

Study the Incorporation of Time-Dependent Holding Costs into the Model, Reflecting How Costs Increase as the Product Ages, Impacting Overall Profitability

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ABSTRACT

In this study, we developed an inventory model for Indian perishable products by the incorporation of time-dependent holding costs into the model. Demand is influenced by pricing and advertising decisions for Indian perishable products. We take a linear time-dependent holding cost, which shows the insurance and storage cost of perishable products increases over time. Many traditional studies take holding cost as a constant, which cannot match the real scenario. The optimal replenishment cycle is obtained in this inventory model. Numerical and sensitive analysis is done to validate the inventory model for Indian perishable products.

Keywords: Replenishment cycle, perishable inventory, linear time-dependent holding cost, advertising

INTRODUCTION

Effective inventory management of perishable products is crucial due to their limited shelf life, susceptibility to spoilage, and potential for deterioration. These factors significantly impact profitability in a unique context like India, where a substantial amount of agricultural produce and food is wasted because of inadequate inventory practices and improper storage methods for perishable goods. In an earlier inventory, it is assumed that holding costs and demand are constant.

However, in reality, holding costs tend to increase over time, and demand is influenced by various factors, including selling price, timing, inventory levels, advertising, and the freshness of the product. Singh (2016) an inventory model with constant demand and linear holding cost. Hasan and Mashud (2019) an inventory model with price- and advertising-dependent demand and show how market decisions affect the demand of the product. Macías-López et al. (2021) an inventory model where demand depends on freshness, stock, and quadratic holding cost; this optimizes the replacement time and maximizes the profit. Khan et al. (2022) inventory model for non-instantaneous deterioration and practical payment method and take linear time-dependent holding cost Mishra et al. (2013) This optimizes the total inventory cost with time-varying holding costs and partial backlogging Suvetha et al. (2024) time-dependent holding costs and a three-stage production inventory model with trapezoidal demand, which minimizes the total cost and constant cost, demonstrating manufacturing cost in the model Atama and Sani (2024) production inventory with a linear time-dependent production rate but take holding cost as a constant to obtain optimal cycle length and ordering quantity. Adamu and Yakubu (2025) EOQ model with constant holding cost and linear demand: check how items can be affected by inventory decisions and optimal ordering of products. Muniappan et al. (2021) deteriorating products EOQ model under constant holding cost and obtain optimal solution by numerical analysis.

Early studies indicate that both linear and nonlinear holding costs accurately represent the realities of perishable products; however, there remains a gap in applying these models to the conditions of the Indian market. This includes combining the effects of advertising with pricing strategies and conducting sensitivity analysis. The aim

of this research incorporates linear time-dependent holding cost, which shows the insurance and storage cost of perishable products increases over time. The optimal replenishment cycle is obtained in this inventory model.

Assumptions

This inventory model focuses on one perishable item from India.

The holding cost is linear and increases with time.

Zero lead time is taken and the replenishment rate is infinite.

It assumed that horizon planning is infinite. The lead time is still of concern, and it is assumed to be insignificant; therefore, replenishment is set to occur at the very beginning of every cycle.

In this inventory a shortage is not allowed.

Order size remains constant.

Advertising frequency and selling price-dependent demand are taken and represented by

$$D(A, p) = A^\gamma (x - yp) \quad x, y > 0 \text{ and } \gamma > 0$$

Where A is advertising frequency and (x-y p) price dependent demand.

Notation:

Symbol	Description
p	Selling price in INR
A	Advertising frequency
T	Replenishment cycle length
θ	Rate at which inventory deteriorates
Ca	Expenditure per Advertising in INR
x, y	Price-demand parameters
c	Purchase cost in INR
Cd	Deterioration cost in INR
γ	Advertising Elasticity
Q	Size of order
O	Ordering cost per order in INR
I(t)	At time t quantity of inventory available
H(t)	Holding cost depend on time
D (A, p)	Demand depends on advertising frequency and price of item

Mathematical Model Formulation:

In this mathematical inventory model, demand is considered a function of advertising and selling price for perishable products.

