda}{\mu} \cdot \frac{1}{(\mu - \lambda)} = \frac{\rho}{\mu - \lambda}$$

Average number of students in waiting line for service,

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$\therefore L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \lambda \cdot \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\lambda}{\mu} \cdot \frac{\lambda}{(\mu - \lambda)} = \rho \cdot \frac{\lambda}{\mu - \lambda} = \rho \cdot \frac{\lambda}{\mu(1 - \frac{\lambda}{\mu})} = \rho \cdot \frac{\lambda}{\mu(1 - \rho)} = \rho \cdot \frac{\lambda}{\mu} \cdot \frac{1}{(1 - \rho)} = \frac{\rho^2}{1 - \rho}$$

Average number of students in the system (in waiting line and being served),

$$L = \frac{\rho}{1 - \rho}$$

Applications of the M/M/1 Mathematical Queuing Model

Cases of how the M/M/1 queueing model are applied in a university library setting:

Case 1: Assume that students arrive at a university library to check out books or seek assistance from library staff. The arrival rate of students follows an exponential distribution with an average of 6 students per hour. Once a student starts receiving service (checking out books or getting assistance), the service time also follows an exponential distribution with an average of 10 minutes per student.

Solution 1

Table 1: Mathematical Translations for the Case 1

S/N	Parameters	Formula	Workings
1	Arrival rate (λ)	λ	6 students per hour
2	Service rate (μ)	μ	6 students per hour

3	Utilization factor (ρ)	$\frac{\lambda}{\mu}$ [Zhang, 2020]	$\frac{6}{6} = 1$
4	Average number of students in the queue (L_q)	$\frac{\rho^2}{1 - \rho}$	$\frac{1^2}{1 - 1} = \frac{1}{0} = \infty = \text{Infinity}$
5	Average number of students in the system (L)	$\frac{\rho}{1 - \rho}$	$\frac{1}{1 - 1} = \frac{1}{0} = \infty = \text{Infinity}$
6	Average time a student spends in the queue (W_q)	$\frac{\rho}{\mu - \lambda}$	$\frac{1}{6 - 6} = \frac{1}{0} = \infty = \text{Infinity}$
7	Average time a student spends in the system (W)	$\frac{1}{\mu - \lambda}$	$\frac{1}{6 - 6} = \frac{1}{0} = \infty = \text{Infinity}$
8	Probability that there is no student in the system = P_0	$1 - \rho$	$1 - \rho = 1 - 1 = 0$

Case 2: Assume that students arrive at a university library to borrow books or seek assistance from library staff. The arrival rate of students follows an exponential distribution with an average of 8 students per hour. Once a student starts receiving service (borrowing books or getting assistance), the service time also follows an exponential distribution with an average of 6 minutes per student.

Solution 2

Table 2: Mathematical Translations for the Case 2

S/N	Parameters	Formula	Workings
1	Arrival rate (λ)	λ	8 students per hour
2	Service rate (μ)	μ	10 students per hour
3	Utilization factor (ρ)	$\frac{\lambda}{\mu}$	$\frac{8}{10} = 0.8 < (1)$
4	Average number of students in the queue (L_q)	$\frac{\rho^2}{1 - \rho}$	$\frac{0.8^2}{1 - 0.8} = 3.2 \approx 3$ students
5	Average number of students in the system (L)	$\frac{\rho}{1 - \rho}$	$\frac{0.8}{1 - 0.8} = 4$ students
6	Average time a student spends in the queue (W_q)	$\frac{\rho}{\mu - \lambda}$	$\frac{8}{10 - 8} = 0.4 \text{ hour} \equiv 24 \text{ minutes}$
7	Average time a student spends in the system (W)	$\frac{1}{\mu - \lambda}$	$\frac{1}{10 - 8} = 0.5 \text{ hour} \equiv 30 \text{ minutes}$
8	Probability that there is no student in the system = P_0	$1 - \rho$	$1 - 0.8 = 0.2$

Table 3: QM for Windows for Arrival rate (λ): 8 students per hour and Service rate (μ): 10 students per hour

