

State Estimation Method for Power System with a Large Number of Branches Using Voltage and Current as State Variables

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Abstract- A power system state estimation is a key tool for a reliable and efficient management of electrical power. This paper presents based on new state variables a state estimation method for power system with a large number of branches. In this method, the node voltage and the active current and reactive current of branch are chosen as the state variables. Therefore, measurement functions of ampere and voltage are significantly simplified and no measurements transformation is needed in this method. The observability analysis indicates the proposed method can be used in power system with very limited measurements. Numerical tests on power system with the radial and meshed network show the real-time branch current measurements can significantly improve the accuracy and computation speed of estimation results.

Keywords: Newton–Raphson (NR) algorithm, branch current measurements, power system state estimation.

I. Introduction

ENERGY shortage has been a severe issue nowadays, so the power system state estimation is very important for a reliable and efficient management of electrical power. It is important for the energy management system (EMS) applications in a power system control center that the measurements allow monitoring of this power system. With the increase interest in integration of more distributed generations (DGs) to the power system, the EMS center needs to have real-time and accurate estimate values of loads, voltage magnitudes, generator and DGs power outputs, etc. required for a good management and planning of the network. This is only possible if all state variables (voltage magnitudes and angles at each bus) of the power system are known. However, only few measurements which usually contain errors are available at this center. [2] State estimation is an important tool for EMS, which makes it possible to estimate the state variables of the power system based on the limited measurements. State estimation algorithms fit measurements made on the system to a mathematical model in order to provide a reliable data base for other monitoring, security assessment and control functions.[3] Much research has been undertaken on state estimation techniques, and online state estimators are being implemented in many power network. In the application of state estimation, consideration must be given to a number of practical difficulties. In node voltage based method, the node voltages are selected as state variables. And the power measurement and branch current magnitude measurements are transformed into equivalent branch and bus injection current vector to make the Jacobian matrix constant and improve calculation efficiency. Due to the measurement transformation, the weights of the real measurements and their corresponding transformed measurements are not equivalent, which may lead to extra errors in the estimation results. [1, 2] In branch current vector based method, compared with the conventional node-voltage-based methods, the branch current vectors are selected as state variables in this type of methods. The voltage measurements are omitted and the power measurements and load pseudo-measurements are transformed into equivalent branch current vector and injection current vector by pre-calculated node voltage vectors. This method has good performance both in computation speed and memory requirements. The main shortage of the method is measurements transformation is needed, which may introduce extra errors.

[3, 4] In Ref. [13], some special state variables are chosen to overcome this shortage, but this method can only be used for radial network. In branch power based method, the whole problem is decomposed into a series of sub-problems and each sub-problem will deal with only single-branch state estimation. It is extremely efficient, but the solution of these methods cannot guarantee to global optimum. Because of the limited number of real-time measurements in the distribution system, pseudo-measurements (especial the loads' pseudo-measurements) are necessary for distribution system state estimator.[5, 6] In Ref. [7], a Gaussian mixture model is used to formulate load probability distribution functions (PDFs), and the expectation maximization algorithm is used to obtain the parameters of the mixture components. The loads' pseudo-measurement is described as fuzzy number in [8]. Ref. [9] proposed a ARMOR based load estimation model according to the data recorded in automated meter reading (AMR) system. Ref. [10] presents a load modeling technique which incorporates the customers' billing information. In Ref. [11], the power on sending end of branch and the square of branch current magnitude are chosen as the state variables, which make the measurement functions simple and easy to be solved. The branch current measurements, voltage and power measurements are all directly formulated in this method without measurements transformation. Therefore, it is suitable to be used in distribution system with large number of branch current measurements. In this paper, a novel state estimation method is proposed, in which the active current and reactive current measurements and voltage measurements are directly formulated in a weighted least square(WLS) estimation model without measurements transformation. Due to the special chosen state variables, the measurement functions are quite simple. The proposed method is suitable both for radial and meshed network with large number of branches.

