

Construction of Q- Rung Orthopair (M, N) Uncertainty Level Subgroup Structures

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ABSTRACT

The main objective of this article is to introduce the idea of q-rung orthopair (m, n)-fuzzy subgroups of a finite group. We discuss the concept of q-rung orthopair (m, n) – fuzzy subgroups as a combination of intuitionist fuzzy subgroup and Pythagorean fuzzy subgroup. We provide the definition of q-rung orthopair (m, n) – fuzzy subgroup and examine various properties associated with it. Finally we analyze q-rung orthopair (m, n)- fuzzy cosets, (m, n) – fuzzy normal subgroups and (m, n)- fuzzy level subgroups.

Keywords: intuitionist fuzzy set, Pythagorean fuzzy set, q-rung orthopair (m, n)- fuzzy subgroup, q-rung orthopair (m,n)- fuzzy Coset, normal subgroup, homomorphism.

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INTRODUCTION

To handle uncertainty in real-life problems Zadeh LA [1] proposed the concept of Fuzzy Sets (FSs). In 1971 expanding the notion of FSs, Rosenfeld A [2] defined fuzzy subgroup. In recent years many researchers studied various properties of fuzzy subgroups, t-norm fuzzy subgroups, fuzzy level subgroup, fuzzy subgroups and fuzzy homomorphism, anti-fuzzy subgroups, fuzzy normal subgroup, fuzzy coset and fuzzy quotient subgroup etc [3-14]. In 1986, Atanassov KT [15] introduced Intuitionistic Fuzzy Set (IFS). In 1996, Intuitionistic fuzzy subgroup was first studied by Biswas R [16]. Zhan J et al. [17] introduced intuitionistic fuzzy M-group. Furthermore, researchers developed intuitionistic fuzzy subgroup in many ways [18-20]. In 2013, Yager RR [21] invented Pythagorean Fuzzy Sets (PFSs). In 2018, Naz .S et al. [22] proposed a novel approach to decision-making problem using Pythagorean fuzzy set. In 2019, Akram M et al. [23] applied

complex Pythagorean fuzzy set in decision-making problems. Ejegwa PA [24] gave an application of Pythagorean fuzzy set-in career placements based on academic performance using max-min-max composition. Some results related to it were given by Peng X [25] and Yang Y [26]. First time, Bhunia S et al. [27] proposed “On the characterization of Pythagorean fuzzy subgroups”. In certain situations, the PFSs unable to handle the problems like if membership degree $\eta = 0.8$ and nonmembership degree $\theta = 0.75$ then $2\eta\theta \geq 1$. It is clear that PFSs fail to define this type of sets but, $1 \leq \eta + \theta \leq 2$ where $\eta > 0$. Most recently another amazing generalization of FSs is proposed by Yager RR [28], q-Rung Orthopair Fuzzy Sets (q-ROFSs) which models uncertain and incomplete information better than both IFSs and PFSs with high accuracy. q-ROFSs is more fruitful in many decision-making problems. This concept is perfectly designed to represent imprecision and ambiguity in mathematical way and to produce a formalized tool to handle fuzziness to real problems. Pinar A et al. [29] suggested a q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance. Peng X et al. [30] published Information measures for q-rung orthopair fuzzy sets. The main objective of this article is to introduce the

idea of q-rung orthopair (m, n) -fuzzy subgroups of a finite group. We discuss the concept of q-rung orthopair (m, n) – fuzzy subgroups as a combination of intuitionist fuzzy subgroup and Pythagorean fuzzy subgroup. We provide the definition of q-rung orthopair (m, n) – fuzzy subgroup and examine various properties associated with it. Finally we analyze q-rung orthopair (m, n) - fuzzy cosets, (m, n) – fuzzy normal subgroups and (m, n) - fuzzy level subgroups.

Preliminaries : In this section, we recall a some definitions and ideas which are very helpful for further proven.

Definition 2.1 Let $A: X \rightarrow [0,1]$ be a fuzzy subset of a group $(X, 0)$. Then A is said to be a fuzzy subgroup of $(X,0)$ if the following conditions are exists.

1. $A(w_1, w_2) \geq \min \{A(w_1), A(w_2), \forall w_1, w_2 \in X$
2. $A(w^{-1}) \geq A(w) \forall w \in X$

Definition 2.2 Let $A = \{(x, \alpha(x), \beta(x)/x \in C\}$ be a IFS of a group (C, α) , then A is said to be an IFSG of x if the following conditions hold;

1. $\alpha(w_1, w_2) \geq \min \{\alpha(w_1), \alpha(w_2)\}$ and $\beta(w_1, w_2) \geq \max \{\beta(w_1), \beta(w_2)\} \forall w_1, w_2 \in X$
2. $\alpha(w^{-1}) \geq \alpha(w)$ and $\beta(w^{-1}) \geq \beta(w) \forall w \in X$

In 2017, yager defined q-rung orthopair fuzzy set as a speculation of IFS.

Definition 2.3: Let X be a crisp set. A q-rung orthopair fuzzy set A on X is defined by $A = \{(x, \alpha(x), \beta(x)) / x \in C\}$ where $\alpha(x) \in [0,1]$ and $\beta(x) \in [0,1]$ are the membership and non-membership degree of $x \in X$ respectively, which satisfy the condition $D \leq \alpha^2(x) + \beta^2(x) \leq 1, (q \geq 1) \forall W \in X$ and its determinacy $\Pi_A = \sqrt{1 - \alpha^2(x) + \beta^2(x)}$.

Definition 2.4: Basic operations on q-rung orthopair fuzzy set: Let $A = \{(x, \alpha_1(x), \beta_1(x)) / x \in X\}$ and $B = \{(x, \alpha_2(x), \beta_2(x)) / x \in X\}$ be two q-ROFs of X -then the following holds.

