

The Minimum Initial Capital of The Discrete-Time Surplus Process as A Kind of Investment for Insurer

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Abstract— This paper examines the minimum initial capital an insurer needs to maintain to prevent insolvency, given that the probability of ruin does not surpass 1% and 5% of the discrete-time surplus process with an investment in a mutual fund. The simulation method is employed to determine the probability of ruin, while the regression technique is utilized for calculating the minimum initial capital.

Keywords— Minimum Initial Capital, Discrete-Time Surplus Process

Notation

1. $\{Y_n, n \in \mathbb{N}\}$ denotes a collection of claim sizes, where Y_n arrives at time $T_n, n \in \mathbb{N}$, with the condition $T_1 \leq T_2 \leq \dots \leq T_n, \{Y_n, n \in \mathbb{N}\}$ constitutes a collection of independent and identically distributed random variables (i.i.d.).
2. c is a premium rate.
3. $U_n, n \in \mathbb{N}$ represents a discrete-time surplus process.
4. $\Phi(u)$ denotes the probability of ruin, defined as

which indicates the probability of U falling below zero for the first time, where u represents initial capital.

I. Introduction

Minimum initial capital reserves for insurers are crucial for maintaining their solvency. The minimum initial capital will be analyzed through the probability of ruin. Numerous researchers examine the discrete-time surplus process from various perspectives. The classical discrete-time surplus compound binomial process for insurance is defined as

$$U(n) = u + cn - \sum_{i=1}^n Y_i,$$

The results yield the exact ruin probability for finite and infinite horizons (Gerber, 1988). (Chan and Zhang, 2006) examined the discrete time surplus process characterized by daily claims, represented as

$$U(n) = u + cn - \sum_{i=1}^n Y_i, U(0) = u; n \in \mathbb{N}, \quad (1)$$

under the stipulation that Y_i follows a geometric distribution and Y_i is an exponential random variable. They suggested a recursive formula for the probabilities of ruin. (Sattayatham et al., 2013) expanded the Chan and Zhang model by allowing Y_i to be an any random variable, rather than being restricted to geometric or exponential distributions. The recursive formulation of the probability of ruin is expressed as

$$\Phi_n(u) = \Phi_1(u) + \int_{-\infty}^{u+c} \Phi_n(u+c-y) dF_{Y_1}(y). \quad (2)$$

Note that the process (1) involves claims occurring daily, represented as $T_n = n$. If we define

$$Z_n = T_n - T_{n-1}, n \in \mathbb{N} \text{ and } T_0 = 0,$$

the model (1) may be expressed as

$$U_n = U_{n-1} + cZ_n - Y_n, U_0 = u. \quad (3)$$

Specifically, when $Z_n = 1$, the claims occur daily, and model (3) transforms into

$$U_n = U_{n-1} + c - Y_n, U_0 = u. \quad (4)$$

(Nyrhinen, H., 2001) and (Cai, J. and Dickson, D. 2004) examined the discrete-time excess process under a constant interest rate r and derived Lundberg's determines on the probabilities of ruin. The model under examination is expressed as

$$U_n = U_{n-1}(1+r) - Y_n, U_0 = u. \quad (5)$$

In a discrete-time surplus process incorporating proportional reinsurance and investment, as discussed by (Luesamai, A. and Chongcharoen, S., 2018) and (Luesamai, A., 2021) establish lower and upper boundaries for the probability of ruin utilizing the methodologies of (Gajek, L., 2005) and (Rudź, M., 2015).

From equation (1), (2), (4), and (5), the probability of ruin is challenging to articulate as an explicit function; researchers typically derive it as lower and upper bounds or represent it in recursive forms. Our research will examine the surplus process occurring at defined intervals, characterized by daily claims, as represented by the equation

$$U_n = U_{n-1}(1 + r) + c - Y_n, U_0 = u. \quad (6)$$

In the present scenario, r denotes the permissible return rate for an insurer investing in the K-Banking mutual fund, specifically inside the SET Banking Sector Index, which has a one-year return of $r = 12.29\%$ annually. Due to the difficulty in calculating the probabilities of ruin, we obtain them via a simulation method. We subsequently utilize the regression method to ascertain the minimum initial capital.

II. Methodology

Parameter Estimation

The claims data is sourced from an insurer in Thailand, as illustrated in Figure 1.

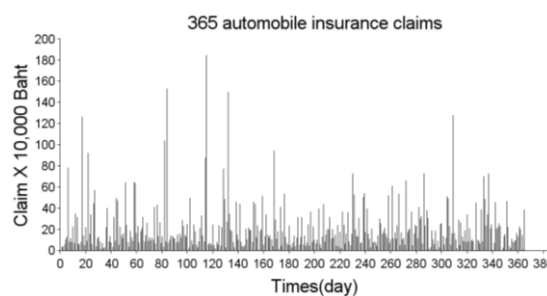


Fig. 1. 365 claims of automobile

The data can be considered as following a log-normal distribution with $\mu = 11.834$ and $\sigma = 0.95532$ at the 95% confidence level.