II. Methodology

The following symbols are used in this paper:

I_{Pij} : branch i-j active current vector,

I_{Qij} : branch i-j reactive current vector,

P_{ij}, Q_{ij} : active and reactive power vector on the sending end of branch i-j (i represents the sending node),

P_{ji}, Q_{ji} : active and reactive power vector on the receiving end of branch i-j (j represents the receiving node),

U_i : voltage magnitude vector of node i,

P_i, Q_i : three phase injection active and reactive power vector of node i,

Z_{ij} : impedance matrix of branch i-j with phases,

X_{ci} : shunt reactance matrix at node i including line charging and any shunt capacitive or inductive reactance.

The node voltage and the active current and reactive current of branch i-j are chosen as the state variables in this method.

To make more readable, a superscript x is used to mark the state variables and a superscript m is used to mark the measurement.

WLS estimation model

The general estimation consists of estimating the state vector based on a set of measurements z in the presence of an error e . A mathematical model describing the functional relations between z , x and e is described in Eq. (1) [2, 13, 14]. This model is expressed in the form of a set of nonlinear equations which relates the measurements z and the true state vector x .

$$Z_i = f_i(X) + e_i \quad (1)$$

where Z_i is the i -th measurement (measurement vector of dimension m), and $f_i(X)$ is the nonlinear function relating state variables with measurements. This is usually the power injections or power flow equations. X is the state vector of dimension n , e_i is the i -th measurement noise vector, and m is the number of measurements.

Power state estimation is therefore based on minimizing the sum of all errors between available measurements and calculated measurement variables contained in the function $f_i(X)$. For the safe operation of the power system, boundary constraints are set for each state variable. The problem of state estimation is usually formulated as a weighted least squares (WLS) problem, which is solved by efficient numerical techniques. The weighted least square (WLS) estimate can be found by minimizing the following objective function [15]:

$$\min J(X) = \sum_{i=1}^m W_i (Z_i - f_i(X))^2 \quad (2)$$

where σ_i^2 is the variance of i -th measurement, and w_i is the weight of the measurement i .

i , m , e_i , f_i are the same as defined previously;

w_i is the weight of the measurement i .

$$W_i = \begin{cases} \frac{1}{\sigma_i^2} & , |Z_i - f_i(X)|/\sigma_i \leq \nu \\ 0 & , |Z_i - f_i(X)|/\sigma_i > \nu \end{cases}$$

Estimation model for radial network

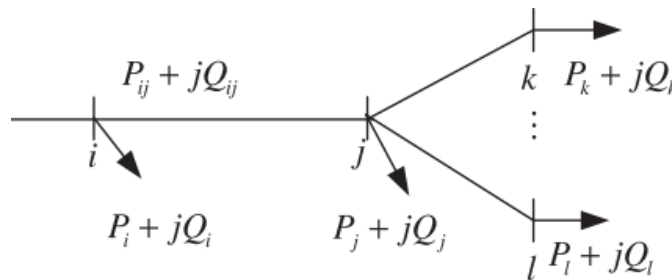


Fig 1: Radial distribution network

1) Branch current measurement function

$$((I_{ij})^m)^2 = (I_{Pij}^x)^2 + (I_{Qij}^x)^2$$

2) Function of power measurement on sending end of branch

$$(P_{ij})^m = U_i^x \times I_{Pij}^x$$

$$(Q_{ij})^m = U_i^x \times I_{Qij}^x$$

3) Functions of power measurement on receiving end of branch

$$(P_{ji})^m = U_i^x \times I_{Pij}^x - r_{ij} \times (I_{Pij}^x)^2$$

$$(Q_{ij})^m = U_i^x \times I_{Qij}^x - x r_{ij} \times (I_{Qij}^x)^2$$

4) Node injection power measurement functions

$$(P_j)^m = U_i^x \times I_{Pij}^x - r_{ij} \times (I_{Pij}^x)^2 - U_j^x \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^n I_{Pjk}^x$$

$$(Q_j)^m = U_i^x \times I_{Qij}^x - r_{ij} \times (I_{Qij}^x)^2 - U_j^x \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^n I_{Qjk}^x + jU_j^2 X_{ci}$$

5) Voltage measurement function

$$(U_i)^m = (U_i)^x$$

6) Voltage constraint measurement function

$$0 = U_i^x \times I_{Pij}^x - r_{ij} \times (I_{Pij}^x)^2 - U_j^x \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^n I_{Pjk}^x$$

$$0 = U_i^x \times I_{Qij}^x - r_{ij} \times (I_{Qij}^x)^2 - U_j^x \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^n I_{Qjk}^x + j(U_j^x)^2 X_{ci}$$

