

Soret-Dufour Effect on Variable Viscosity MHD Flow Through Porous Medium

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DOI: <https://doi.org/10.51583/IJLTEMAS.2026.150500006>

Received: 11 April 2026; Accepted: 16 April 2026; Published: 22 May 2026

ABSTRACT

In this paper we have explored the Soret and Dufour effects on unsteady viscous incompressible and electrically conducting MHD flow through porous medium past an inclined infinite plate with variable viscosity fluid and periodic suction or injection of fluid through the plate with exponential increment in temperature and exponentially decrement in concentration. The developed or governing physical model of the flow field has been converted into the mathematical model. Then mathematical model of system of equations is represented by set of governing non linear partial differential equations which have been non dimensionalized by the help of non dimensional numbers. The associated dimensionless governing equation of flow field is solved numerically by Crank Nicolson finite difference implicit method for different values of governing flow parameters. The effects of various responsible flow parameters have been discussed through graphs and table. The effects and their numerical corresponding variations have been concord with physical nature of the problem. The velocity profile, concentration profile and temperature profile are shown through graphs for different values of flow parameters. The obtained numerical values of the skin friction, Nusselt number and Sherwood number with their variations for different values of flow parameters have been tabulated.

Keywords: Soret-Dufour effects, variable viscosity, Periodic Suction or Injection, Crank Nicolson finite difference implicit method.

INTRODUCTION

Soret-Dufour effect on MHD flows arises in many areas of engineering and applied physics. The studies of such flow have application in MHD generators, chemical-engineering, nuclear reactors, geothermal energy, and reservoir engineering and astrophysical studies. The assumption of the pure fluid is rather impossible in nature. The presence of foreign mass in the fluid plays a vital role in flow of fluid. Thermal diffusion or Soret effect is one of the mechanisms in the transport phenomena in which molecules are transported by temperature gradient in a multi-component mixture driven. The inverse phenomena of thermal diffusion are known as Dufour effect, if multi component mixture were initially at the same temperature, are allowed to diffuse into each other, there arises a difference of temperature in the system. In this article we have explored the Soret and Dufour effects on unsteady viscous incompressible and electrically conducting MHD flow through porous medium past an inclined infinite plate with variable viscosity fluid and periodic suction or injection of fluid through the plate with exponential increment in temperature and exponentially decrement in concentration. Sparrow and Cess (1961) analyzed the effect of magnetic field on free convection heat transfer, Dursunkaya et al. (1992) studied Diffusion thermo and thermal diffusion effect in transient and steady natural convection from vertical surface. Raptis et al. (1999) discussed radiation and free convection flow past a moving plate. A. Postelnicu (2004) analyzed the Influence of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous media considering Soret and Dufour effects. Alam et al. (2005) Investigated Dufour effect and Soret effect on

MHD free convective heat and mass transfer flow past a vertical flat plate embedded in porous medium. Bhavana et al. (2013) analyzed the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source. Effect of Variable Permeability and Variable Magnetic field on MHD Flow past an inclined plate with exponential Temperature and Mass Diffusion with Chemical reaction through Porous media by N. Pandya and M. S. Quraishi (2018).

In all of these studies mentioned above the viscosity of the fluid was assumed to be constant. However, we known that the physical property may change significantly. Therefore to predict correctly the flow behavior with respect to variable fluid viscosity, it is necessary to view the variation of viscosity with dependent variable and this variation would also play a vital rule.

The effect of temperature dependent viscosity in the boundary layer flow, heat and mass transfer is analyzed Free Convection Flow over an Uniform-Heat-Flux Surface with Temperature-Dependent Viscosity by Jang, J. -Y. and C. -N. Lin (1988). The Effect of Variable Viscosity on Flow and Heat Transfer to a Continuous Moving Flat Plate by Pop I, R. S. R. Gorla and M. Rashidi (1992). Effects of Variable Viscosity and Viscous Dissipation on the Flow of Third Grade Fluid in a Pipe by Massoudi, M. and I. Christie (1995). The Effect of Temperature-Dependent Viscosity on Free-Forced Convective Laminar Boundary Layer Flow Past a Vertical Isothermal Flat Plate by Kafoussias, N. G. and E. W. Williams (1995). Mixed Convection Boundary Layer Flow on a Continuous Flat Plate with Variable Viscosity by Hady, F. M., A. Y. Bakier and R. S. R. Gorla (1995). Mixed Convection Flow from a Vertical Flat Plate with Temperature Dependent Viscosity by Hossain, Md. A. and Md. S. Munir (2000).

Flow of Viscous Incompressible Fluid with Temperature Dependent Viscosity and Thermal Conductivity Past a Permeable Wedge with Uniform Surface Heat Flux by Hossain, Md. A., Md. S. Munir and D. A. S. Rees (2000). Effect of Variable Viscosity on Free Convection over a Non-Isothermal axisymmetric Body in a Porous Medium with Internal Heat Generation by Begai S. (2004). Plume Formation in Strongly Temperature-Dependent Viscosity Fluid over a Very Hot Surface by Ke, Y. and V. S. Solomatonov (2004). The Effect of Variable Viscosity on Mixed Convection Heat Transfer along a Vertical Moving Surface by Ali, M. E. (2006). The Influence of Variable Viscosity and Viscous Dissipation on the Non-Newtonian Flow: Analytical Solution by Hayat, T., R. Ellahi and S. Asghar (2007). Variable Viscosity Effects on Hydromagnetic Boundary Layer Flow along a Continuously Moving Vertical Plate in the Presence of Radiation by Mahmoud, M. A. A. (2007).

Effect of Variable Viscosity on Combined Forced and Free Convection Boundary-Layer Flow over a Horizontal Plate with Blowing or Suction by Mahmoud M. A. A. (2007). Effect of Variable Viscosity on the Peristaltic Transport of a Newtonian Fluid in an Symmetric Channel by Hayat T. and N. Ali (2008). Effect of Thermal Radiation and Variable Fluid Viscosity on Free Convective Flow and Heat Transfer Past a Porous Stretching Surface by Mukhopadhyay S. and G. C. Layek (2008). Thermal Radiation Effect on Unsteady MHD Free Convection Flow Past a Vertical Plate with Temperature-dependent Viscosity by Mostafa A. A. Mahmoud (2009). Analysis of mhd carreau fluid flow over a stretching permeable sheet with variable viscosity and thermal conductivity by Tariq Abbas and et al. (2020). Analytical treatment of MHD flow and chemically reactive Casson fluid with Joule heating and variable viscosity effect by Haroon Ur Rasheed and et al. (2022). Siti Suzilliana Putri Mohamed Isa and et al. (2023) have discussed Soret-Dufour Effects on Heat and Mass Transfer of Newtonian Fluid Flow over the Inclined Sheet and Magnetic Field.

