ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

Applications of Class of Symmetric Mesokurtic Error Distributions in Auto Regressive Models

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DOI: https://doi.org/10.51583/IJLTEMAS.2025.1409000099

Abstract: This paper examines a family of symmetric mesokurtic distributions proposed by Sebastian and James (2002), employed as error distributions in Auto Regressive models. As maximum likelihood equations become intractable under such settings, the Modified Maximum Likelihood Estimation (MML) approach is adopted to obtain parameter estimates. The asymptotic properties are discussed, and simulation experiments are carried out to evaluate the estimators. Results from the simulation show that the proposed estimates perform better than least squares estimators with respect to standard errors for small sample sizes, and the relative efficiency tends to one as n increases.

I. Introduction

Parameter estimation in Auto Regressive (AR) models has been widely studied in statistical literature. Error terms in such models are typically considered under two categories: Gaussian and Non-Gaussian. Durbin (1960) and Weiss (1990) studied Gaussian error structures, while Huber (1981), Tikku et al. (1986, 1999, 2000) explored Non-Gaussian errors. In particular, Tikku et al. (2000) examined AR models with a family of non-normal symmetric innovations, applying the Modified Maximum Likelihood (MML) method because the Maximum Likelihood (ML) equations were analytically intractable. Following their approach, this study applies MML estimation when error terms belong to the symmetric mesokurtic distributions introduced by Sebastian and James (2002).

Consider the time series model:

$$y_t = \sum_{j=1}^{p} a_j y_{t-j} + \varepsilon_t \qquad t = 1,2,\dots,n \tag{1}$$

Here, ε_t the errors are independent and identically distributed (i.i.d.) with mean zero and variance σ^2. Traditionally, they are assumed to follow a normal distribution with mean zero and variance . However, many studies (e.g., Ganadesikan (1977)) highlight deviations from normality in practical data. Violations of normality affect the robustness of estimators, as pointed out by Huber (1981), Martin and Yohai (1985), and Tikku et al. (1986).

In this paper, instead of normal errors, the error distribution is considered from the symmetric mesokurtic family introduced by Sebastian and James (2002). The MML method, as outlined in Tikku and Suresh (1992), is used since it ensures robustness along with other desirable properties. The family of probability density functions is given by:

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}}\{1 + KH_6(z)\}e^{-\frac{z^2}{2}}, -\infty < x < \infty\ , \sigma > 0 \tag{2}$$

where , $Z = \frac{X-\mu}{\sigma}$, $K = \frac{\alpha}{720}$, and $0 \le \alpha < 6.9$. Here, $H_6(z) = Z^6 - 15Z^4 + 45Z^2 - 15$ is the sixth-degree Hermite polynomial. As shown by Sebastian and James (2002), this distribution is symmetric with Pearson's measure of kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2} = 3$, where μ_k is the kth central moment of Z. For small values of α, the distribution resembles normal, while for larger values it becomes trimodal with two side humps, making it useful in the presence of outliers. Detailed treatment of this family is available in Sebastian and James (2002, 2005). Parameter estimation using MML in this setting is given in Sebastian and James (2001). In this work, ε_t are assumed i.i.d. according to equation (2).

2. AR(1) Model

Consider the AR(1) model:

$$y_t = ay_{t-1} + \varepsilon_t \tag{3}$$

where, $|a| < 1$, $E(\varepsilon_t) = 0$, with $V(\varepsilon_t) = \sigma^2$

Assume $y_0 = \frac{\varepsilon_0}{\sqrt{1-a^2}}$ and ε_0 , ε_1, …, ε_n are i.i.d. with distribution:

$$f(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}}\left\{1 + KH_6\left(\frac{\varepsilon}{\sigma}\right)\right\}e^{-\frac{\varepsilon^2}{2\sigma^2}}, -\infty < \varepsilon < \infty\ , \sigma > 0 \tag{4}$$

Since , $\varepsilon_t = y_t - ay_{t-1}$, $t = 1,2,\dots,\ n$, and letting $Z_t = \frac{\varepsilon_t}{\sigma}$, the log-likelihood equations of AR(1) are:

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

$lnL = C - nln(\sigma) + \sum_{i=1}^{n} ln[1 + KH_6(z_t)] - \sum_{i=1}^{n} \frac{Z_t^2}{2}$

where C is a constant. Differentiating with respect to and gives:

$$\frac{\partial lnL}{\partial a} = \frac{1}{\sigma}[\sum_{i=1}^{n} y_{t-1} g(Z_t) - \sum_{i=1}^{n} y_{t-1} Z_t] \quad \text{and} \tag{5}$$

$$\frac{\partial lnL}{\partial \sigma} = \frac{1}{\sigma}\left[n + \sum_{i=1}^{n} Z_t + g(Z_t) - \sum_{i=1}^{n} {Z_t}^2\right], \tag{6}$$

where , $g(z_t) = \frac{6KH_5(Z_t)}{1+KH_6(Z_t)}$.

Equating $\frac{\partial lnL}{\partial a}$ and $\frac{\partial lnL}{\partial \sigma}$ to zero, we get the likelihood equations as

$$\sum_{i=1}^{n} y_{t-1} g(Z_t) - \sum_{i=1}^{n} y_{t-1} Z_t = 0 \quad \text{and} \tag{7}$$

$$n + \sum_{i=1}^{n} Z_t \; g(Z_t) - \sum_{i=1}^{n} {Z_t}^2 = 0 \tag{8}$$

3. Information matrix of a and σ

Differentiating (5) with respect to and taking expectations, we get

$$E\left(\frac{\partial^2 lnL}{\partial a^2}\right) = \frac{-n}{1-a^2}\left[1 + E\left\{\frac{6KH_5(Z_t)}{1+KH_6(Z_t)}\right\}^2\right]$$

From (6), we get

$$E\left(\frac{\partial^2 lnL}{\partial \sigma^2}\right) = \frac{-2n}{\sigma^2}\left[1 + E\left\{\frac{6KZ_tH_5(Z_t)}{1+KH_6(Z_t)}\right\}^2\right]$$

It can be shown that $E\left(\frac{\partial^2 lnL}{\partial a \partial \sigma}\right) = 0$ and $E\left(\frac{\partial^2 lnL}{\partial \sigma \partial a}\right) = 0$. Thus, the Fisher information matrix of a and σ is:

$$I = \begin{bmatrix} \frac{-n}{1-a^2}(1 + I_a) & 0 \\ 0 & \frac{2n}{\sigma^2}(1 + I_\sigma) \end{bmatrix} \tag{9}$$

where, $I_a = E_\phi\left\{\frac{6KH_5(Z_t)}{1+KH_6(Z_t)}\right\}^2$ and $I_\sigma = 18E_\phi\left\{\frac{KZ_tH_5(Z_t)}{1+KH_6(Z_t)}\right\}^2$.

Here E_ϕ denotes expectation with respect to a normal distribution. The inverse of this matrix I^{-1} provides the asymptotic variance–covariance matrix of the MLE of a and σ.

4. MML Estimates

Equations (7) and (8) are analytically intractable and iterative solutions are hampered by multiple roots, slow convergence, or divergence (Lee et al., 1980; Vaughan, 1992). Therefore, the MML approach of Tikku et al. (1992) is applied. The robustness and efficiency of MML estimates over least squares have been established (Tikku et al., 2000).

By noticing that y_{t-1} and Z_t are independent and the complete sums are invariant under ordering, order Z_t for given a in an increasing order and denote the ordered Z values by

$$Z_{(i)} = \frac{y_{[i]} - a y_{[i]-1}}{\sigma}, \quad 1 \le i \le n,$$

where $(y_{[i]}, y_{[i]-1})$ is that pair $(y_t,\ y_{t-1})$ values which determine $Z_{(i)}$ $(1 \le i \le n)$. Or, $y_{[i]}$ is called the i^{th} concomitant of $Z_{(i)}$. By equating equations (5) and (6) to zero, we can write

$$\frac{1}{n}\frac{\partial lnL}{\partial a} = \frac{1}{n\sigma}[\sum_{i=1}^{n} y_{t-1} g(Z_t) - \sum_{i=1}^{n} y_{t-1} Z_t] = 0$$

$$\frac{1}{n}\frac{\partial lnL}{\partial \sigma} = \frac{1}{n\sigma}\left[n + \sum_{i=1}^{n} Z_t + g(Z_t) - \sum_{i=1}^{n} {Z_t}^2\right] = 0$$