Estimation model for meshed network

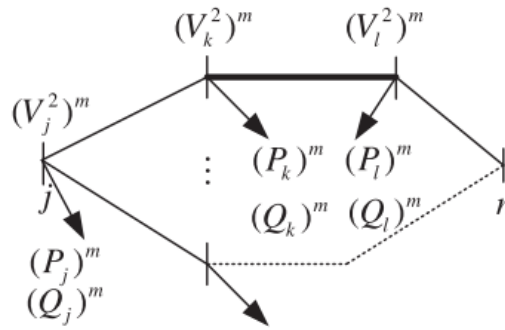


Fig 2: Meshed distribution network

1) Node injection power measurement functions

$$(P_l)^m = U_k^x \times I_{Pkl}^x - r_{kl} \times (I_{Pkl}^x)^2 + U_n^x \times I_{Pnl}^x - r_{nl} \times (I_{Pnl}^x)^2$$

$$(Q_l)^m = U_k^x \times I_{Qkl}^x - r_{kl} \times (I_{Qkl}^x)^2 + U_n^x \times I_{Qnl}^x - r_{nl} \times (I_{Qnl}^x)^2 + jU_l^2 X_{cl}$$

2) Voltage measurement function

$$(U_i)^m = (U_i)^x$$

3) Voltage constraint measurement function

$$0 = U_k^x \times I_{Pkl}^x - r_{kl} \times (I_{Pkl}^x)^2 - (U_n^x \times I_{Pnl}^x - r_{nl} \times (I_{Pnl}^x)^2)$$

$$0 = U_k^x \times I_{Qkl}^x - r_{kl} \times (I_{Qkl}^x)^2 - (U_n^x \times I_{Qnl}^x - r_{nl} \times (I_{Qnl}^x)^2) + jU_l^2 X_{cl}$$

Observability analysis

For a radial network with n branches, there are $3 \times n + 1$ state variables in this estimation model which has more n variables than the conventional estimation method. Actually these extra unknown variables do not need extra measurements to guarantee system observable. In the radial network, extra $n + 1$ voltage constraint measurement functions (defined in Eq. (9)) are included. Thus, $2 \times n$ measurement functions are at least needed to solve the proposed estimation model. If all the load pseudo-measurements (including the zero injection) are generated, $2 \times n + 2$ measurement functions are provided. If the roots of feeders have voltage measurements, three voltage measurement functions are provided. Then the observability requirement is satisfied. The real-time branch current measurements, other power and voltage measurements can be used to improve the accuracy of estimation results in this method.

For a meshed network with n branches, n nodes, $3 \times n$ state variables are used in this estimation method. In the meshed network, extra n voltage constraint measurement functions (defined in Eq. (9)) are included. Thus, $2 \times n$ measurement functions are at least needed to solve the proposed estimation model.

Therefore, this observability requirement can be easily satisfied in meshed distribution system in field application. And other real-time measurements can be used to improve the accuracy of estimation results. This state estimation model can also be used in identifying best location to place meters [15, 16].

The state estimation algorithm

Fig. 3 shows the state estimation algorithm for this work.