The object of this work is to investigate the effect of Soret and Dufour with variable fluid viscosity so we have set the viscosity as a function of dependent variable distance from the plate so that viscosity depends upon distance from the plate on MHD unsteady flow over an inclined infinite plate through porous medium with periodic suction or injection of fluid through the plate. Heat and mass transfer have been taken exponentially increasing and decreasing respectively. The dimensionless governing equation of flow field is solved numerically by Crank Nicolson finite difference implicit technique for different values of governing flow parameters. The velocity profile, concentration profile and temperature profile are shown through graphs for different values of flow parameters.

Table 1: Nomenclature

α_1	Some constant for viscosity parameter
u	The velocity of fluid in x direction
g	Gravitational acceleration due to gravity
β	The volumetric coefficient of thermal expansion
β^*	Coefficient of volume expansion for mass transfer
ν	Kinematic viscosity of ambient fluid
μ_∞	Ambient viscosity of the fluid
ρ_∞	Ambient fluid density
B	Magnetic parameter
K	Permeability of porous medium
σ	Electrical conductivity of the fluid
T	Dimensional temperature
T_∞	Temperature of ambient fluid
C_∞	Concentration of ambient fluid
D	Mass diffusion coefficient
D_m	Effective chemical molecular diffusivity,
D_T	Thermal diffusion coefficient
k	Thermal conductivity of the fluid
C_p	Specific heat at constant pressure
K_T	Thermal diffusion ratio
C	Dimensional concentration.
T_w	Temperature on plate
C_w	Concentration on plate

Governing Mathematical Model

An unsteady flow of a variable viscosity of incompressible electrically conducting fluid initially at rest past infinite inclined plate with constant temperature and constant mass diffusion in the presence of magnetic field with periodic suction or injection are studied. Since viscosity may depends on flow parameters like pressure, Temperature etc. In this work Velocity, Temperature and Mass transfer are varying here with the distance from plate so we have taken viscosity as a function of distance which exponentially decreases with distance from plate. The plate is inclined at angle ' λ ' from vertical, through porous medium. x-axis is taken along the plate and y-axis is taken normal to it. Since the plate is electrically non conducting so induce magnetic field is neglected. Initially the plate and fluid are at the same temperature and concentration level is zero and plate is at rest.

Now at time $t > 0$, the plate is at rest along x-direction against gravitational field, y-axis is normal to the plate, the plate temperature and concentration raised exponentially increasing and decreasing respectively. A transverse magnetic field of uniform strength B is assumed normal to the direction of flow. The transversely applied magnetic field and magnetic Reynolds numbers are very small and hence induced magnetic field is negligible, Cowling (1957). Due to infinite length in x-direction, the flow variables are functions of y and t only. Under the above consideration, governing equations for this unsteady problem by the help of Navier-Stokes equations are given by

Momentum equation

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v = -v_0 \text{ (a constant)}$$

$$\frac{\partial u}{\partial t} - v \frac{\partial u}{\partial y} = \frac{1}{\rho_\infty} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + g \beta \cos \lambda (T - T_\infty) + g \beta^* \cos \lambda (C - C_\infty) - \frac{\sigma B^2 u}{\rho_\infty} - \frac{\mu u}{\rho_\infty K}$$

Since $\frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial y}$, therefore above equation becomes

$$\frac{\partial u}{\partial t} - v \frac{\partial u}{\partial y} = \frac{1}{\rho_\infty} \left(\mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial y} \right) + g \beta \cos \lambda (T - T_\infty) + g \beta^* \cos \lambda (C - C_\infty) - \frac{\sigma B^2 u}{\rho_\infty} - \frac{\mu u}{\rho_\infty K} \quad (1)$$

Energy equation

$$\rho_\infty C_p \left(\frac{\partial T}{\partial t} - v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{\rho D_m K_T}{c_s} \frac{\partial^2 C}{\partial y^2} \quad (2)$$

Equation of mass transfer

$$\frac{\partial C}{\partial t} - v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\text{Where } \mu = \mu_\infty e^{-\alpha_1 y} \quad (4)$$

As per the modeled flow governing equations (1) to (4), Velocity, Temperature and Mass transfer are the functions with respect to dependent variable y , that is with respect to the distance from plate. Temperature is taken exponentially decreasing with time. Moreover variable viscosity has been considered as a function of distance that is viscosity exponentially decreases with distance from plate. Concentration has been taken exponentially decreasing.

Considering the suction or injection velocity v_0 as a periodic function of time define as $v_0 = 1 - \epsilon \cos At$. Where $\epsilon (<1)$ is the amplitude of suction or injection velocity and A is some constant.

Where α_1 is some constant for viscosity parameter, u is the velocity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the coefficient of volume expansion for mass transfer, v is the kinematic viscosity of ambient fluid, μ_∞ is ambient viscosity, ρ_∞ is the ambient fluid density, B is magnetic parameter, K is the permeability of porous medium, σ is the electrical conductivity of the fluid, T is the dimensional temperature, T_∞ is temperature of ambient fluid, C_∞ is concentration of ambient fluid, D is mass diffusion coefficient, D_m is effective chemical molecular diffusivity, D_T is thermal diffusion coefficient, k is the thermal conductivity of the fluid, C_p is specific heat at constant pressure, K_T is thermal diffusion ratio, C is the dimensional concentration. T_w and C_w are the temperature and concentration on plate. Initial and boundary conditions are given as:

$$\left. \begin{aligned} t \leq 0; \quad u = 0, \quad v_0 = 1 - \epsilon \cos At \quad T = 0, \quad C = 0 \quad \forall y \\ t > 0; \quad u = 0, \quad T = T_\infty + (T_w - T_\infty)e^{At}, \quad C = C_\infty + (C_w - C_\infty)e^{-At} \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (5)$$

In order to form a non dimensional partial differential equation, introducing following non dimensional quantities:

$$\left. \begin{aligned} \bar{u} = \frac{u}{u_0}, \quad \bar{t} = \frac{tu_0^2}{\nu}, \quad \bar{y} = \frac{yu_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}, \quad G_m = \frac{\nu g \beta^* (C_w - C_\infty)}{u_0^3}, \\ G_r = \frac{\nu g \beta (T_w - T_\infty)}{u_0^3}, \quad S_c = \frac{\nu}{D}, \quad \bar{K} = \frac{u_0^2}{\nu^2} K, \quad S_r = \frac{D_m K_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}, \quad P_r = \frac{\mu C_p}{k}, \\ M = \frac{\sigma B^2 \nu}{\rho_\infty u_0^2}, \quad \bar{\mu} = \frac{\mu}{\mu_\infty}, \quad \alpha = \frac{\nu \alpha_1}{u_0}, \quad \frac{\mu_\infty}{\rho_\infty} = \nu D_u = \frac{D_m K_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}, \quad A = \frac{u_0^2}{\nu} \end{aligned} \right\} \quad (6)$$

G_r , G_m , S_r , D_u , S_c , P_r and M are Thermal Grashof number, Solutal Grashof number, Soret number, Dufour number Schmidt number, Prandtl number and Magnetic parameter respectively. Where A is some constant.