Parameter	Value		Parameter	Value	Minutes	Seconds
M/M/1 (exponential service times)			Average server utilization	.8		
Arrival rate(λ)	8		Average number in the queue(L_q)	3.2		
Service rate(μ)	10		Average number in the system(L)	4		
Number of servers	1		Average time in the queue(W_q)	.4	24	1440
			Average time in the system(W)	.5	30	1800
			Probability (% of time) system is empty (P_0)	.2		

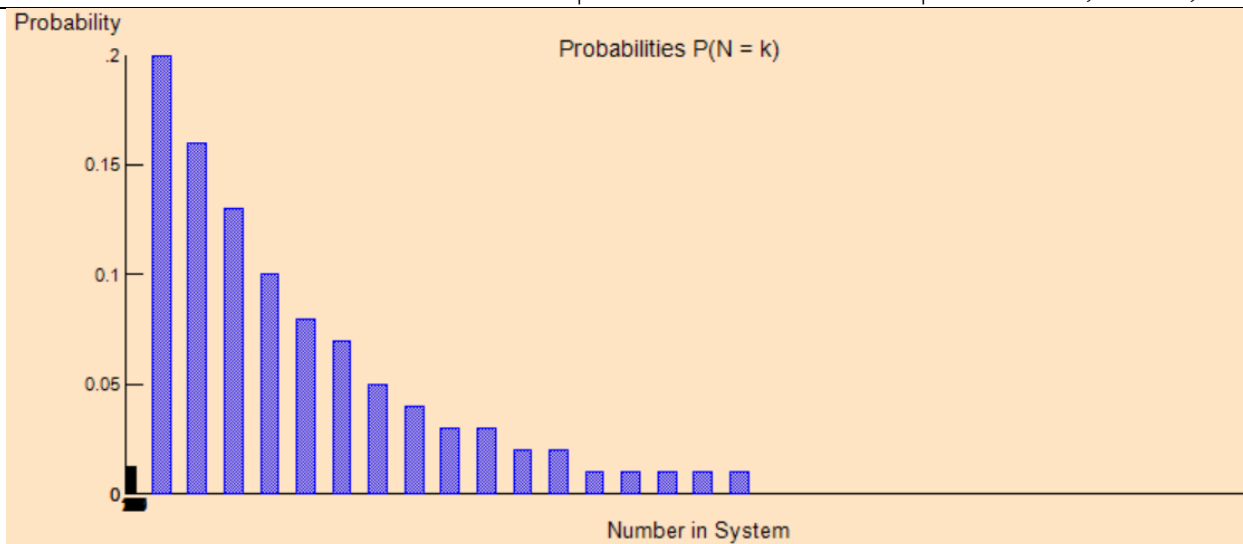


Figure 2: QM for Windows Probability for Arrival rate (λ): 8 students per hour Service rate (μ): 10 students per hour