Replacing, Z_t, y_t and y_{t-1} by $Z_{(i)}$, $y_{[i]}$ and $y_{[i]-1}$ respectively we get,

$$\frac{1}{n}\frac{\partial lnL}{\partial a} = \frac{1}{n\sigma}\left[\sum_{i=1}^{n} y_{[i]} g(Z_{(i)}) - \sum_{i=1}^{n} y_{[i]-1} Z_{(i)}\right] = 0$$

$$\frac{1}{n}\frac{\partial lnL}{\partial \sigma} = \frac{1}{n\sigma}\left[n + \sum_{i=1}^{n} Z_{(i)} + g(Z_{(i)}) - \sum_{i=1}^{n} {Z_{(i)}}^2\right] = 0$$

Let $t_{(i)} = E(Z_{(i)})$, $1 \le t \le n$. Replace, $g(Z_{(i)})$ in the above equations by a linear function

$$g(Z_{(i)}) = \alpha_i + \beta_i Z_{(i)}$$

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

Now use Taylor's series expansion to write

$$g(Z_{(i)}) \approx g(t_{(i)}) + (Z_{(i)} - t_{(i)}) \frac{\partial g(Z_t)}{\partial Z_t}|_{t=t_{(i)}}$$

Here, $\alpha_i = \frac{6KH_5(t_{(i)})+(1+6KH_5(t_{(i)})t_{(i)})}{1+KH_6(t_{(i)})}$ and $\beta_i = \left[\frac{6KH_5(t_{(i)})}{1+KH_6(t_{(i)})}\right]^2$

Denote the likelihood equation obtained by linearizing $g(Z)$ as L^*. Due to the symmetry of the p.d.f., it is easy to verify that $t_{(i)} = -t_{n-i+1}$ and hence $\alpha_i = -\alpha_{n-i+1}$ and hence $\beta_i = -\beta_{n-i+1}$ with $\sum \alpha_i = 0$. Hence, we can write,

$\frac{1}{n}\frac{\partial lnL}{\partial a} \approx \frac{1}{n}\frac{\partial lnL^*}{\partial a} = 0$ which gives

$$\frac{1}{n\sigma}\left[\sum_{i=1}^{n} Z_{(i)} y_{[i]-1} - \sum_{i=1}^{n}(\alpha_i + \beta_i Z_{(i)}) y_{[i]-1}\right] = 0 \quad (10)$$

$\frac{1}{n}\frac{\partial lnL}{\partial \sigma} \approx \frac{1}{n}\frac{\partial lnL^*}{\partial \sigma} = 0$ which gives

$$\frac{1}{n\sigma}\left[-n - \sum_{i=1}^{n}(\alpha_i + \beta_i Z_{(i)}) Z_{(i)} + \sum_{i=1}^{n} {Z_{(i)}}^2\right] = 0 \quad (11)$$

Solving equation (10) we get

$$\hat{a} = K - D\sigma$$

where, $K = \frac{\sum w_i y_{[i]-1} y_{[i]}}{\sum w_i {y^2}_{[i]-1}}$ and $D = \frac{\sum y_{[i]-1}\alpha_i}{\sum w_i {y^2}_{[i]-1}}$ with $w_i = 1 - \beta_i$

Solving the equation (11) we get

$$\hat{\sigma} = \frac{-B+\sqrt{B^2+4nC}}{2n} \quad (12)$$

where, $B = \sum_{i=1}^{n}(\alpha_i y_{[i]} - K y_{[i]-1})$, $C = \sum_{i=1}^{n} w_i (y_{[i]} - K y_{[i]-1})^2$

To compute $\hat{a}$ and $\hat{\sigma}$ we need the values of $t_{(i)}$, $(1 \le i \le n)$. For large n, one can obtain the values by equating,

$$\int_{-\infty}^{t_{(i)}} f(Z)dZ = \frac{i}{n+1}, 1 \le i \le n$$

The MML estimates can be computed from the equations (11) and (12) using a suitable iterative procedure. We can use the least square estimates of α and σ as the initial estimates to compute the MML estimates.