By the substitution of above non dimensional quantities we get the non dimensional form of equations (1) to (5) as:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - (1 - \epsilon \cos t) \frac{\partial \bar{u}}{\partial \bar{y}} = \left(\frac{\partial \bar{\mu}}{\partial \bar{y}} \right) \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right) + \bar{\mu} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + G_r \theta \cos \lambda + G_m C \cos \lambda - M \bar{u} - \bar{\mu} \frac{\bar{u}}{\bar{K}} \quad (7)$$

$$\frac{\partial \theta}{\partial \bar{t}} - (1 - \epsilon \cos t) \frac{\partial \theta}{\partial \bar{y}} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial \bar{y}^2} + D_u \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (8)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} - (1 - \epsilon \cos t) \frac{\partial \bar{C}}{\partial \bar{y}} = \frac{1}{S_c} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + S_r \frac{\partial^2 \theta}{\partial \bar{y}^2} \quad (9)$$

And

$$\bar{\mu} = \exp(-\alpha \bar{y}) \quad (10)$$

The initial and boundary condition Eq. (5) in non dimensional form as following:

$$\left. \begin{aligned} \bar{t} \leq 0; \quad \bar{u} = 0, \quad \theta = 0, \quad \bar{C} = 0 \quad \forall \bar{y} \\ \bar{t} > 0; \quad \bar{u} = 0, \quad \theta = e^{\bar{t}}, \quad \bar{C} = e^{-\bar{t}} \quad \text{at } \bar{y} = 0 \\ \bar{u} \rightarrow 0, \quad \theta \rightarrow 0, \quad \bar{C} \rightarrow 0 \quad \text{as } \bar{y} \rightarrow \infty \end{aligned} \right\} \quad (11)$$

For the sake of convenience dropping the bars from equations (7) to (11) then we have the system as

$$\frac{\partial u}{\partial t} - (1 - \epsilon \cos t) \frac{\partial u}{\partial y} = \left(\frac{\partial \mu}{\partial y} \right) \left(\frac{\partial u}{\partial y} \right) + \mu \frac{\partial^2 u}{\partial y^2} + G_r \theta \cos \lambda + G_m C \cos \lambda - M u - \mu(y) \frac{u}{K} \quad (12)$$

$$\frac{\partial \theta}{\partial t} - (1 - \epsilon \text{Cost}) \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} + D_u \frac{\partial^2 C}{\partial y^2} \quad (13)$$

$$\frac{\partial C}{\partial t} - (1 - \epsilon \text{Cost}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (14)$$

and

$$\mu = \text{Exp}(-\alpha y) \quad (15)$$

With initial and boundary conditions as

$$\left. \begin{aligned} t \leq 0; \quad u = 0, \theta = 0, C = 0 \quad \forall y \\ t > 0; \quad u = 0, \theta = e^t, C = e^{-t} \quad \text{at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (16)$$

Now, it is useful to calculate physical quantities for primary interest, these are coefficient of skin-friction ' τ ' at the wall along x-axis, Nusselt number Nu and Sherwood number Sh. Dimensionless forms of these physical quantities are:

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0}, Nu = - \left(\frac{\partial \theta}{\partial y} \right)_{y=0}, Sh = - \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (17)$$

METHOD OF SOLUTION

Equations (12) to (14) are non-linear partial differential equations, are solved using initial boundary conditions Eq. (16), it is solved here by Crank-Nicolson implicit finite difference method for numerical solution. By this method the transform form of Equations (12) to (14) in form of finite differences with indices are expressed as:

$$\begin{aligned} & \frac{u_{i,j+1} - u_{i,j}}{\Delta t} - (1 - \epsilon \text{Cost}) \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right) = \\ & \text{Exp}[-\alpha \Delta y] \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{2(\Delta y)^2} \right) \\ & - \alpha \text{Exp}[-\alpha \Delta y] \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right) + G_r \text{Cos}\lambda \left(\frac{\theta_{i,j+1} + \theta_{i,j}}{2} \right) \\ & + G_m \text{Cos}\lambda \left(\frac{C_{i,j+1} + C_{i,j}}{2} \right) - M \left(\frac{u_{i,j+1} + u_{i,j}}{2} \right) - \frac{\text{Exp}[-\alpha \Delta y]}{K} \left(\frac{u_{i,j+1} + u_{i,j}}{2} \right) \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} - (1 - \epsilon_{Cost}) \left(\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} \right) = \\ D_u \left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) \\ \frac{1}{P_r} \left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right) \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{C_{i,j+1} - C_{i,j}}{\Delta t} - (1 - \epsilon_{Cost}) \left(\frac{C_{i+1,j} - C_{i,j}}{\Delta y} \right) = \\ \frac{1}{S_c} \left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) \\ + S_r \left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right) \end{aligned} \quad (20)$$

Corresponding boundary and initial conditions from Equation (16) are now as:

$$\left. \begin{aligned} u_{i,0} &= 0, & \theta_{i,0} &= 0, & C_{i,0} &= 0 & \forall i \\ u_{0,j} &= 0, & \theta_{0,j} &= e^{j \cdot \Delta t}, & C_{0,j} &= e^{-j \cdot \Delta t} & \forall j \\ u_{n,j} &\rightarrow 0, & \theta_{n,j} &\rightarrow 0, & C_{n,j} &\rightarrow 0 \end{aligned} \right\} \quad (21)$$

Here, index 'i' refers to y and j to time, $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$. For known values of u , θ and C at t , we calculate these values for $(t + \Delta t)$ as follows, after substitution of i and $j = 1, 2, 3 \dots n-1$, where 'n' corresponds to ∞ . Now system of equations (19) and (20) are solved by Thomas Algorithm as discussed in Carnahan et al. (1969). Then θ and C are known for all values of y at $t + \Delta t$.

Replacing values of θ and C in equation (18) and solved by same with initial and boundary conditions (21), we have solutions for u till desired time t . Crank-Nicolson implicit finite difference method is second order method ($O(\Delta t^2)$) in time also and has no limitation for space and time steps, that is, the method is unconditionally stable. Computation has been executed for $\Delta y = 0.1$, $\Delta t = 0.002$ and repeated till $y(\max) = 4$, $y(\max)$ is corresponds to infinity.