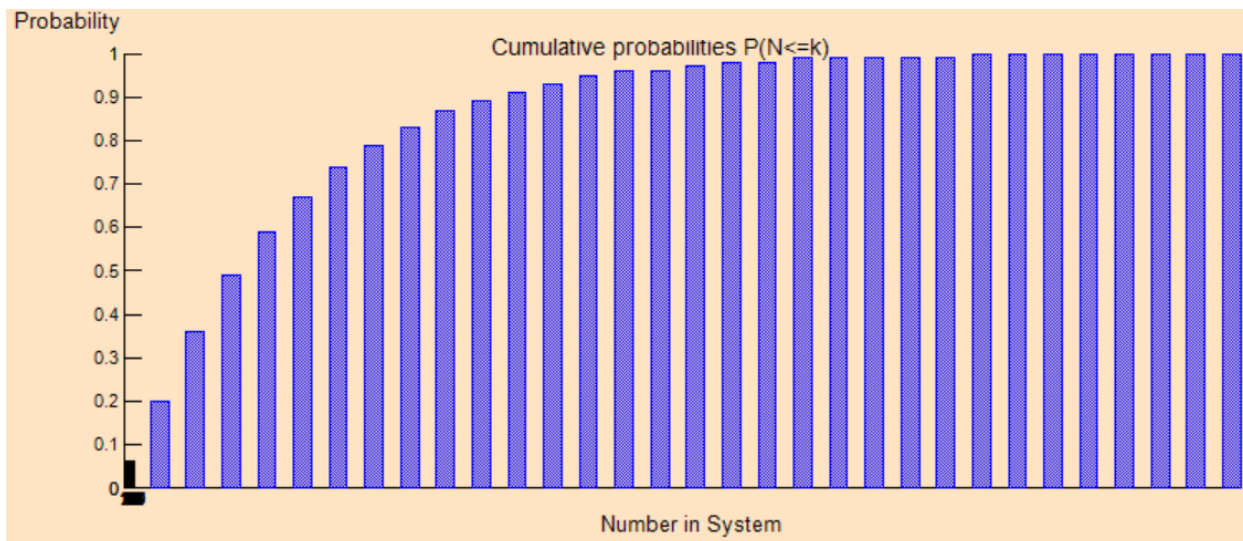


Figure 3: QM for Windows Cumulative Probability for Arrival rate (λ): 8 students per hour Service rate (μ): 10 students per hour

Case 3: Consider a university library where students arrive to borrow books. Assume that the arrival rate of students follows an exponential distribution with an average of 10 students per hour. Once a student begins the borrowing process, the service time also follows an exponential distribution with an average of 5 minutes per student.

Solution 3

Table 4: Mathematical Translations for the Case 3

S/N	Parameters	Formula	Workings
1	Arrival rate (λ)	λ	10 students per hour
2	Service rate (μ): $\frac{1}{\text{service time}}$	μ	$\frac{60 \text{ minutes}}{5 \text{ minutes}} = 12 \text{ students per hour}$
3	Utilization factor (ρ)	$\frac{\lambda}{\mu}$	$\frac{10}{12} \approx 0.8333 < (1)$
4	Average number of students in the queue (L_q)	$\frac{\rho^2}{1 - \rho}$	$\frac{0.8333^2}{1 - 0.8333} = 4.1655 \approx 4 \text{ students}$

5	Average number of students in the system (L)	$\frac{\rho}{1 - \rho}$	$\frac{0.8333}{1 - 0.8333} = 4.9988 \approx 5$ students
6	Average time a student spends in the queue (W_q)	$\frac{\rho}{\mu - \lambda}$	$\frac{0.8333}{12 - 10} \approx 0.4167$ hour
7	Average time a student spends in the system (W)	$\frac{1}{\mu - \lambda}$	$\frac{1}{12 - 10} = 0.5$ hour $\equiv 30$ minutes
8	Probability that there are no student in the system = P_0	$1 - \rho$	$1 - 0.8333 = 0.1667$

Table 5: QM for Windows for Arrival rate (λ): 10 students per hour and Service rate (μ): 12 students per hour

Parameter	Value		Parameter	Value	Minutes	Seconds
M/M/1 (exponential service times)			Average server utilization	.83		
Arrival rate(λ)	10		Average number in the queue(L_q)	4.17		
Service rate(μ)	12		Average number in the system(L)	5		
Number of servers	1		Average time in the queue(W_q)	.42	25	1500
			Average time in the system(W)	.5	30	1800
			Probability (% of time) system is empty (P_0)	.17		

