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A Production and Distribution Model for a Declining Inventory Item

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Abstract: We create an integrated production-distribution model to address a failing two-tier supply chain. The supplier's production batch size is limited to an integer multiple of the discrete delivery lot amount for the buyer. Cost functions for suppliers, buyers, and the complete supply chain have been established. This leads to determining optimal policies for both individuals and the integrated supply chain. We provide a method for making optimum supply chain decisions while minimising overall system costs. Our technique is presented with a numerical example.

Keywords: Integrated Supply Chain, Deterioration and Inventory.

I. Introduction

It is not commonplace for inventory products including milk, fruit, blood, pharmaceuticals, and vegetables to degrade over time. Studying the behaviour of decaying and degrading products is crucial for developing effective inventory control rules that account for this behaviour.

Ghare and Schrader (1963) were the first writers to investigate the impact of decay on inventory items. The word "inventory decay" refers to the occurrence of direct spoiling, physical depletion, and degradation. They created a generic EOQ model with continuous demand and exponential decline that can be solved repeatedly but not directly. Covert and Philip (1973) and Tadikamalla (1978) applied Ghare and Schrader's approach to Weibull and Gamma distribution-based degradation patterns, respectively. All models assumed rapid replenishment and did not account for backorders or shortages. Misra (1975) created EOQ models with limited production rates, but did not enable backorders. His models incorporate both changing and constant degradation rates. Shah (1977) created models for exponential and Weibull degradation scenarios, allowing for backorders with rapid replacement. Mak's (1982) manufacturing lot size inventory model includes backorders for products with exponential decay. Heng et al. (1991) combined Misra and Shah's ideas and created a model for degrading products with finite production rates and backorders.

Degradation inventory model research has gained popularity in recent years, with models incorporating various degradation patterns, demand functions, and back-ordering procedures. In most circumstances, degradation is considered to represent a constant proportion of total inventory on hand. The Weibull distribution has been used to represent item degradation (Chakrabarty et al., 1998), with some focus on deteriorating objects with expiry dates (Hsu et al., 2006; Loetal., 2007). Recent research in this field has examined time-varying, stock-level, and price-dependent demand. Several studies, including Wee (1993, 1995), Hariga et al. (1997), Bhunia and Maiti (1998), Chung and Tsai (2001), Wang (2002), Balkhi (2004), Teng and Chang (2005), Yang (2005), Shah et al. (2005), Chang et al. (2006), Hou (2006), Wu et al. (2006), Manna and Chaudhuri (2006), Pal et al. (2006), and Lo et al. (2007) provide examples of inventory models with various assumptions on demand patterns and deterioration.

Existing inventory models for decaying products are typically EOQ-based and take into account many supply chain sub-systems separately. While collaboration between suppliers and purchasers has gained attention in the literature, few integrated inventory techniques for degrading commodities have been established to far. Rau et al. (2003) created a multi-echelon inventory model for a deteriorating item to determine the ideal joint total cost from an integrated perspective, involving suppliers, producers, and buyers. Yang and Wee (2003) created a mathematical model for multi-item manufacturing lot sizing with deteriorating goods.

Existing literature recommends several inventory models for decaying objects. Few have been designed specifically for the JIT environment. Yang and Wee (2003) created an integrated multi-product lot-size inventory model for degrading products in a JIT scenario. Both lot-splitting of materials from suppliers to producers and lot-splitting of completed goods from producers to numerous customers are studied. However, the ideal solutions are not provided. Furthermore, the paper's differential equations are no longer relevant for tiny numbers of purchasers, such as one. This study focusses on the issue of a single supplier and customer dealing with a deteriorating inventory item in a JIT context.

Almost all writers employ calculus-based methodologies to solve inventory models for degrading objects. When cost functions for the supplier and consumer are computed separately, these procedures grow more difficult. When a lot is delivered from a supplier to a buyer at a certain "delivery points" the supplier's inventory level changes abruptly, impacting the supplier's inventory function. These inflexions make it challenging to apply traditional optimisation approaches. This work uses an algebraic technique to calculate cost functions for suppliers, buyers, and the whole supply chain, assuming a minor degradation rate that may be ignored.

This work aims to address a gap in the research on inventory models for degrading products in JIT contexts. This might be viewed as an extension and generalisation of Kim and Ha's (2003) study.

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II. Assumptions and Notation

Assumptions

(a) The operating environment is deterministic.

(b) The supplier's production rate and the buyer's demand rate are constant.