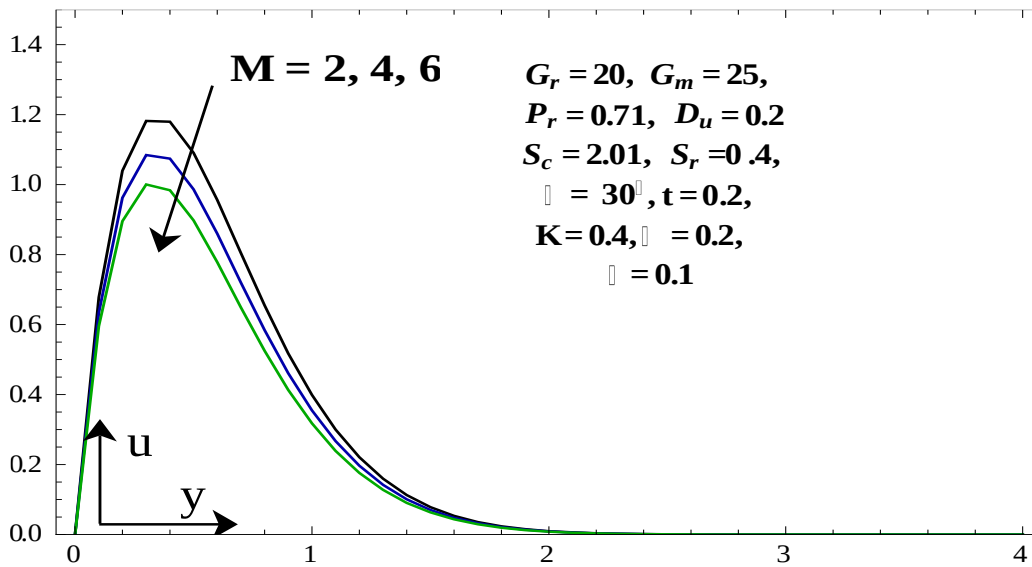


Fig. 1. Velocity Profile for different values of ' M '

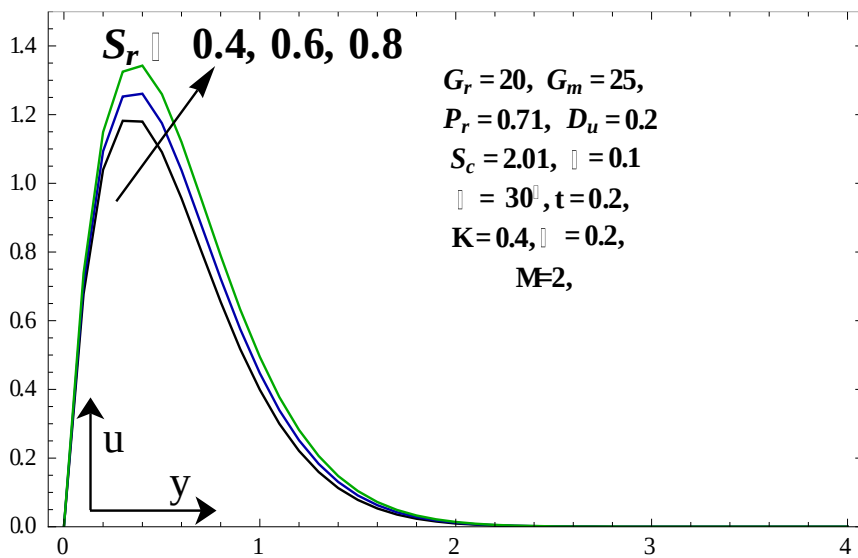


Fig. 2. Velocity Profile for different values of ' S_r '

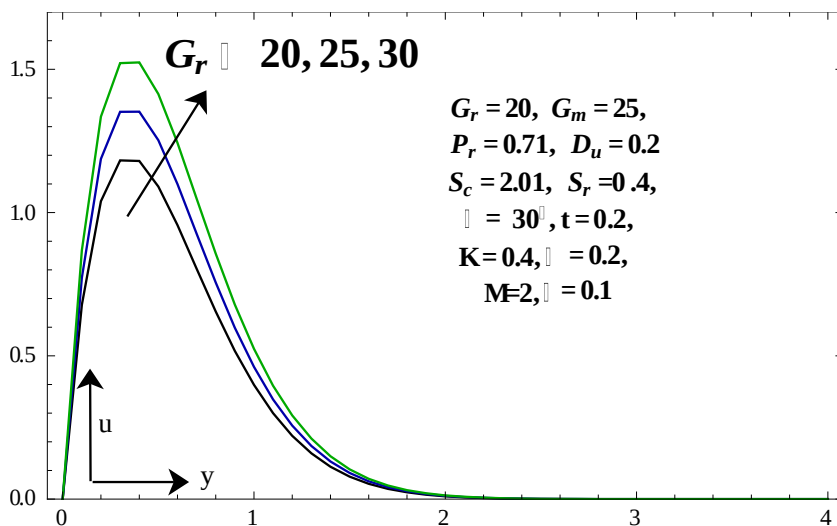


Fig. 3. Velocity Profile for different values of ' G_r '

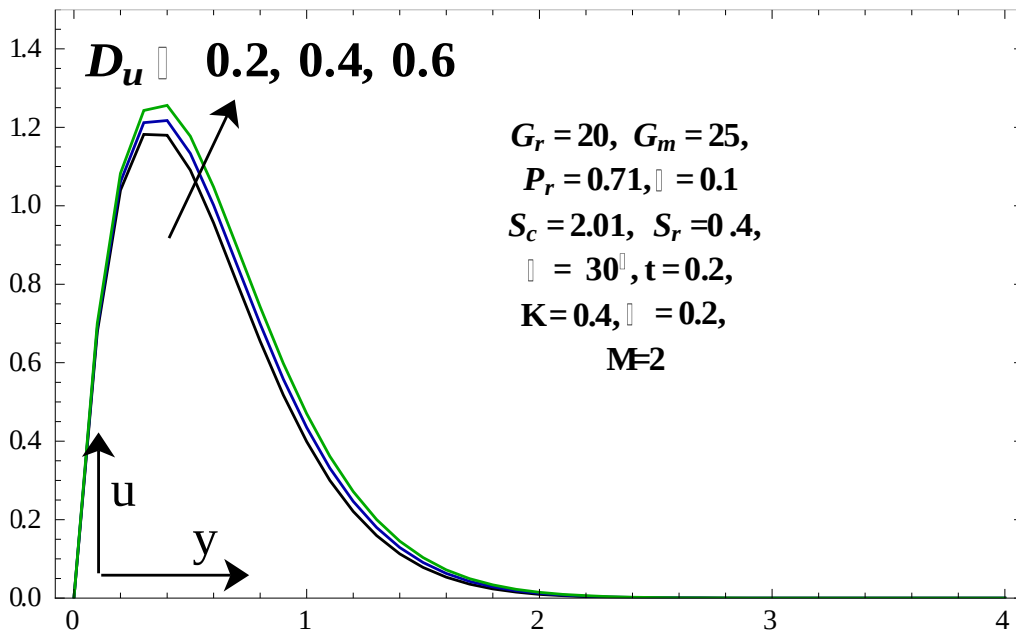


Fig. 4. Velocity profile for different values of ' D_u '