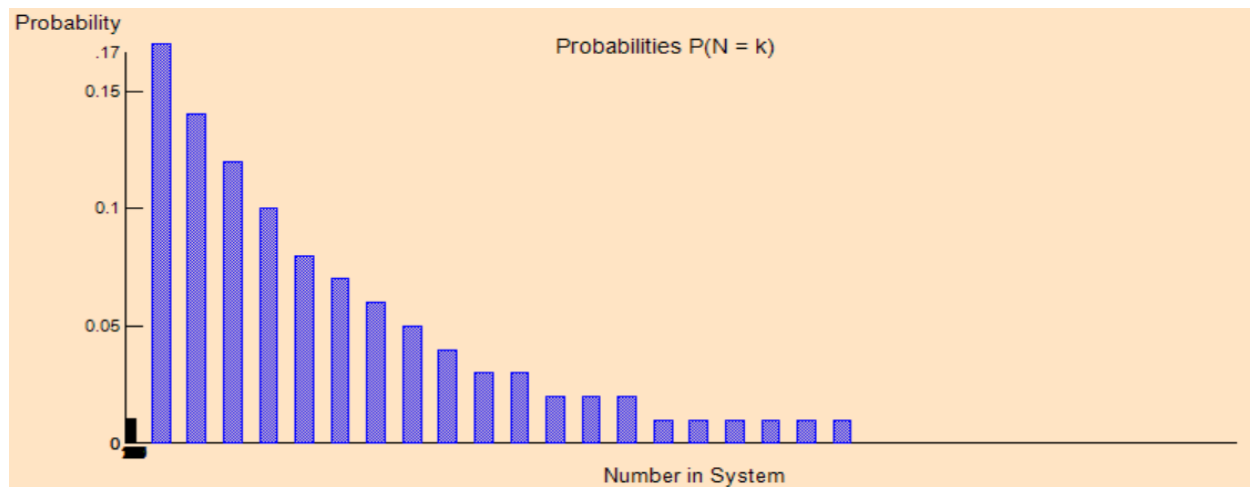


Figure 4: QM for Windows Probability for Arrival rate (λ): 10 students per hour Service rate (μ): 12 students per hour

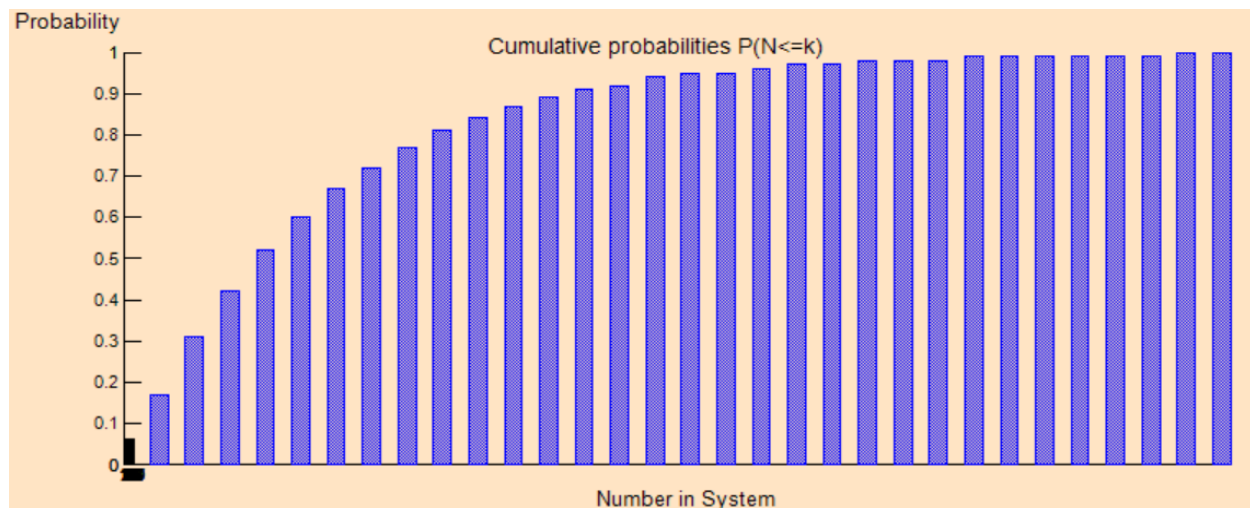


Figure 5: QM for Windows Cumulative Probability for Arrival rate (λ): 10 students per hour Service rate (μ): 12 students per hour

Case 4: In a university library, students arrive randomly to check out books with an average arrival rate of 5 students per hour. The time taken to check out a book follows an exponential distribution with an average service rate of 10 minutes per student. Calculate various performance measures using the M/M/1 queuing model.

Solution 4:

Table 6: Mathematical Translations for the Case 4

S/N	Parameters	Formula	Workings
1	Arrival rate (λ)	λ	5 students per hour
2	Service rate (μ): $\frac{1}{\text{service time}}$	μ	$\frac{1}{10 \text{ minutes} = 1/6 \text{ hour}} = 6 \text{ students per hour}$
3	Utilization factor (ρ)	$\frac{\lambda}{\mu}$	$\frac{5}{6} \approx 0.8333 (<1)$
4	Average number of students in the queue (L_q)	$\frac{\rho^2}{1 - \rho}$	$\frac{0.8333^2}{1 - 0.8333} = 4.1655 \approx 4 \text{ students}$
5	Average number of students in the system (L)	$\frac{\rho}{1 - \rho}$	$\frac{0.8333}{1 - 0.8333} = 4.9988 \approx 5 \text{ students}$
6	Average time a student spends in the queue (W_q)	$\frac{\rho}{\mu - \lambda}$	$\frac{0.8333}{6 - 5} \approx 0.8333 \text{ hour}$
7	Average time a student spends in the system (W)	$\frac{1}{\mu - \lambda}$	$\frac{1}{6 - 5} = 1 \text{ hour} \equiv 60 \text{ minutes}$
8	Probability that there are no student in the system = P_0	$1 - \rho$	$1 - 0.8333 = 0.1667$