5. General AR(1) Model

If one start with the general model

$$y_t = \mu + a y_{t-1} + \varepsilon_t, t = 1,2,\dots,n \quad (13)$$

where, μ is the unknown location, we can estimate μ using MML estimation procedure as follows:

The modified likelihood equations for estimating μ is

$$\frac{\partial lnL^*}{\partial \mu} = \frac{1}{\sigma}\left[\sum_{i=1}^{n} Z_{(i)} + \sum_{i=1}^{n}(\alpha_i + \beta_i Z_{(i)}) Z_{(i)}\right]$$

Solving the equation $\frac{\partial lnL^*}{\partial \mu} = 0$, μ the fact that $\sum \alpha_i = 0$ we get

$$\hat{\mu} = \frac{\sum_{i=1}^{n} w_i (y_{[i]} - a y_{[i]-1})}{w}, \text{ where, } w = \sum w_i$$

Now the estimation of a and σ should be adjusted for μ as follows:

$$x_{[i]} = y_{[i]} - \frac{1}{w}\sum w_i y_{[i]}$$

The MML estimators of a and σ are modified by replacing $y_{[i]}$ by $x_{[i]}$ and $y_{[i]-1}$ by $x_{[i]-1}$. Hence, we can compute $\hat{\mu}$ using the formula

$$\hat{\mu} = \frac{\sum_{i=1}^{n} w_i (x_{[i]} - a x_{[i]-1})}{w} \quad (14)$$

6. Extension to AR(q) Model

The theory of MML estimation given above for an AR(1) model can be easily extended to the case of an AR(q) model.

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

$y_t = \sum_{j=1}^{q} a_j y_{t-j} + \epsilon_t$ for $t = 1,2,\dots,n$

The MML estimates of a_j $(1 \le j \le q)$ and σ are the solutions of the $q+1$ equations

$G_{j1}a_1 + G_{j2}a_2 + \dots + G_{jq}a_q = K_j + D_j\sigma$ for $1 \le j \le q$ and

$-n\sigma^2 - \sum_{i=1}^{n} \alpha_i\left(y_{[i]} - \phi_1 y_{[i]-1} - \phi_2 y_{[i]-2} - \dots - \phi_q y_{[i]-q}\right) + \sum_{i=1}^{n} w_i y_{[i]}\left(y_{[i]} - \phi_1 y_{[i]-1} - \phi_2 y_{[i]-2} - \dots - \phi_q y_{[i]-q}\right) = 0$

Where, $\left(y_{[i]}, y_{[i]-1}, \dots, y_{[i]-q}\right)$ is that observation $\left(y_1, y_2, \dots, y_{i-q}\right)$ which involves $Z_{(i)}$, $(1 \le i \le n)$.

Now define, $G_{jl} = \sum w_i\, y_{[i]-j} y_{[i]-l}$ for $G_{jl} = G_{lj}$ for $j \ne l$ and

$K_j = \sum \beta_i\, y_{[i]} y_{[i]-j}$ and $D_i = \sum \alpha_i\, y_{[i]-j}$, or $1 \le j \le q$

The MML estimates are $A^{-1} = C^{-1}(K + D\hat{\sigma})$ and $\hat{\sigma} = \frac{-B+\sqrt{B^2+4nC}}{2n}$ with $A = \left(a_1, a_2, \dots, a_q\right)'$, $K = \left(K_1, K_2, \dots, K_q\right)'$ and $C = \left(C_{ij}\right)$

Simulation Study

The following table gives the results of the simulation study based on 10000 Monte Carlo simulations. I have simulated mean and variance of $\hat{a}$ and $\hat{\sigma}$ for n=20,50 and 300 and for $\alpha = 1,2,3,4,5,6$. Values given the bracket are their simulated standard errors. It shows that the MML estimates outperform the least square estimates in terms of the standard errors for small n and their relative efficiency approaches one for large n.