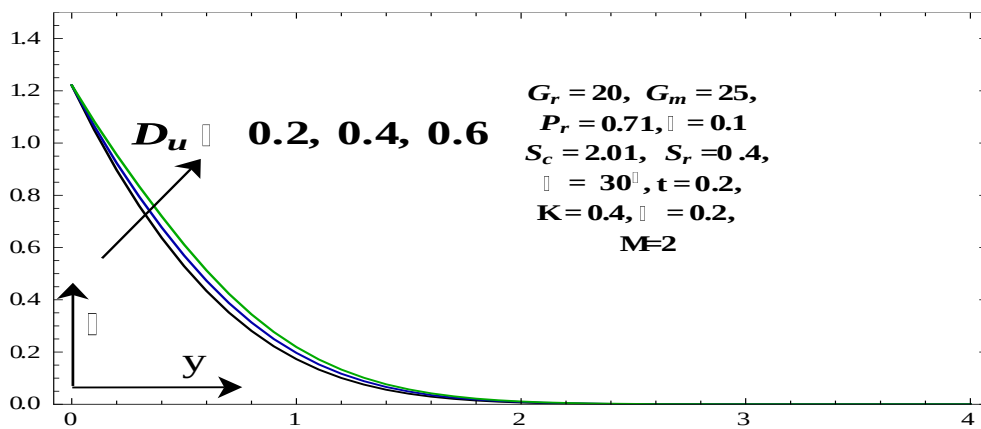


Fig. 5. Temperature Profile for different values of ' D_u '

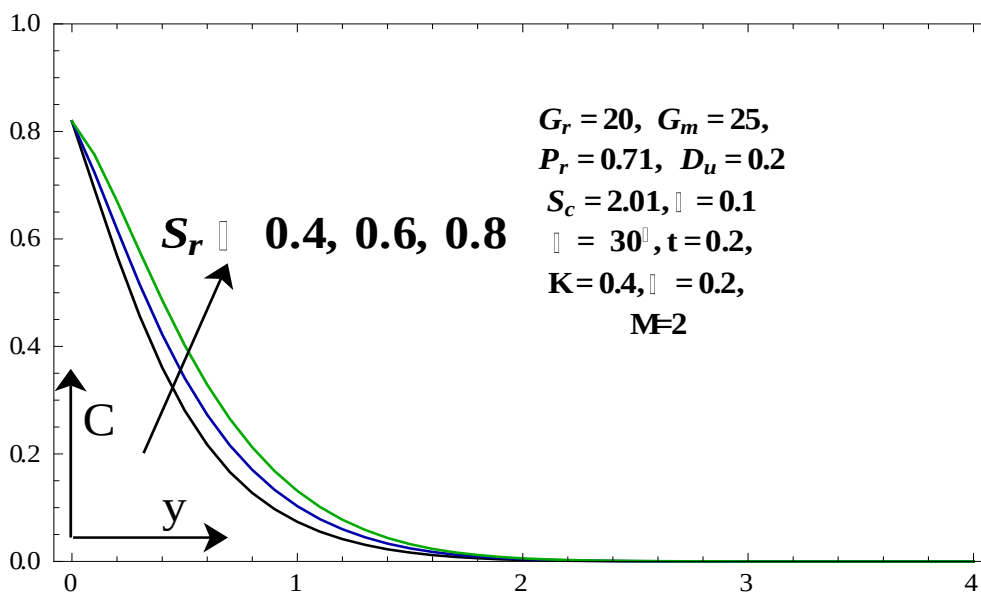


Fig. 6. Concentration Profile for different values of ' S_r '

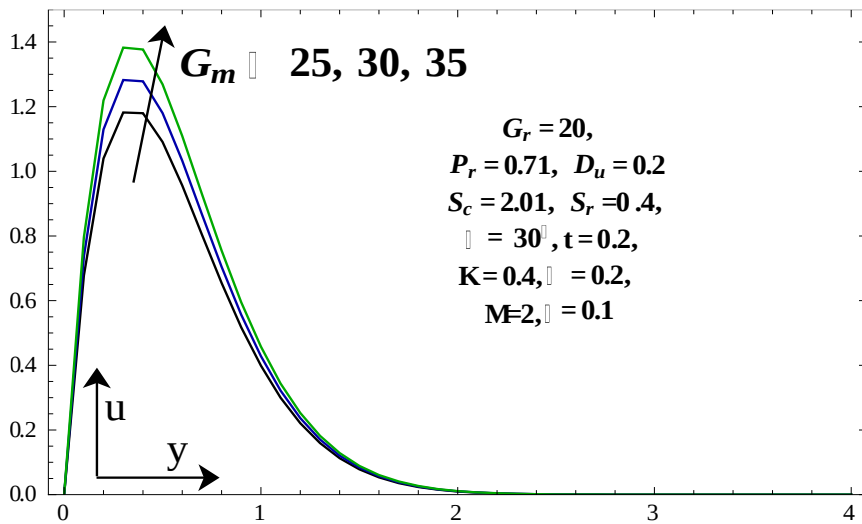


Fig. 7. Velocity profile for different value of 'Gm'

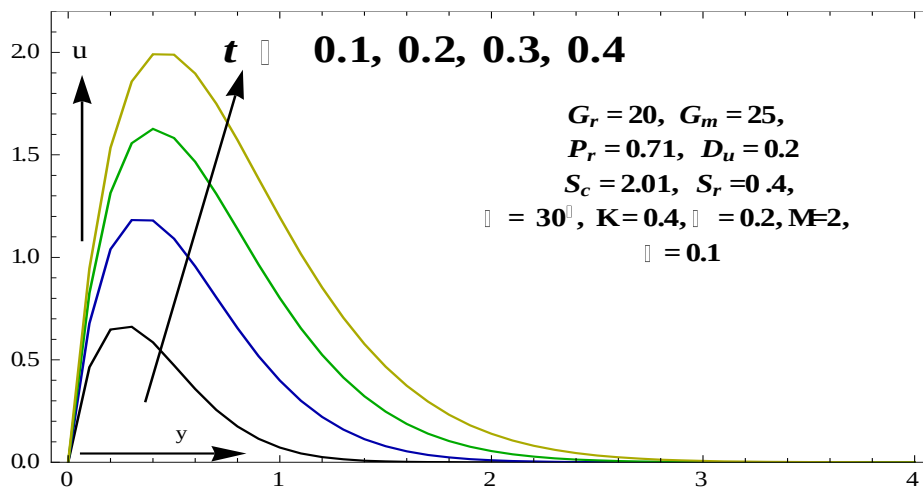


Fig. 8. Velocity profile for different value of 't'