Table 7: QM for Windows for Arrival rate (λ): 5 students per hour and Service rate (μ): 6 students per hour

Parameter	Value	Parameter	Value	Minutes	Seconds
M/M/1 (exponential service times)		Average server utilization	.83		
Arrival rate(λ)	5	Average number in the queue(L_q)	4.17		
Service rate(μ)	6	Average number in the system(L)	5		
Number of servers	1	Average time in the queue(W_q)	.83	50	3000
		Average time in the system(W)	1	60	3600
		Probability (% of time) system is empty (P_0)	.17		

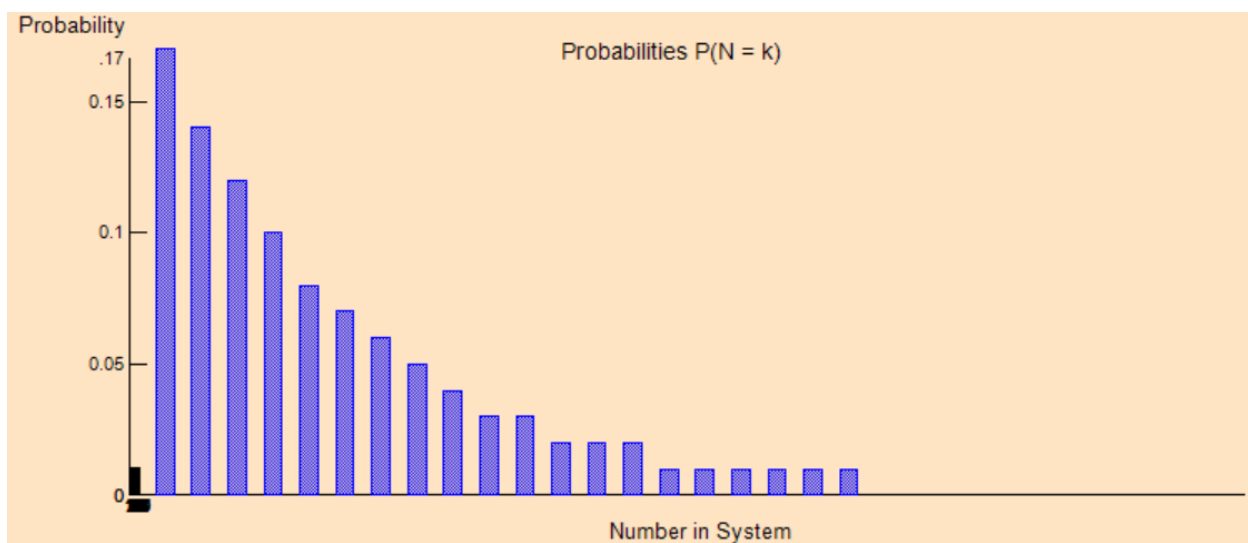


Figure 6: QM for Windows Probability for Arrival rate (λ): 5 students per hour Service rate (μ): 6 students per hour