Table-1: Simulated performance of the MML estimates for different values of α and n.

n=20, σ= 1, a =0.2						
$\alpha =$	1	2	3	4	5	6
a_0	0.1316	0.1315	0.1314	0.1314	0.1313	0.1313
	(0.0449)	(0.0450)	(0.0454)	(0.0452)	(0.0453)	(0.0453)
$\hat{a}$	0.1825	0.1828	0.1836	0.1844	0.1858	0.186
	(0.0430)	(0.0432)	(0.0435)	(0.04360)	(0.0437)	(0.0437)
σ_0	0.999	0.999	0.9991	0.9991	0.9992	0.9992
	(0.0263)	(0.0260)	(0.0259)	(0.0258)	(0.0255)	(0.0255)
$\hat{\sigma}$	0.9701	0.975	0.9731	0.9691	0.969	0.969
	(0.0240)	(0.0238)	(0.0237)	(0.0237)	(0.0235)	(0.0235)
n=20, σ= 1, a =0.5						
$\alpha =$	1	2	3	4	5	6
a_0	0.3893	0.389	0.3888	0.3888	0.3887	0.3885
	(0.0430)	(0.0430)	(0.0435)	(0.0435)	(0.0434)	(0.0438)
$\hat{a}$	0.4597	0.4606	0.462	0.462	0.4621	0.4629
	(0.0370)	(0.0371)	(0.0369)	(0.0369)	(0.0372)	(0.0371)
σ_0	1.0849	1.0849	1.0848	1.0848	1.0845	1.0849
	(0.0460)	(0.0458)	(0.0457)	(0.0456)	(0.0455)	(0.0455)
$\hat{\sigma}$	0.9665	0.9658	0.965	0.9648	0.9648	0.9647
	(0.0240)	(0.0240)	(0.0239)	(0.0235)	(0.0230)	(0.0229)
n=20, σ= 1, a=0.8						

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

$\alpha =$	1	2	3	4	5	6
a_0	0.6537	0.6535	0.6536	0.6534	0.654	0.6541
	(0.0362)	(0.0363)	(0.0363)	(0.0365)	(0.0368)	(0.0368)
$\hat{a}$	0.7379	0.738	0.7395	0.739	0.7392	0.7395
	(0.0240)	(0.0240)	(0.0241)	(0.0243)	(0.0243)	(0.0245)
σ_0	1.2423	1.1322	1.1471	1.148	1.15	1.1498
	(0.0259)	(0.0260)	(0.0260)	(0.0264)	(0.0265)	(0.0264)
$\hat{\sigma}$	0.9691	0.9684	0.969	0.9691	0.9698	0.9699
	(0.0240)	(0.0240)	(0.0241)	(0.0245)	(0.0245)	(0.0246)

		n=50, σ= 1, a =0.2					
	$\alpha =$	1	2	3	4	5	6
	a_0	0.179	0.1789	0.1789	0.1789	0.1788	0.1788
		(0.0193)	(0.0193)	(0.0193)	(0.0194)	(0.0194)	(0.0194)
	$\hat{a}$	0.193	0.1935	0.1935	0.194	0.1944	0.1952
		(0.018)	(0.018)	(0.0181)	(0.0181)	(0.0179)	(0.0178)
	σ_0	1.0137	1.0136	1.0136	1.0132	1.013	1.0129
		(0.011)	(0.0109)	(0.0109)	(0.0108)	(0.0107)	(0.0106)
	$\hat{\sigma}$	0.993	0.9877	0.998	0.998	0.9881	1.0079
		(0.0097)	(0.0098)	(0.0099)	(0.010)	(0.010)	(0.010)
		N=50, σ= 1, a =0.5					
	$\alpha =$	1	2	3	4	5	6
	a_0	0.461	0.4609	0.4609	0.4608	0.4609	0.4609
		(0.0165)	(0.0165)	(0.0165)	(0.0165)	(0.0166)	(0.0166)
	$\hat{a}$	0.4822	0.4825	0.483	0.4835	0.484	0.485
		(0.0150)	(0.0356)	(0.0148)	(0.0149)	(0.01450)	(0.0146)
	σ_0	1.0356	1.0347	1.0375	1.0367	1.0365	1.0356
		(0.011)	(0.0111)	(0.0110)	(0.0112)	(0.0111)	(0.0113)
	$\hat{\sigma}$	0.9977	0.9977	0.998	0.998	0.9881	1.0079
		(0.0097)	(0.0098)	(0.0091)	(0.0093)	(0.0092)	(0.0095)
		n=50, σ= 1, a =0.8					
	$\alpha =$	1	2	3	4	5	6
	a_0	0.7694	0.7693	0.7693	0.7695	0.7698	0.7699
		(0.0110)	(0.0110)	(0.0111)	(0.0115)	(0.0114)	(0.0113)
	$\hat{a}$	0.7713	0.7717	0.772	0.772	0.7715	0.7726
		(0.0086)	(0.0086)	(0.0087)	(0.0087)	(0.0088)	(0.0089)
	σ_0	1.0856	1.1	1.101	1.0856	1.0856	1.0856