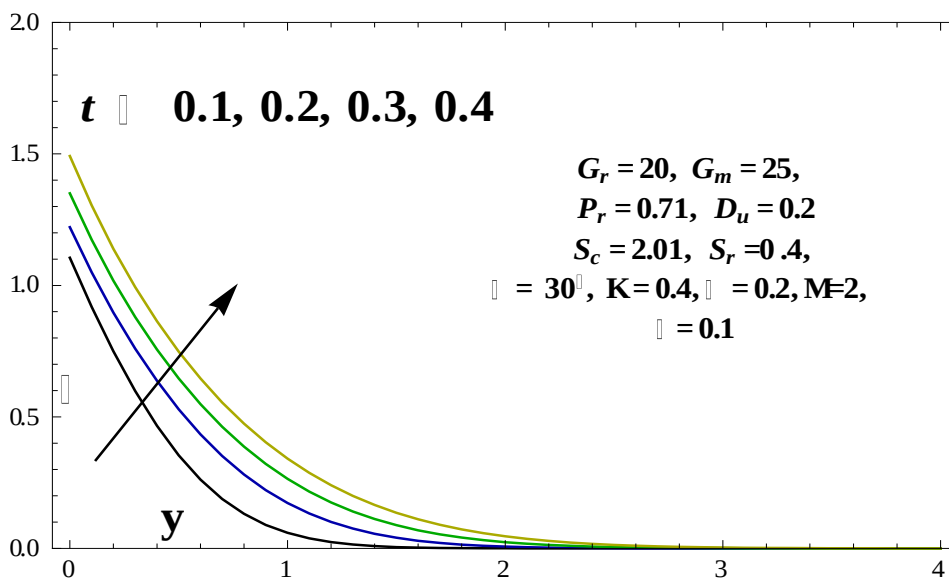


Fig. 9. Temperature profile for different values of 't'

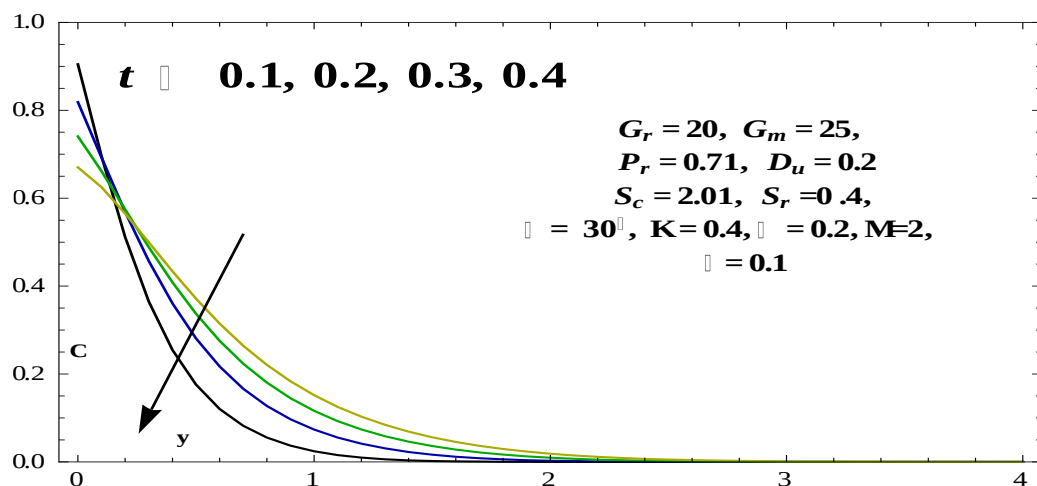


Fig. 10. Concentration profile for different values of 't'

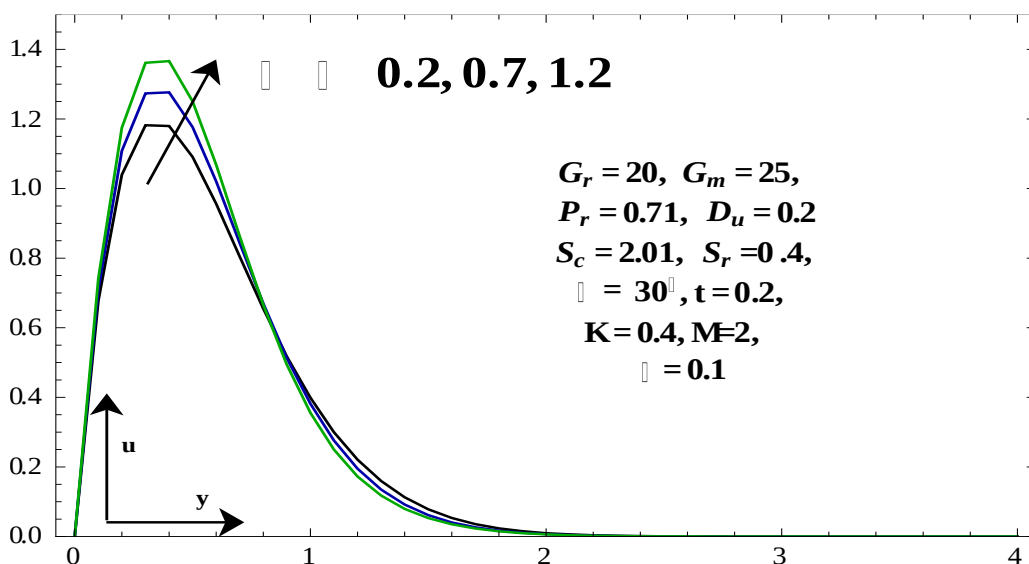


Fig. 11. Velocity profile for different positive values of 'α'

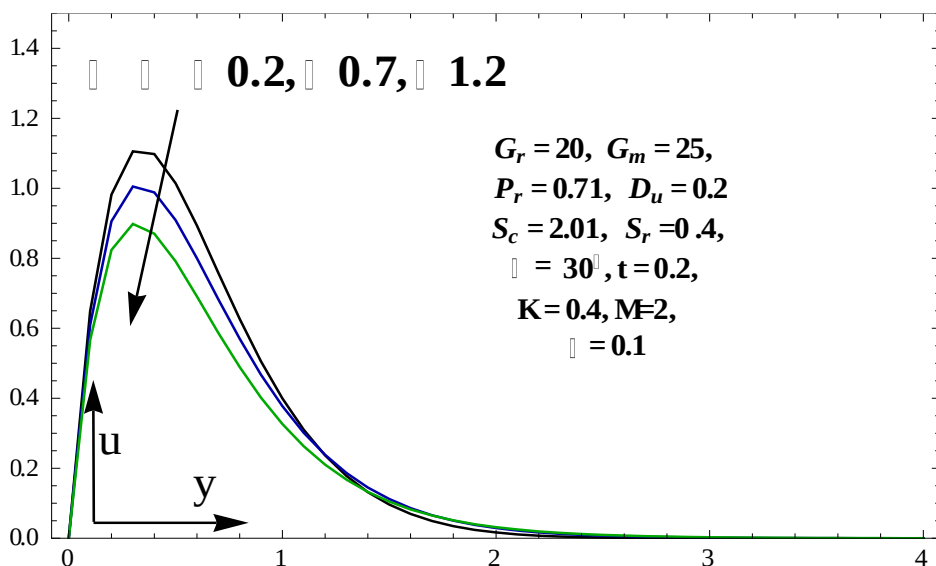


Fig. 12. Velocity profile for different negative value of 'α'