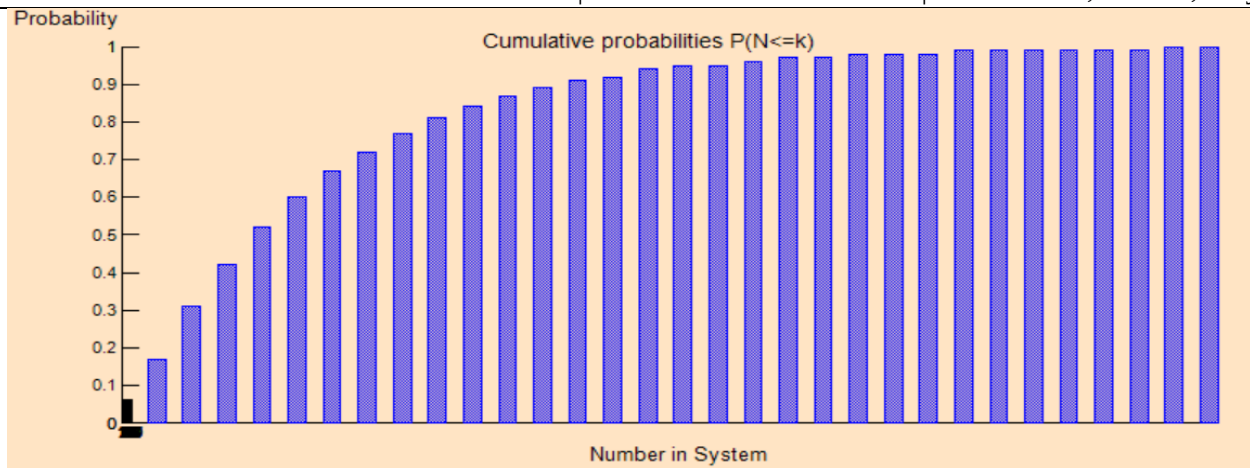


Figure 7: QM for Windows Probability for Arrival rate (λ): 5 students per hour Service rate (μ): 6 students per hour

Table 8: A table showing Awaiting time in queue and the Utilization factors for Service rate (number of customers/ unit time), $\mu = 10$ (fixed), $\lambda = 1:1:9$.

$\mu = 10$		
$W = \frac{1}{\mu - \lambda}$	$\rho = \frac{\lambda}{\mu}$	$W_q = \frac{\rho}{\mu - \lambda}$
0.111111	0.1	0.0111111
0.125	0.2	0.025
0.142857	0.3	0.0428571
0.166667	0.4	0.0666667
0.2	0.5	0.1
0.25	0.6	0.15
0.333333	0.7	0.2333333
0.5	0.8	0.4
1	0.9	0.9