Demand rate is defined as

$$D(A, p) = A^\gamma (x - yp) \quad x, y > 0 \text{ and } \gamma > 0$$

By considering assumptions, differential equations are:

$$\frac{dI(t)}{dt} = -D(A, p) - \theta I(t), \quad T \geq t \geq 0$$

$$\frac{dI(t)}{dt} = -A^v(x - yp) - \theta I(t), \quad T \geq t \geq 0 \quad (1)$$

By solving differential equation (1), $I(t)$ expressed as

$$I(t) = \frac{A^v(x - yp)}{\theta} (e^{\theta(-t+T)} - 1) \quad (2)$$

With boundary condition $I(t) = Q, I(T) = 0$ (3)

$$Q = \frac{A^v(x - yp)}{\theta} (e^{\theta T} - 1) \quad (4)$$

Ordering cost generated per cycle

$$OC = O \quad (5)$$

Production cost generated per cycle

$$Pr C = Cp \int_0^T D(A, p) dt$$

$$Pr C = Cp A^v(x - yp)T \quad (6)$$

Deteriorated items per cycle

$$DC = Cd(Q - \int_0^T D(A, p) dt)$$

$$DC = Cd\left(\frac{A^v(x - yp)}{\theta} (e^{\theta T} - 1) - A^v(x - yp)T\right) \quad (7)$$

The Advertising cost per cycle

$$AC = CaA \quad (8)$$

The Holding cost generated per cycle

$$HC = \int_0^T I(t)(h + \rho t) dt$$

$$HC = \int_0^T \frac{A^v(x - yp)}{\theta} (e^{\theta(-t+T)} - 1) (h + \rho t) dt$$

$$HC = \frac{A^v(x - yp)}{\theta} \left[\frac{h(e^{\theta T} - 1)}{\theta} + \frac{\rho(e^{\theta T} - 1)}{\theta^2} - \frac{\rho T}{\theta} - hT - \frac{\rho T^2}{2} \right] \quad (9)$$

$$Total Cost = OC + Pr C + DC + AC + HC \quad (10)$$

$$\begin{aligned} \text{Total Cost} = & O + Cp A^\gamma(x - yp)T + Cd\left(\frac{A^\gamma(x - yp)}{\theta}(e^{\theta T} - 1) - A^\gamma(x - yp)T\right) + CaA \\ & + \frac{A^\gamma(x - yp)}{\theta} \left[\frac{h(e^{\theta T} - 1)}{\theta} + \frac{\rho(e^{\theta T} - 1)}{\theta^2} - \frac{\rho T}{\theta} - hT - \frac{\rho T^2}{2} \right] \quad (11) \end{aligned}$$

The Total cost generated per cycle

$$\begin{aligned} Tc = & \frac{1}{T} \left[O + Cp A^\gamma(x - yp)T + Cd\left(\frac{A^\gamma(x - yp)}{\theta}(e^{\theta T} - 1) - A^\gamma(x - yp)T\right) + CaA \right. \\ & \left. + \frac{A^\gamma(x - yp)}{\theta} \left[\frac{h(e^{\theta T} - 1)}{\theta} + \frac{\rho(e^{\theta T} - 1)}{\theta^2} - \frac{\rho T}{\theta} - hT - \frac{\rho T^2}{2} \right] \right] \quad (12) \end{aligned}$$

For optimizing total cost per unit time, the necessary and sufficient conditions are

$$\frac{\partial Tc}{\partial T} = 0 \quad \text{and} \quad (13)$$

$$\frac{\partial^2 Tc}{\partial T^2} > 0 \quad \text{for all } 0 < T \quad (14)$$

It satisfies all necessary conditions and gives an optimal solution.

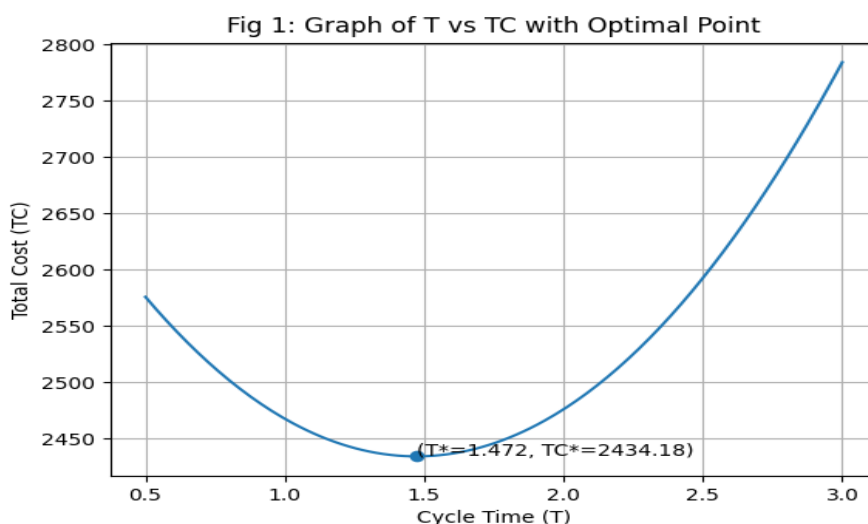
Numerical Applications:

We use numerical values based on realistic assumptions regarding Indian perishable items, such as tomatoes. The selected parameter values meet all the requirements of the inventory model and yield optimal solutions. The parameter values are taken from the secondary sources like government retail price report, earlier Indian perishable product studies.