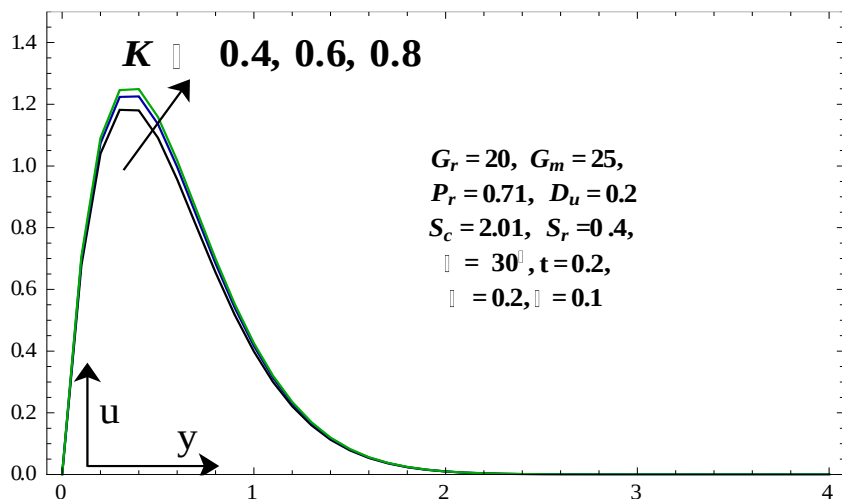


Fig. 13. Velocity profile for different values of 'K'

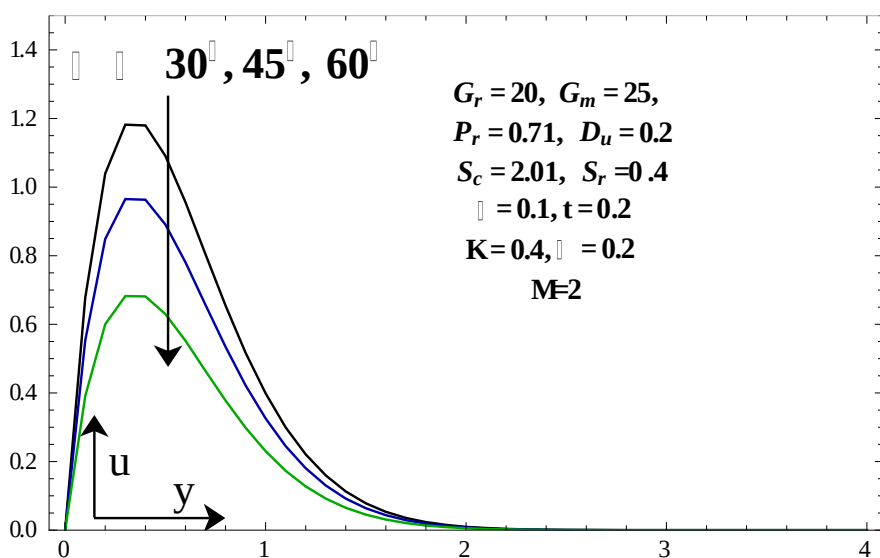


Fig. 14. Velocity profile for different value of 'λ'

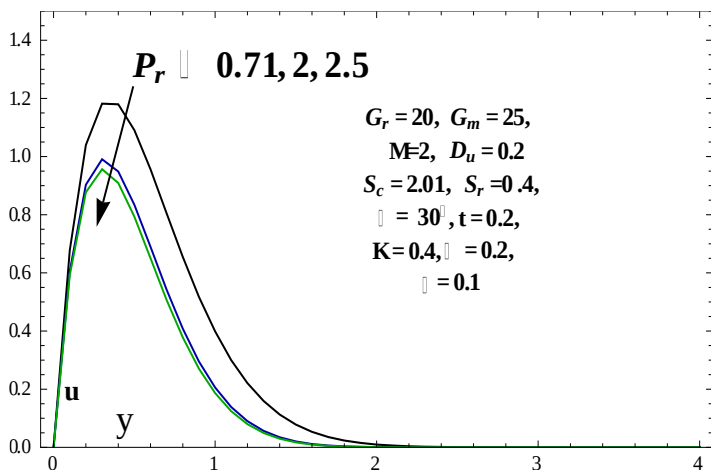


Fig. 15. Velocity profile for different value of 'Pr'

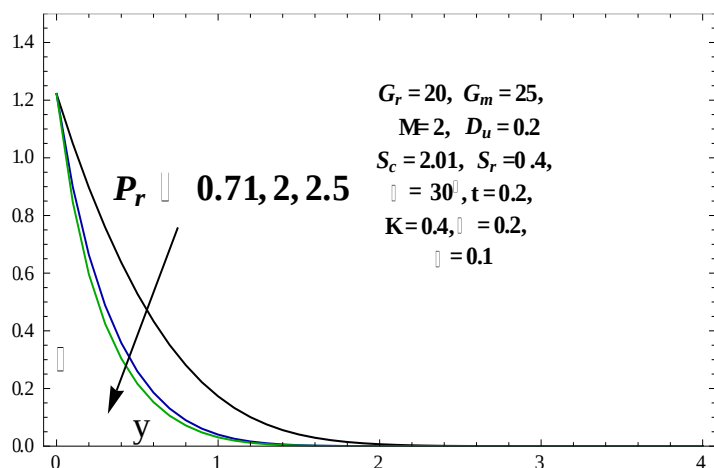


Fig. 16. Temperature profile for different value of ' Pr '

RESULTS AND DISCUSSION

In this paper we studied Soret and Dufour effects on MHD flow through porous medium past an inclined infinite plate with variable viscosity fluid and periodic suction or injection of fluid through the plate with exponential increment in temperature and exponentially decrement in concentration. The dimensionless governing equation of flow field is solved numerically by Crank Nicolson finite difference implicit method for different values of governing flow parameters. The effects of various flow parameters discussed through graphs and table. The velocity profile, concentration profile and temperature profile shown through graphs for different values of flow parameters. The consequences of the relevant parameters break down the flow field and discussed with the help of graphs of velocity profile, temperature profiles and concentration profiles.