(c) The inventory item's deterioration is a constant fraction of its on-hand inventory.

(d) The production rate is greater than the demand rate.

(e) The buyer pays transportation and order handling costs.

(f) The cost of the deteriorating item is constant.

(g) Shortages are not permitted.

(h) The deterioration rate is small enough to justify a square or higher power.

Notation.

1. for the complete supply chain:

N = number of deliveries per batch cycle;

Q = production lot size per batch cycle (units);

T = total cycle duration (in time units);

q = delivery lot size (units);

d = item degradation rate;

C_d = cost of deterioration per unit ($).

2. for the Supplier:

P = represents production rate (units/time unit),

C = represents setup cost ($/setup),

H_S = represents inventory holding cost ($/unit/time unit), and

S_{sup} = represents the area under the supplier's inventory curve.

3. for the buyer:

D = demand rate in units/time unit;

A = ordering cost in $/order;

HB = inventory holding cost in $/unit/time unit;

F = fixed transportation cost per delivery.

V = represents the variable cost of order handling and receiving ($),

S = represents the area under the buyer's inventory level curve.

III. Model development.

The proposed scenario involves a supplier delivering specified quantities of a product to a buyer's warehouse at predetermined intervals. These deliveries arrive in the warehouse precisely when the previous delivery's products have been exhausted. Figures 1 and 2 exhibit inventory time-plots for the buyer and supplier, respectively.

The entire cycle time (T) is split into two components: T_1, the time the supplier produces the product, and T_2, the period the provider does not create the product. Additionally, we designated T_3 as the interval between two successful deliveries. This analysis considers the following costs:

(a) Setup cost per unit time for the supplier = C/T,

(b) Holding cost per unit time for the supplier = $H_S S_{sup}/T$,

(c) Deterioration cost per unit time for the supplier = $C_d d S_{sup}/T$,

(d) Ordering cost per unit for the buyer = A/T,

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(e) Holding cost per unit time for the buyer = $H_B S_{buy}/T$,

(f) Transportation and handling cost per unit time for the buyer = $N_F + VN_q/T$,

(g) Deterioration cost per unit time for the buyer = $C_d dS_{buy}/T$.

3.1. The Buyer's Inventory Cost Model

The delivery lot size is $q = x + DT_3$, where x is the number of degraded units.

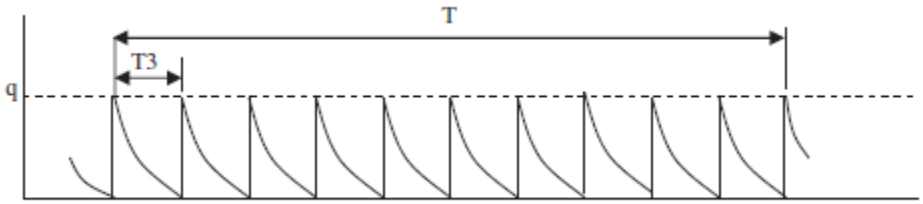


Fig. 1. The buyer's inventory level.

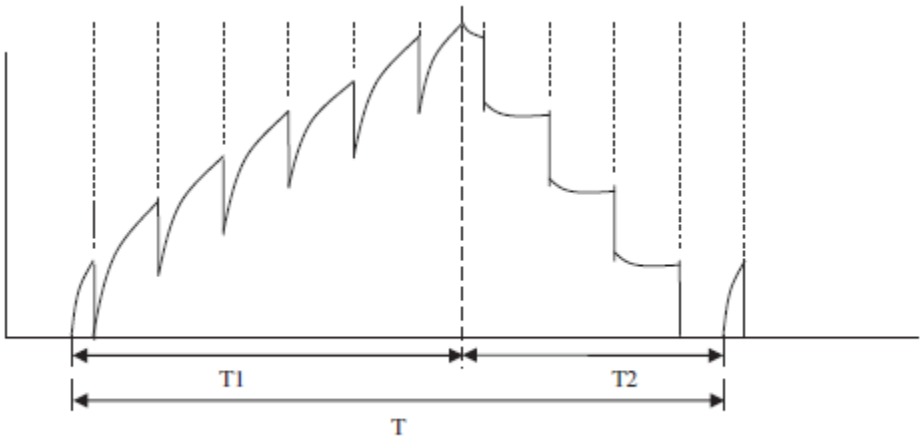


Fig. 2. The supplier's inventory level.