Simulation Results

This section presents the probabilities of ruin of discrete-time surplus process (6). We start by establishing the initial capital $u = 0,10000, \dots, 1200000$ baht and calculate the premium rate c using the expected value premium principle described by

$$c = (1 + \theta) \frac{EY_i}{EZ_i}$$

Given that claims occur daily, we have $EZ_i = 1$. Therefore,

$$c = (1 + \theta)EY_i,$$

where θ represents a safety loading. The claims Y_i follow a log-normal distribution, hence

$$\begin{aligned} EY_i &= \exp\left(\mu + \frac{1}{2}\sigma^2\right) \\ &= \exp\left(11.834 + \frac{1}{2}(0.95532)^2\right) \\ &= 217579.18913. \end{aligned}$$

The number of simulations was established at 20,000. Figure 2 illustrates the flowchart of the simulation. The results of simulation as shown in Figure 3.

Figure 3 demonstrates the correlation between initial capital and the probability of ruin when safety loading θ values range from 0.1 to 1.0. The blue curve denotes $\theta = 0.1$, the green curve indicates $\theta = 0.2$, and the lowest curve corresponds to $\theta = 1.0$, respectively.

Minimum initial capital

Figure 3 illustrates that the relationship between initial capital u and the probability of ruin y is characterized by an exponential function for every instance of θ , represented as

$$y = ae^{-bu}. \quad (7)$$

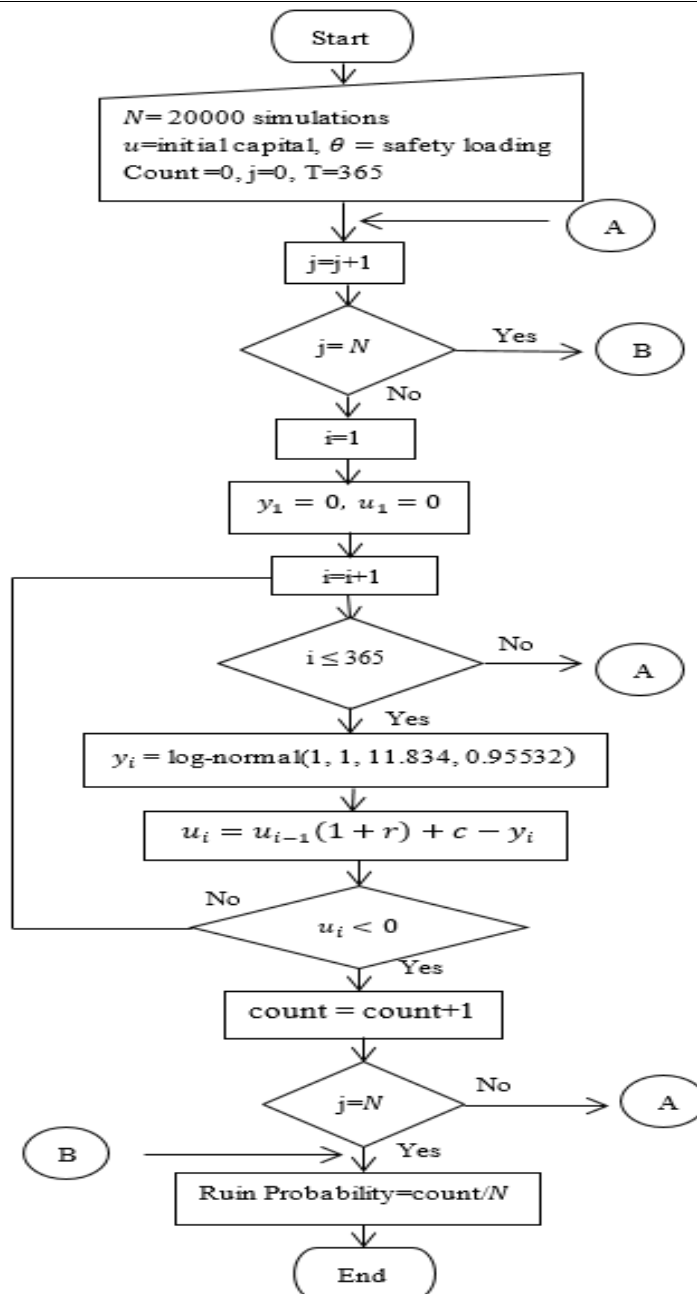


Fig. 2. The flowchart illustrating the simulation of eq. (5).