The parameter values are taken $O=220$, $Cp=15$, $Cd=5$, $Ca=18$, $\theta=0.07$, $A=14$, $x=85$, $y=28$

, $\gamma=28$, $h=1.3$ and $\rho=0.35$ from solving equation (13) we obtain optimal solution $T^*=1.472$

By putting T^* in equation (12) we obtain $Tc^*=2434.18$.



The graphical representation shows that total cost function is convex in nature and gives optimal replenishment cycle solution.

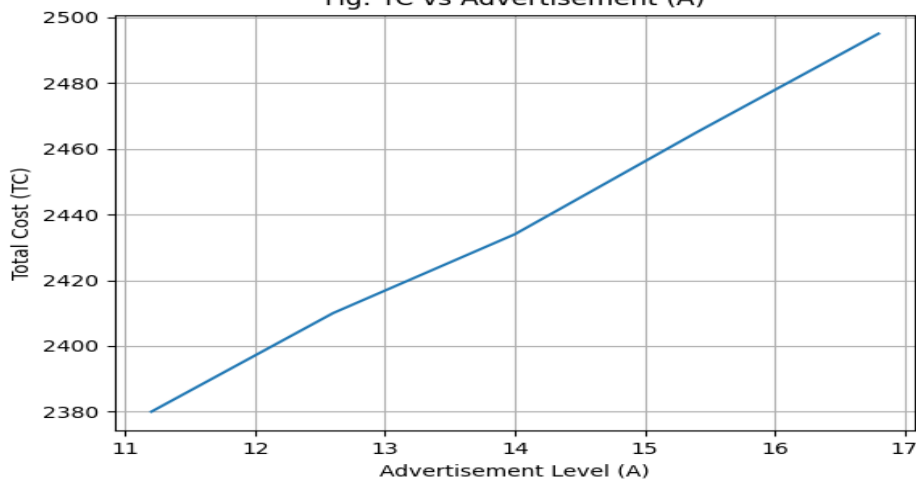
Sensitivity Analysis

We perform sensitive analyses by changing parameters. O, Cp, Cd, h, Ca, A, p, and keeping other parameters original. The optimal values are found when we change one parameter at a time and the other remains the same as the original values, which are taken from the above numerical example.