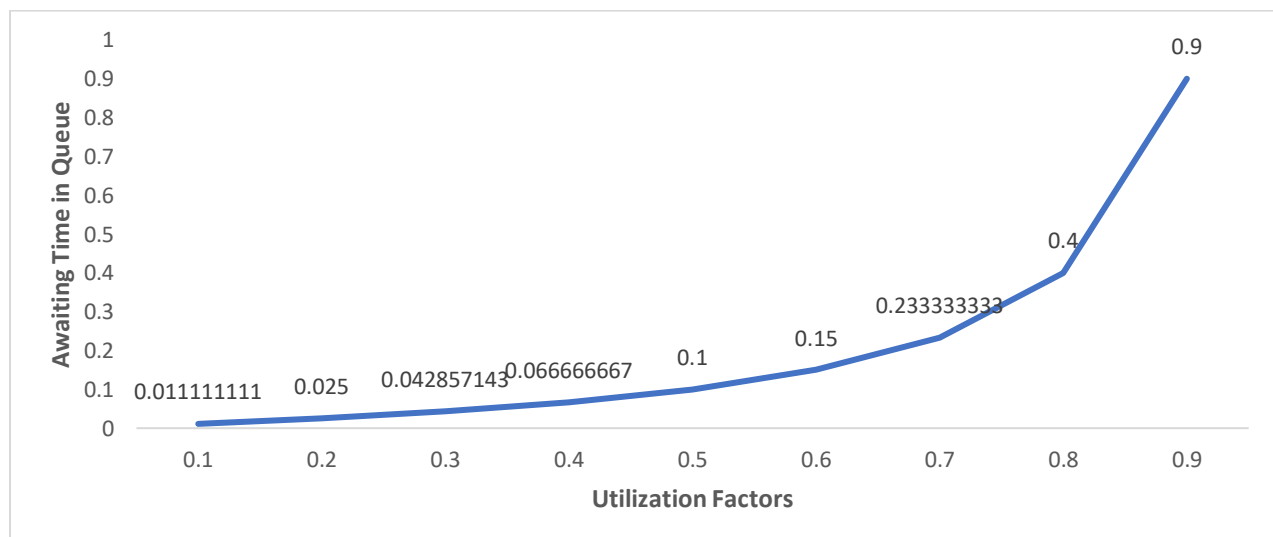


Figure 8: Awaiting time in queue vs. Utilization factors

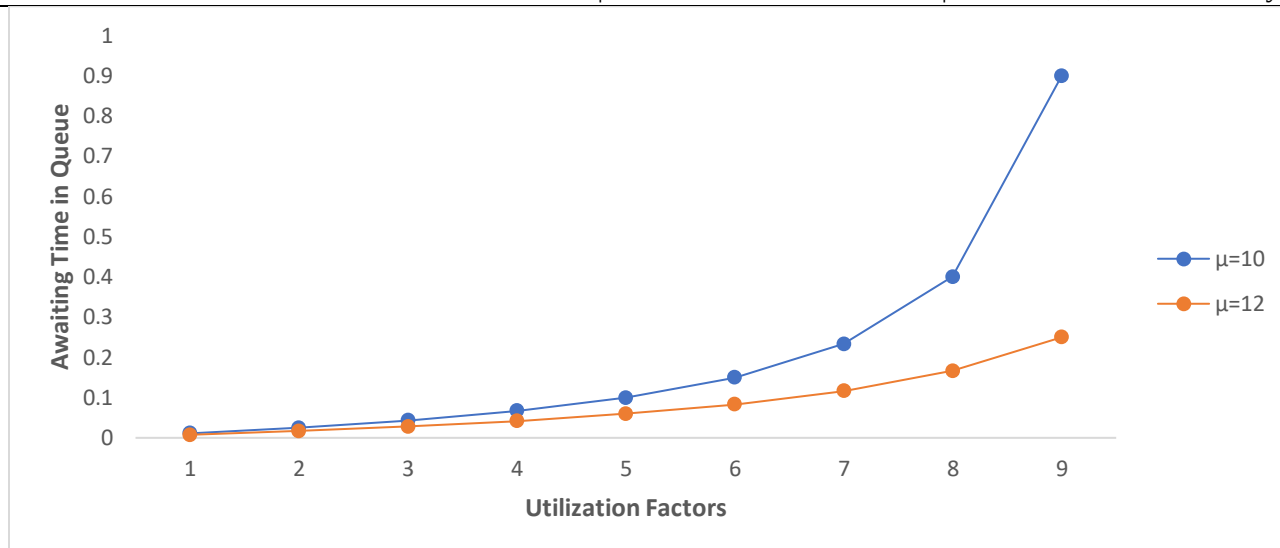


Figure 9: Awaiting time in queue vs. Utilization factors

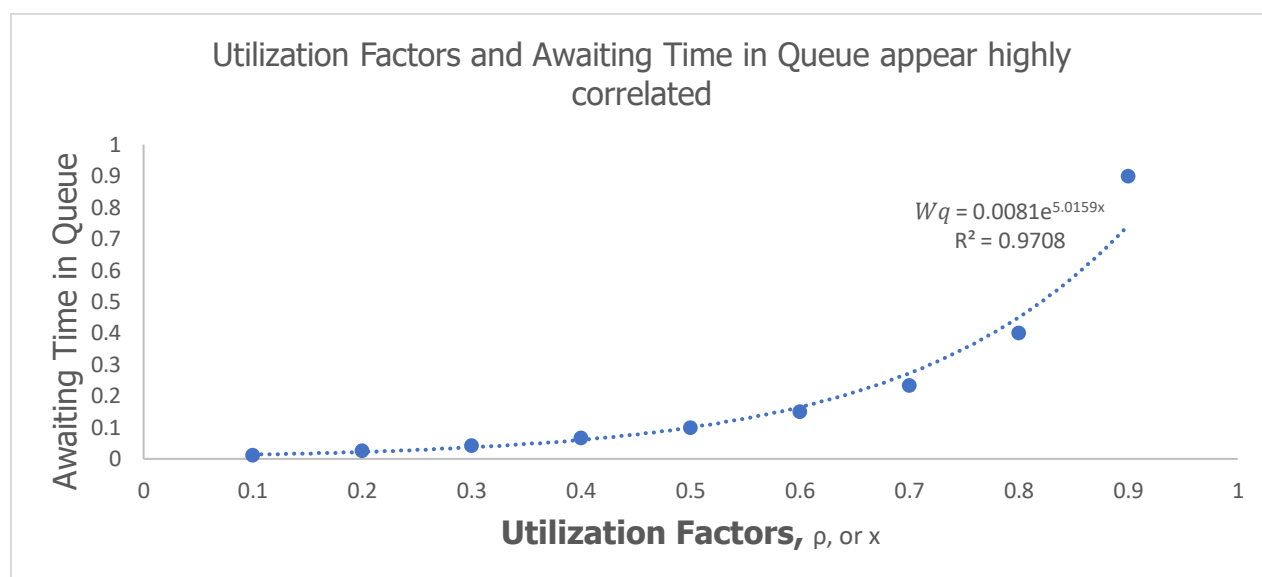


Figure 10: Correlation between Utilization Factors and Awaiting Time in Queue

IV. Discussion of Results

The following interpretations of the Cases are drawn:

Interpretation 1: In Case 1:

Given that the utilization factor (ρ) is equal to 1, it indicates that the system is operating at capacity as the arrival rate of students matches the service rate of the library staff. This leads to an infinite number of students in the system and an infinite amount of time spent in the queue and in the system.

Interpretation 2: In the Case 2:	Interpretation 3: In the Case 3:
The utilization factor (ρ) is 0.8, indicating that the system is operating at high utilization but still below full capacity.	With an arrival rate of 10 students per hour and a service rate of 12 students per hour, the system is slightly overloaded as the utilization factor (ρ) is greater than 0.8. This indicates that, on average, are 4.17 (≈ 4) students waiting in the queue.
The average number of students in the queue is 3.2 (≈ 3) students.	On the average, there are 5 students in the system.

The average number of students in the system is 4 students.	iii. Students are expected to spend about 0.4167 hour (≈ 25 minutes) waiting in the queue before receiving service.
Students are expected to spend an average of 0.4 hour (24 minutes) in the queue before receiving service and 0.5 hour (30 minutes) in the system.	iv. Students are expected to spend around 0.5 hour (30 minutes) in the entire system
The probability of idle server = 20%	v. The probability of idle server = 17%

Interpretation 4: In Case 4:

- On average, there are approximately 4.17 (≈ 4) students waiting in the queue to be served.
- On average, there are approximately 5 students in the system at any given time.
- On average, a student spends approximately 0.8333 hour (≈ 50 minutes) waiting in the queue.
- Moreover, on average a student spends 1 hour (60 minutes) in the system service time.
- The probability of idle server = 17%