Fig. 2, 3, 4, 7, 8, 11 and 13 depicts that the velocity ' u ' increases with increase in S_r , Gr , D_u , G_m , t , α and K . Fig. 1, 12, 14 and 15 show that velocity decreases with increase in M , α , λ and Pr . Temperature profile θ in Fig. 5 and 9 show increment with increase in G_m and t respectively while Temperature decreases with increase in Pr . Concentration profile ' C ' increases with S_r in Fig. 6 and decreases with ' t ' Fig.10

Table 2. Depicts that increase in G_r , G_m , α , S_r , t , K and D_u the Skin friction coefficient ' τ ' increases while increase in M , Pr , λ and $-\alpha$ then Skin friction ' τ ' decreases.

Table 2. Also shows that increase in Pr and S_r , Nusselt Number ' Nu ' increases while it decreases with increase in D_u . Nusselt number first decreases and then increases with respect to time. Sherwood Number ' Sh ' increases with increase in D_u while it decreases with increase in Pr , S_r and t .

CONCLUSION

This study investigated the MHD flow through a porous medium past an inclined infinite plate, accounting for Soret and Dufour effects with variable viscosity and periodic suction/injection. The governing equations were solved using the **Crank-Nicolson** implicit finite difference method. The following conclusions are drawn from the results:

Flow Field Characteristics (Velocity Profile)

- **Enhancement of Flow:** The fluid velocity u increases significantly with an increase in the Soret number, Dufour number, Grashof number Gr Solutal Grashof number G_m . This is attributed to the enhancement of thermal and solutal buoyancy forces. Additionally, an increase in the permeability parameter K reduces Darcian resistance, therefore the flow accelerates.

- **Retardation of Flow:** The velocity is found to decrease with an increase in the magnetic parameter M , Prandtl number Pr , and inclination parameter λ . The decrease with M is due to the development of the resistive Lorentz force, which acts as a drag against the fluid motion.
- **Variable Viscosity Effect:** The study reveals that the fluid velocity is highly sensitive to viscosity changes. As the viscosity parameter α increases or decreases—representing a decrease or increase in the fluid's internal resistance—the velocity profile shows a marked increment or decrement. This confirms that treating viscosity as a variable rather than a constant is essential for accurately predicting the flow behavior of real fluids properties.

Thermal and Concentration Distributions

Temperature Profile: The temperature of the fluid increases with higher values of the Dufour number diffusion-thermo effect and time. Conversely, an increase in the Prandtl number leads to a thinning of the thermal boundary layer, resulting in decreased fluid temperature.

Concentration Profile: The concentration increases with the Soret number due to thermo-diffusion effects and decreases with time.

Surface Gradients (Skin Friction, Nusselt, and Sherwood Numbers)

Skin Friction: The skin friction coefficient at the plate increases with higher values of Gr , Gm , Sr , Du , and K , reflecting the acceleration of the fluid near the boundary. It decreases with increasing magnetic field strength M and Prandtl number Pr .

Nusselt number: The **Nusselt number** increases with Pr and Sr , indicating enhanced heat transfer at the surface. However, it decreases as the **Dufour number** Du increases.

Sherwood number: The **Sherwood number** is enhanced by the Dufour effect but is suppressed by an increase in the Prandtl number, Soret number, and time. This highlights the strong coupling between heat and mass diffusion in the presence of cross-diffusion effects.

Conflicts of Interest: The authors declare no conflicts of interest.

We are thankful to reviewers and Editors of the journal, and software company (Mathematica) for developing the techniques that help in the computation and editing. There is no financial support for this research from any organization from anywhere.

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Table 2: Skin friction coefficient τ , Nusselt number Nu and Sherwood number Sh for different values of parameters

Gr	Gm	α	M	Pr	Sr	λ	t	K	Du	τ	Nu	Sh
20	25	0.2	2	0.71	0.4	300	0.2	0.4	0.2	6.77909	1.71408	1.26437
25	25	0.2	2	0.71	0.4	300	0.2	0.4	0.2	7.73222	1.71408	1.26437
30	25	0.2	2	0.71	0.4	300	0.2	0.4	0.2	8.68536	1.71408	1.26437
20	30	0.2	2	0.71	0.4	300	0.2	0.4	0.2	7.37239	1.71408	1.26437
20	35	0.2	2	0.71	0.4	300	0.2	0.4	0.2	7.96570	1.71408	1.26437
20	25	0.7	2	0.71	0.4	300	0.2	0.4	0.2	7.11135	1.71408	1.26437
20	25	1.2	2	0.71	0.4	300	0.2	0.4	0.2	7.41848	1.71408	1.26437
20	25	0.2	4	0.71	0.4	300	0.2	0.4	0.2	6.34006	1.71408	1.26437
20	25	0.2	6	0.71	0.4	300	0.2	0.4	0.2	5.95834	1.71408	1.26437
20	25	0.2	2	2.00	0.4	300	0.2	0.4	0.2	6.05806	3.22211	0.424553
20	25	0.2	2	2.50	0.4	300	0.2	0.4	0.2	5.91942	3.74963	0.105401
20	25	0.2	2	0.71	0.6	300	0.2	0.4	0.2	7.07635	1.75024	0.956001
20	25	0.2	2	0.71	0.8	300	0.2	0.4	0.2	7.38356	1.78967	0.622159
20	25	0.2	2	0.71	0.4	450	0.2	0.4	0.2	5.53510	1.71408	1.26437
20	25	0.2	2	0.71	0.4	600	0.2	0.4	0.2	3.91391	1.71408	1.26437
20	25	0.2	2	0.71	0.4	300	0.1	0.4	0.2	4.63594	1.86770	2.09706
20	25	0.2	2	0.71	0.4	300	0.3	0.4	0.2	8.25896	1.75878	0.795038
20	25	0.2	2	0.71	0.4	300	0.4	0.4	0.2	9.45491	1.87330	0.453853
20	25	0.2	2	0.71	0.4	300	0.2	0.6	0.2	6.96962	1.71408	1.26437
20	25	0.2	2	0.71	0.4	300	0.2	0.8	0.2	7.06911	1.71408	1.26437
20	25	0.2	2	0.71	0.4	300	0.2	0.4	0.4	6.88755	1.56573	1.35475
20	25	0.2	2	0.71	0.4	300	0.2	0.4	0.6	7.00158	1.38527	1.46660
20	25	-0.2	2	0.71	0.4	300	0.2	0.4	0.2	6.49224	1.71408	1.26437
20	25	-0.7	2	0.71	0.4	300	0.2	0.4	0.2	6.10267	1.71408	1.26437
20	25	-1.2	2	0.71	0.4	300	0.2	0.4	0.2	5.67172	1.71408	1.26437