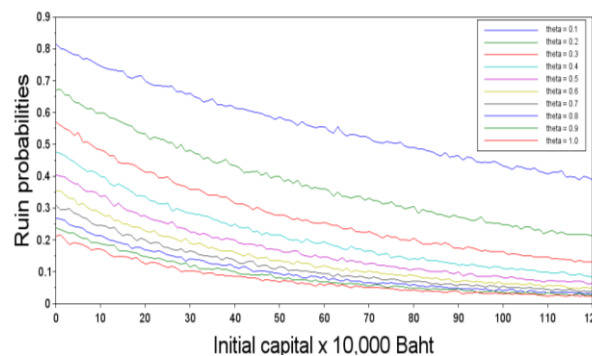


Figure 3. The correlation between initial capital and the probability of ruin.

Regression method

To employ least squares linear regression, we utilize the logarithmic function on both sides of the equation (7)

$$\ln y = -bu + \ln a$$

and define

$$l = \ln y, m = -b, \text{ and } v = \ln a.$$

We obtain the linear equation

$$l = mu + v.$$

By applying least squares linear regression, we have

$$m = \frac{n \sum_{i=1}^n u_i \ln y_i - (\sum_{i=1}^n u_i) (\sum_{i=1}^n \ln y_i)}{n \sum_{i=1}^n u_i^2 - (\sum_{i=1}^n u_i)^2}$$

and

$$v = \frac{(\sum_{i=1}^n u_i^2)(\sum_{i=1}^n \ln y_i) - (\sum_{i=1}^n u_i)(\sum_{i=1}^n u_i \ln y_i)}{n \sum_{i=1}^n u_i^2 - (\sum_{i=1}^n u_i)^2}$$

Given that $m = -b$ and $v = \ln a$, it follows that

$$b = -\frac{n \sum_{i=1}^n u_i \ln y_i - (\sum_{i=1}^n u_i) (\sum_{i=1}^n \ln y_i)}{n \sum_{i=1}^n u_i^2 - (\sum_{i=1}^n u_i)^2}$$

and

$$a = \exp \left(\frac{(\sum_{i=1}^n u_i^2)(\sum_{i=1}^n \ln y_i) - (\sum_{i=1}^n u_i)(\sum_{i=1}^n u_i \ln y_i)}{n \sum_{i=1}^n u_i^2 - (\sum_{i=1}^n u_i)^2} \right).$$

From (7), if the insurance requirements are that the probability of ruin does not exceed α , then

$$ae^{-bu} \leq \alpha.$$

By applying the logarithmic function to both sides and resolving the inequality, we derive

$$u \geq \frac{1}{b} (\ln a - \ln \alpha).$$

Therefore, the minimal initial capital (MIC) is

$$\text{MIC} = \frac{1}{b} (\ln a - \ln \alpha).$$

III. Results

This section displays the MIC in response to the insurer's requirement that the probability of ruin should not exceed $\alpha = 0.01$ and $\alpha = 0.05$, as illustrated in Tables 1 and 2, respectively.

Table 1. The MIC (baht) with $\alpha = 0.01$.

θ	a	b	MIC
0.1	0.788738	0.00000060	7279749.15
0.2	0.643205	0.00000096	4337373.41
0.3	0.527789	0.00000121	3277777.97
0.4	0.439963	0.00000139	2722378.85
0.5	0.371625	0.00000152	2378486.28
0.6	0.318307	0.00000163	2122963.93
0.7	0.274975	0.00000169	1961003.34
0.8	0.240077	0.00000176	1805895.83
0.9	0.211397	0.00000179	1704555.50
1.0	0.185808	0.00000182	1605565.30

Table 2. The MIC (baht) with $\alpha = 0.05$.

θ	a	b	MIC
0.1	0.788738	0.00000060	4597352.62
0.2	0.643205	0.00000096	2660875.59
0.3	0.527789	0.00000121	1947663.99
0.4	0.439963	0.00000139	1564509.85
0.5	0.371625	0.00000152	1319645.54
0.6	0.318307	0.00000163	1135578.70
0.7	0.274975	0.00000169	1008673.21
0.8	0.240077	0.00000176	891442.47
0.9	0.211397	0.00000179	805428.18
1	0.185808	0.00000182	721258.75

IV. Conclusions

Table 1 demonstrates that to keep the probability of ruin at or below 1%, a reserve of 7,279,749.15 baht is required with a safety loading θ of 0.1. The safety loading θ of 0.02 requires a reserve of 4,337,373.41 baht. The premium c is defined by the equation $c = (1 + \theta)EY_i$. An increase in the safety loading θ leads to a higher premium rate c , consequently decreasing the MIC. The results shown in Table 2 align with those in Table 1, demonstrating that an increase in the safety loading θ is associated with a reduction in the MIC. Table 2 indicates that, given insurer acceptance, the MIC keeps the probability of ruin at or below 5%. For all instances of identical safety loading θ , the MIC presented in Table 2 must be lower than the MIC in Table 1.

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