The quantity q is broken into two parts: consumption (DT_3) and the number of deteriorating items (x). If the rate of degradation is low, we can use the square root and higher powers. Thus, x represents the degradation of q units throughout the time T_3.

$$q = DT_3 + \frac{dqT_3}{2}$$

Substituting, $T_3 = T/N$, We obtain,

$$T = \frac{2Nq}{2D + dq} \qquad \text{..... (1)}$$

Also, given the overall degeneration on the buyer's end, we have

$$dS_{buy} = Nq - DT$$

In other words,

$$S_{buy} = \frac{Nq - DT}{d} \qquad \text{..... (2)}$$

Equations (1) and (2) show that the buyer's cost function may be represented as

$$TC_{buy}(q,N) = \frac{A}{T} + \frac{H_B S_{buy}}{T} + \frac{NF + VNq}{T} + \frac{C_d dS_{buy}}{T}$$
$$= \left(\frac{D}{Nq} + \frac{d}{2N}\right)(A + NF + VNq) + \frac{q}{2}(H_B + C_d d)$$

..... (3)

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The Supplier's Inventory Cost Model

To describe the number of degraded units at the supplier's end, we can use

$$y = dS_{sup}$$

The supply chain's total number of deteriorating units is

$$Y + dqT/2. \text{ Since } Q = Nq + y, T1 = (Nq+y)/P.$$

Taking into account the starting inventory and average inventory across the supply chain, we have

$$y + \frac{dqT}{2} = dT\left(\frac{D}{P}q + \frac{Nq+y}{2}\frac{P-D}{P}\right)$$

Leading to,

$$S_{sup} = \frac{y}{d} = qT\left(\frac{D}{P} - \frac{1}{2} + \frac{N}{2} - \frac{DN}{2P}\right) \quad \text{.....(4)}$$

Based on equations (1) and (4), the supplier's cost function may be expressed as

$$TC_{sup}(q,N) = \frac{C}{T} + \frac{H_S S_{sup}}{T} + \frac{C_d d S_{sup}}{T}$$
$$= \left(\frac{D}{Nq} + \frac{d}{2N}\right)C + (H_S + C_d d)q\left(\frac{D}{P} - \frac{1}{2} + \frac{N}{2} - \frac{DN}{2P}\right) \quad \text{..... (5)}$$

Integrated Inventory Cost Model

Average Cost Function of Integrated Inventory Model

the average total cost for the integrated inventory model for the full supply chain is calculated by combining the average expenses of the supplier and customer.

$$TC(q,N) = TC_{buy} + TC_{sup}$$
$$= \left(\frac{D}{Nq} + \frac{d}{2N}\right)(A + C + NF + VNq)$$
$$+ \frac{q}{2}\left[(H_B + C_d d) + (H_S + C_d d)\left\{\frac{(2-N)D}{P} + N - 1\right\}\right] \quad \text{..... (6)}$$

Note that d = 0 changes the above equation to the JIT model without degradation, as presented by Kim and Ha (2003). It is evident that for every given N, TC is convex and minimised.

$$q = \sqrt{\frac{2D(A+C+NF)}{\left[(H_B + C_d d) + (H_S + C_d d)\left\{\frac{(2-N)D}{P} + N - 1\right\} + dV\right]N}} \quad \text{..... (7)}$$

Also, for every given q, TC is convex in N and is minimised.

$$N = \sqrt{\frac{(2D + dq)(A+C)P}{(H_S + C_d d)q^2(P-D)}} \quad \text{..... (8)}$$

Optimal interval for N.

Since $N \geq 0$, according to Equation (7), q approaches its upper bound when N = 1.

$$q \leq \sqrt{\frac{2DP(A+C+F)}{P(H_B + C_d d) + (H_S + C_d d)D + dVP}} \quad \text{..... (9)}$$

As N rises, the associated q drops. Hence, from Equation (7), we get

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$$q \geq \sqrt{\frac{2PD(A+C+NF)}{[(H_B+C_d d)+dV][2D+N(P-D)]N}} \quad \text{…… (10)}$$

Furthermore, since $N \geq 1$, Eq. (8) leads to,

$$1 \leq N \leq N_U = \frac{(H_B+C_d d)+dV}{(H_S+C_d d)}\frac{A+C}{F}+\sqrt{\frac{2D[(H_B+C_d d)+dV]+d(A+C)P}{(H_S+C_d d)(P-D)}} \quad \text{…….. (11)}$$

Solution Procedure

For the integrated model, we propose the following search-based solution procedure:
Step 1: Determine the ideal interval for N using inequality (11).