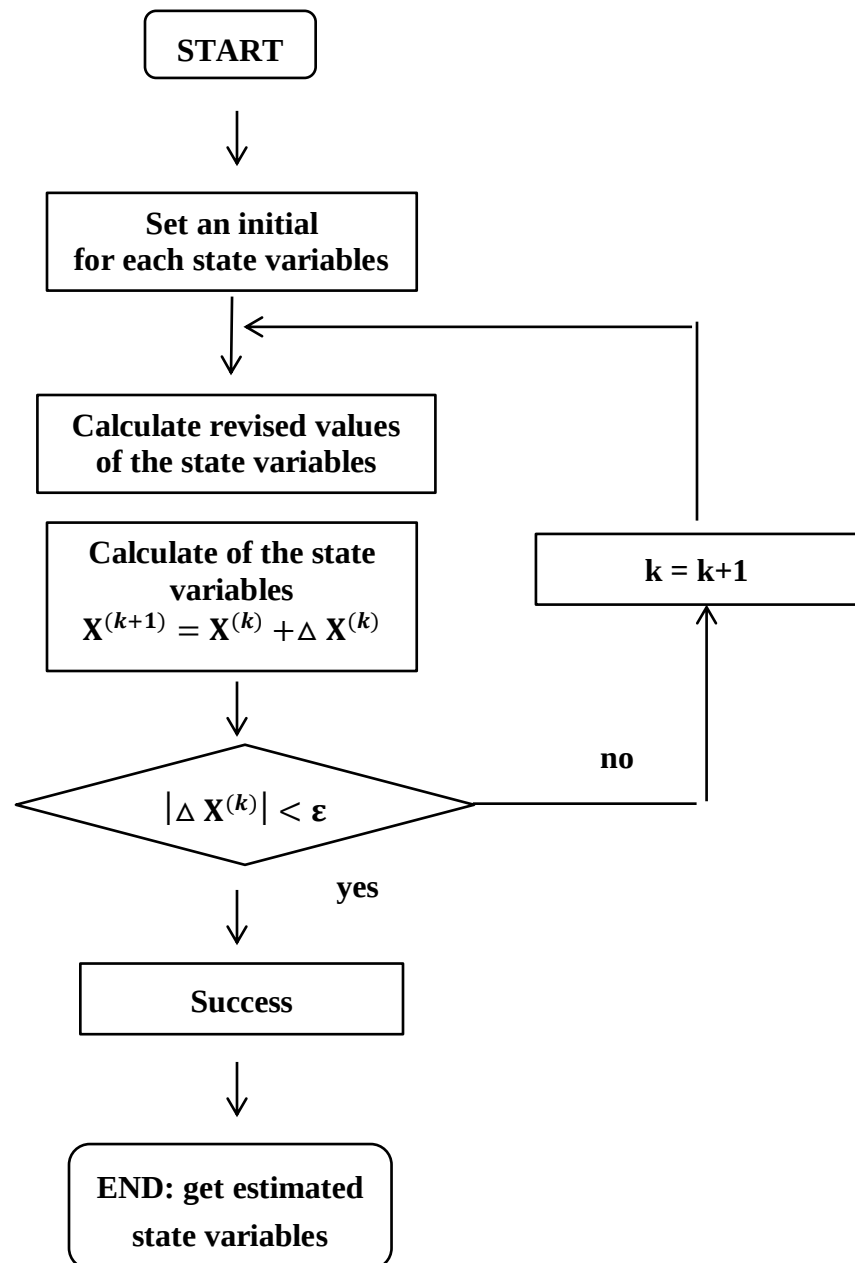


Fig 3: State estimation algorithm

Comparison of convergence for previous algorithm and proposed in the paper algorithm is plotted as the number of iteration.

Comparison of accuracy for previous algorithm and proposed in the paper algorithm is plotted as the objective function and the maximum state estimation error.

Table 1 shows the state estimation results of the two algorithms for IEEE 14-bus test system.

Table1. The results of the two algorithms for IEEE 14-bus test system

	previous algorithm	proposed algorithm
maximum state estimation error, P12	0.0245	0.0156
objective function	40.33	33.31
number of iterations	12	8

In case of previous algorithm the maximum state estimation error is 0.0245 at bus 14.

In case of proposed algorithm the maximum state estimation error is 0.0156 at bus 14.

In case of previous algorithm the objective function is 40.33.

In case of proposed algorithm the objective function is 33.31.

In case of previous algorithm the number of iterations is 12.

In case of proposed algorithm the number of iterations is 8.