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

		(0.012)	(0.0125)	(0.0123)	(0.0122)	(0.0120)	(0.0121)
	$\hat{\sigma}$	0.9931	0.9943	0.9974	0.9954	0.9934	0.998
		(0.0099)	(0.010)	(0.010)	(0.0101)	(0.01021)	(0.0104)

n=300, σ= 1, a =0.2						
$\alpha =$	1	2	3	4	5	6
a_0	0.1956	0.19561	0.1999	0.1997	0.1957	0.1989
	(0.0031)	(0.0031)	(0.0031)	(0.0030)	(0.0031)	(0.0030)
$\hat{a}$	0.0009	0.0008	0.0008	0.0009	0.0008	0.0007
	(0.0030)	(0.0030)	(0.0030)	(0.0030)	(0.0030)	(0.0030)
σ_0	1.0191	0.9991	1.001	1.002	0.9998	1.0019
	(0.0019)	(0.0019)	(0.0018)	(0.0019)	(0.0019)	(0.0019)
$\hat{\sigma}$	1.0012	0.9877	0.998	0.998	0.9881	1.0079
	(0.0017)	(0.0017)	(0.0017)	(0.0016)	(0.0016)	(0.0016)
n=300, σ= 1, a =0.5						
$\alpha =$	1	2	3	4	5	6
a_0	0.4921	0.4992	0.5	0.4999	0.4999	0.4992
	(0.0021)	(0.0023)	(0.0024)	(0.0024)	(0.0023)	(0.0024)
$\hat{a}$	0.4974	0.4999	0.5	0.4996	0.5011	0.5
	(0.0024)	(0.0023)	(0.0024)	(0.0022)	(0.0024)	(0.0024)
σ_0	1.1421	1.1249	1.132	1.1505	1.0845	1.1329
	(0.0036)	(0.0036)	(0.0035)	(0.0034)	(0.0030)	(0.0036)
$\hat{\sigma}$	1.0012	0.9999	1.0154	0.999	1	0.9998
	(0.0017)	(0.0018)	(0.0018)	(0.0027)	(0.0018)	(0.0017)
n=300, σ= 1, a =0.8						
$\alpha =$	1	2	3	4	5	6
a_0	0.6537	0.6535	0.6536	0.6534	0.654	0.6541
	(0.0362)	(0.0363)	(0.0363)	(0.0365)	(0.0368)	(0.0368)
$\hat{a}$	0.7379	0.738	0.7395	0.739	0.7392	0.7395
	(0.024)	(0.024)	(0.0211)	(0.0243)	(0.0243)	(0.0245)
σ_0	1.2423	1.4322	1.1471	1.148	1.15	1.1498
	(0.0259)	(0.0260)	(0.0260)	(0.0264)	(0.0265)	(0.0264)
$\hat{\sigma}$	0.9691	0.9684	0.969	0.9691	0.9698	0.9699
	(0.0240)	(0.0240)	(0.0241)	(0.0246)	(0.0245)	(0.0245)

Conclusion

This paper introduced a family of symmetric distributions, which possess many desirable properties as that of normal distribution and besides, the possibility of accommodating heavier tail distributions in time series models. Derivations of the parameter estimates

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XIV, Issue IX, September 2025

of the Auto-Regressive models were done and their optimality properties were studied using simulation. For the estimation, instead of the usual MLE method, MML estimation method was used. The performance of the estimates analyzed using a simulation study and found that estimates outperform the least square estimates in terms of the variances for small n and their relative efficiency approaches one for large n.

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