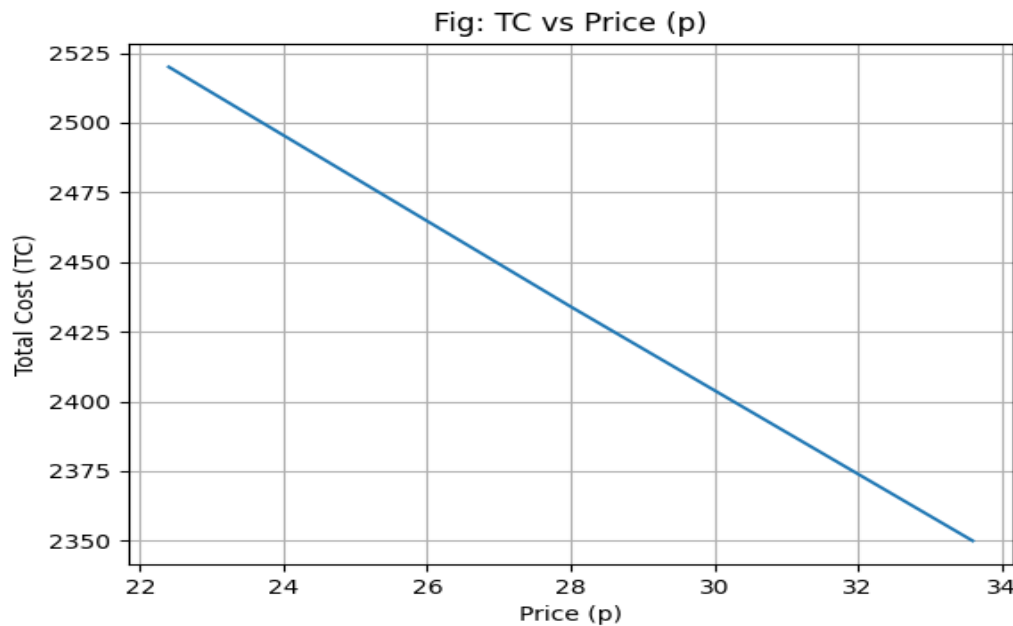
Sensitivity analysis Table

Parameter	Value	% Change of parameter	T'	Tc'
O	176	-20%	1.34	2398
	198	-10%	1.41	2416
	220	0%	1.472	2434
	242	+10%	1.53	2452
	264	+20%	1.58	2471
h	1.04	-20%	1.60	2395
	1.17	-10%	1.53	2415
	1.30	0%	1.47	2434
	1.43	+10%	1.42	2453
	1.56	+20%	1.37	2472
ρ	0.28	-20%	1.50	2428
	0.31	-10%	1.48	2431
	0.35	0%	1.47	2434
	0.38	+10%	1.45	2438
	0.42	+20%	1.43	2442
A	10	-20%	1.53	2430
	12	-10%	1.51	2437
	14	0%	1.47	2434
	16	+10%	1.46	2436
	18	+20%	1.49	2467
p	22	-20%	1.45	2520
	25	-10%	1.46	2477
	28	0%	1.47	2434
	30	+10%	1.48	2392
	33	+20%	1.49	2350

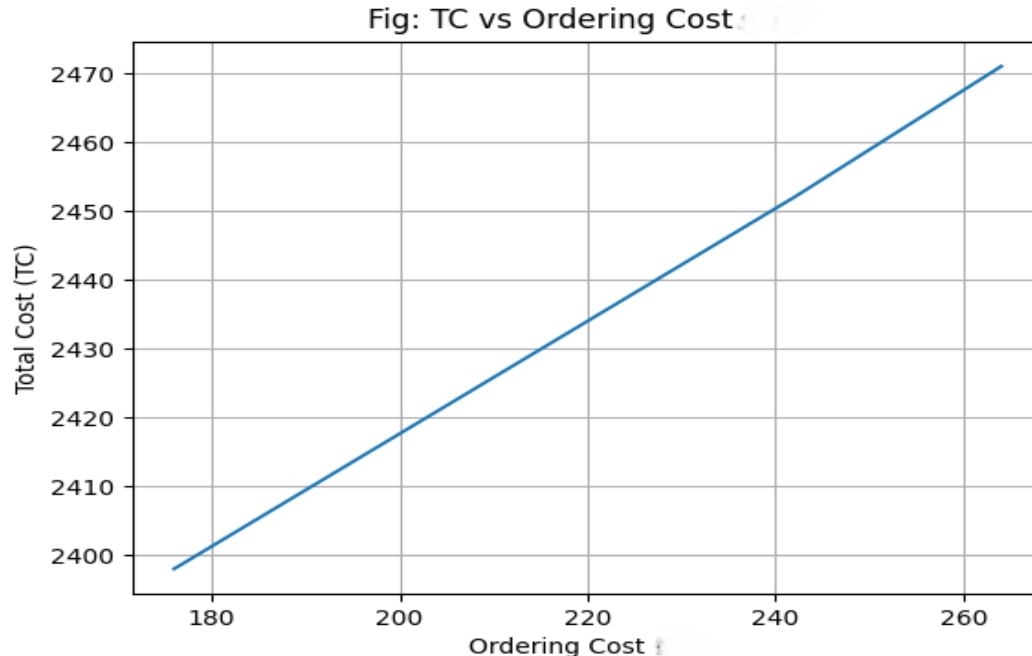
Fig: TC vs Advertisement (A)



We observe from the graph of total cost and advertising that it represents an increasing trend, which means when we increase advertising, then total cost also increases.



We observe from the graph of total cost and price that it represents a decreasing trend, which means when increase selling price, then total cost decreases.



We observe from the graph of total cost and ordering cost that it represents a higher ordering cost than a higher total cost.

CONCLUSION

In this study, we developed an inventory model for Indian perishable products by the incorporation of time-dependent holding costs into the model. The model considers demand as a function of selling price and advertising frequency. The numerical and sensitive analyses represent validation of the inventory model for

Indian perishable products. This study determines the optimal replenishment cycle, which optimizes the total inventory cost for the Indian perishable products.

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