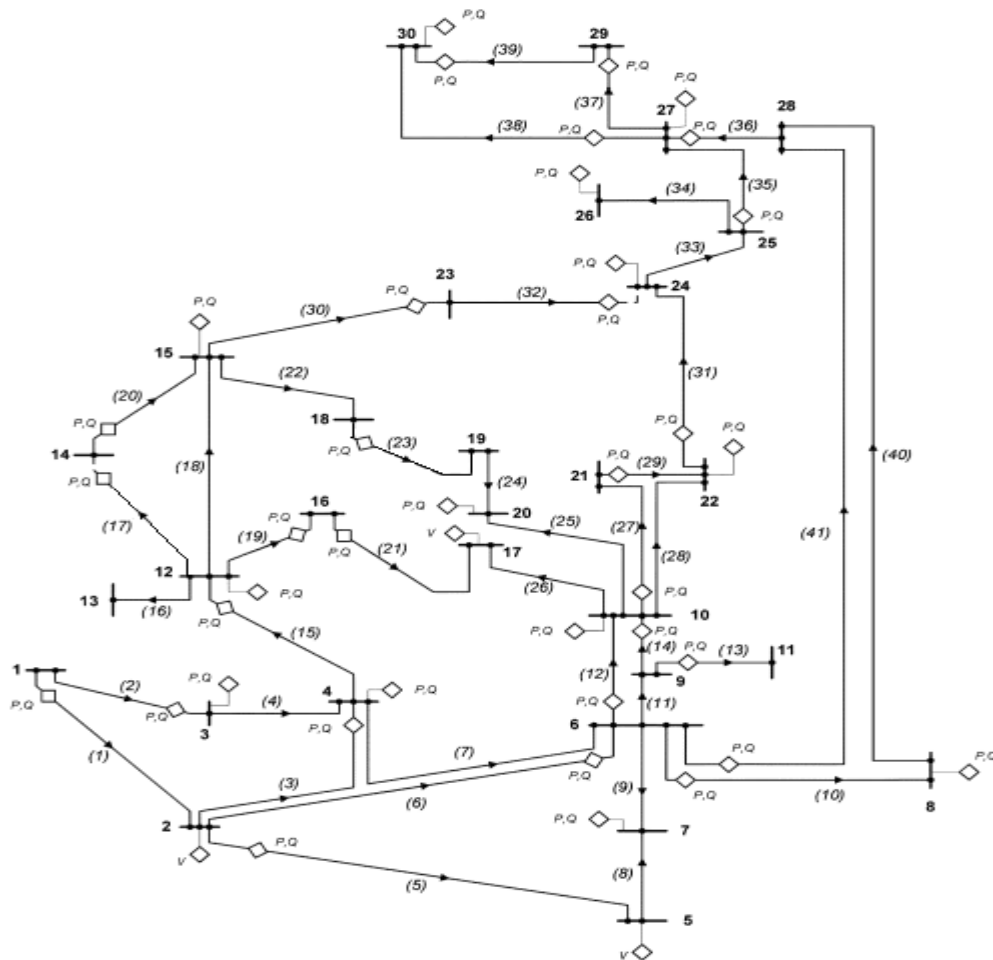


Fig5. IEEE 30-bus network

Table2. The results of the two algorithms for IEEE 30-bus test system

	previous algorithm	proposed algorithm
maximum state estimation error, P12	0.0725	0.0336
objective function	50.32	37.28
number of iterations	15	10

Table 2 shows the state estimation results of the two algorithms for IEEE 30-bus test system.

In case of previous algorithm the maximum state estimation error is 0.0725 at bus 14.

In case of proposed algorithm the maximum state estimation error is 0.0336 at bus 14.

In case of previous algorithm the objective function is 50.32.

In case of proposed algorithm the objective function is 37.28.

In case of previous algorithm the number of iterations is 15.

In case of proposed algorithm the number of iterations is 10.

Through the reduction of the objective function value of the state estimation, it can be seen that the proposed algorithm improves the robustness to bad data.

Through the reduction of the number of iterations of state estimation, it can be seen that the proposed algorithm reduces the computation time of the state estimation.

From Tables 1 and 2, it can be seen that the proposed algorithm satisfy the conditions of accuracy and computational time for power system state estimation.

IV. Conclusion

The branch current measurements, voltage and power measurements are all directly formulated in this method without measurements transformation. Therefore, it is suitable to be used in distribution system with large number of branch current measurements. The node voltage and the active current and reactive current of branch are chosen as the state variables, which make the measurement functions simple and easy to be solved. This method can be used in radial and meshed power systems. This method is designed for this situation and has a good performance. It can be applied to the state estimation of renewable integration and smart grid systems.

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