Step 2: Use Eq. (7) to compute q values for all numbers in this interval (1, NU).

Step 3: Use Eq. (6) to get TC values.

Step 4: Select the optimal N and q pair that results in the minimal TC in step 2.

The following technique allows for easy determination of the optimal q for each integer number N using (7) the computing load of discovering the best (q, N) pair is unlikely to be considerable.

IV. Numerical example

This research uses an enhanced version of the graphic provided by Kim and Ha (2003). The data utilised in this example region is as follows:

1. for the whole supply chain,

C_d = \$50 per unit.

2. for the supplier:

P = 19,200 units per year;

C = \$600 each batch;

H_S = \$6/unit/year.

3. for the buyer:

D = 4800 units per year; A = \$25 per order;

H_B = \$7 per unit/year. F = \$50 per delivery;

V = \$1/unit.

The approach detailed above identifies solutions for varied degradation rates, d, ranging from 0 to 0.2. Figures 3 and 4 illustrate how increasing d affects manufacturing lot size (Q) and total supply chain cost (TC).

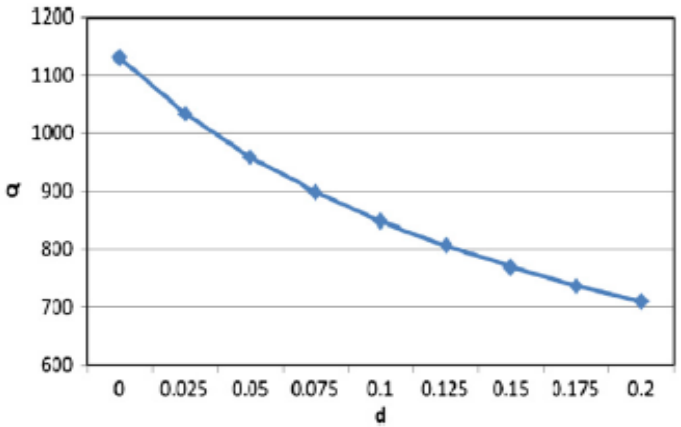


Fig. 3. Production lot size, *Q* vs. *d*.

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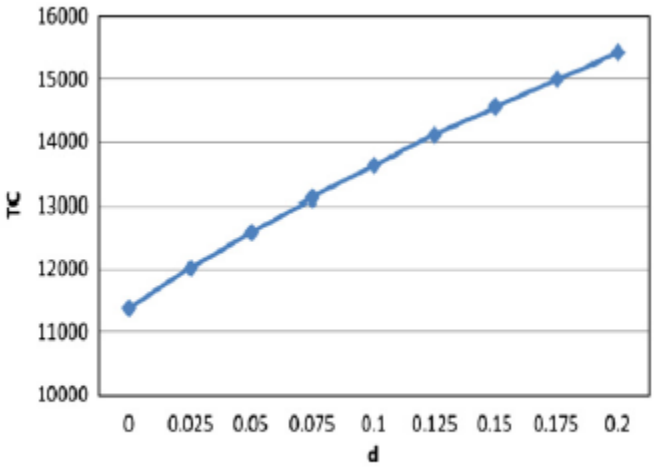


Fig. 4. Annual total system cost, *TC* vs. *d*.

V. Summary and conclusions.

Our work presents an integrated single-supplier, single-buyer inventory model for a degrading JIT scenario. The cost functions for suppliers, buyers, and the integrated supply chain are generated. This study also analyses the ideal interval for deliveries each production batch cycle, which is considered to be an integer. We provide an effective technique for establishing the appropriate delivery lot size for the buyer, the supplier's optimal manufacturing batch size, and the number of deliveries from each batch. Our results indicate that when the degradation rate increases, both the ideal manufacturing lot size and cycle duration decrease (refer to Fig. 3). As the degradation rate increases, production lot size and cycle time decrease to benefit the whole supply chain. Increased degradation rate reduces delivery lot size without reducing the number of deliveries each production batch. The ideal interval for N is not unmanageably wide and does not change with degradation rates over d = 0. Reduced inventory cycle durations for both parties tend to lessen the detrimental impacts of degradation on the supply chain. Not surprisingly, our findings show that the supply chain's average yearly cost rises as the rate of degradation increases (refer to Figure 4). Deterioration impacts inventory cycles, requiring more frequent setups and deliveries than would otherwise be necessary. Future research should examine how enabling backorders in an integrated supply chain affects item degradation.

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