By analyzing these performance measures, the university library can evaluate and optimize its operations, such as staff scheduling, opening hours, and resource allocation, to improve students' satisfaction and efficiency.

Combining figures 8 and 9, the utilization factor which is the ratio of the mean arrival rate to the mean service rate indicates the probability that a new user (student) will have to wait. As the utilization factor increases, the average waiting time in a queue also increases exponentially.

From Figure 10, the correlation between the Utilization factor and the Awaiting time in queue = 0.823493 (82.35%) is a strong positive relationship. The regression equation is $W_q = 0.0081e^{5.0159p}$ with the R-squared value as $R^2 = 0.9708$ (97.08%). The R-squared value measures the proportion of variation in the dependent variable explained by the independent variable; how well the regression model fits the data (Okeke & Chukwuka, 2023).

V. Conclusion

The M/M/1 queuing model has been used in the study of students' service systems, including university library facilities. The following conclusions have been drawn:

- 1. Efficiency and Resource Allocation:** The M/M/1 queuing model helps in understanding the utilization of resources, such as library staff, and optimizing the efficiency of service delivery in university libraries.
- 2. Wait Time and Queue Management:** The M/M/1 queuing model has been used to analyze and predict wait times, queue lengths, and service capacity in university library settings. This information is valuable for managing library operations and improving user satisfaction.
- 3. System Performance Evaluation:** The M/M/1 queuing model has evaluated the performance of university library facilities in terms of service levels, resource utilization, and students waiting times. This analysis leads to recommendations for system improvements and operational changes.
- 4. User Experience and Satisfaction:** Using the M/M/1 model helps in enhancing the user experience of students in university libraries. By reducing wait times and optimizing resource allocation, libraries can improve students' satisfaction and loyalty.

The M/M/1 queuing can be extended to include multiple servers (M/M/c) and finite system capacities (M/M/1/K) to reflect a broader real-world system.

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