1. $A \cup B = \{(x, \alpha_1(x) \cup \alpha_2(x), \beta_1(x) \cap \beta_2(x)) / x \in X\}$
2. $A \cap B = \{(x, \alpha_1(x) \cap \alpha_2(x), \beta_1(x) \cup \beta_2(x)) / x \in X\}$
3. $A^C = \{(x, \alpha_2(x), \alpha_1(x)) / x \in X\}$ and
4. $A \subseteq B$ if a. $\alpha_1(x), \alpha_2(x)$ and b. $\beta_1(x) = \beta_2(x) \forall x \in X$
5. $A = B$ if a. $\alpha_1(x) = \alpha_2(x)$ and b. $\beta_1(x) = \beta_2(x) \forall x \in X$

In this section let G be a finite group. This section elucidates the concept of a q-rung orthopair (m, n) – fuzzy subgroup of G and delivers certain characteristics associated with it.

Definition 2.5 : Let $A = \{(w, \gamma_A(w)) / W \in G\}$ be a q-rung orthopair (m, n) - fuzzy set on G , then A is a q-rung orthopair (m, n) – fuzzy subgroup of G if, for $w_1, w_2 \in G$;

- (i) $(\delta_A(m_1 w_1, w_2))^q \geq \min \{(\delta_A(w_1))^q, (\delta_A(w_1))^q\}$ and
- $(A_A(n_1 w_1, w_2))^q \geq \max \{(A_A(w_1))^q, (A_A(w_1))^q\}$

(i) $(\delta_A(n_1w_1))^q =$ and $(\delta_A(w^{-1}))^q$ and $(A_A(n_1w^{-1}))^q = (A_A(w^{-1}))^q$, for $m, n \in [0, 1]$

Note that if $q=1$ called intuitionistic fuzzy subgroup and $q=2$ Pythagorean fuzzy subgroup, $q=3$ fermatean fuzzy subgroup of G .

Section :3 Standard Results on q -rung orthopair (m, n) Fuzzy subgroup

Theorem 3.1 Let A be a q -rung orthopair (m, n) – fuzzy subgroup of G , then

$$(\delta_A(w^k))^q \geq (\delta_A(w))^q \text{ and } (\Delta_A(w^k))^q \leq (\Delta_A(w))^q \text{ for } k \in \mathbb{N}$$

Proof: By definition 2.5

$$(\delta_A(w^2))^q \geq \min \{ (\delta_A(w))^q, (\delta_A(w))^q \} = (\delta_A(w))^q$$

$$\text{and } (\Delta_A(w^2))^q \leq \max \{ (\Delta_A(w))^q, (\Delta_A(w))^q \} = (\Delta_A(w))^q$$

The proof follows by mathematical induction.

Corollary 3.2 In definition 2.5 since G is a finite group, condition 1 gives condition 2

Proof Let $w \in G$ with order of W .

$$O(W) = S, \text{ then } w^{-1} = w^{n-1}, (\delta_A(m^{-1}))^q \geq (\delta_A(mw^{n-1}))^q \geq (\delta_A(w))^q$$

$$\text{For } w = w^{-1} \text{ it follows that } (\delta_A(mw))^q = (\delta_A(w^{-1}))^q$$

$$\text{Similarly, for non-membership degree } (A_A(mw))^q = (A_A(w^{-1}))^q$$

Theorem 3.3 Let A be a q -rung orthopair (m, n) - fuzzy subgroup of G , ‘ e ’ be the identify element in G . Then $(\delta_A(me))^q \geq (\delta_A(w))^q$ and $(A_A(ne))^q \leq (A_A(w))^q$ for $w \in G$

$$\text{Proof: Let } w \in G, (\delta_A(me))^q = (\delta_A(ww^{-1}))^q \geq \min \{ (\delta_A(w))^q, (\delta_A(w^{-1}))^q \} \geq (\delta_A(w))^q$$

$$\text{Analogously, it can be shown that } (A_A(ne))^q \leq (A_A(w))^q$$

Theorem 3.4 Let A be a q -rung orthopair (m, n) – fuzzy subgroup of G if and only if

$$(\delta_A(m(w_1w_2^{-1})))^q \geq \min \{ (\delta_A(w_1))^q, (\delta_A(w_2))^q \} \text{ and } (\Delta_A(n(w_1w_2)))^q \leq \max \{ (\Delta_A(w_1))^q, (\Delta_A(w_2))^q \}, \text{ for } w_1w_2 \in G \text{ and } \min \in [0, 1]$$

Proof Let A be a q -rung orthopair (m, n) -fuzzy set on a group G .

Suppose A is a q -rung orthopair (m, n) - fuzzy subgroup of G , then for $w_1w_2 \in G$,

$$(\delta_A(m(w_1w_2^{-1})))^q \geq \min \{ (\delta_A(w_1))^q, (\delta_A(w_1^{-1}))^q \} = \min \{ (\delta_A(w_1))^q, (\delta_A(w_1))^q \} \text{ and}$$

$$(\Delta_A(n(w_1, w_2^{-1})))^q \leq \max \{ (\Delta_A(w_1))^q, (\Delta_A(w_2^{-1}))^q \} = \max \{ (\Delta_A(w_1))^q, (\Delta_A(w_2))^q \}$$

Conversly,

$$\text{Let } (\delta_A(m(w_1w_2^{-1})))^q \geq \min \{ (\delta_A(w_1))^q \} \text{ and } (\Delta_A(n(w_1, w_2^{-1})))^q \leq \max \{ (\Delta_A(w_1))^q \},$$

for all $w_1w_2 \in G$. Then

$$(\delta_A(m(w_1w_2)))^q = (\delta_A(m(w_1(w_2))))^q \geq \min \{ (\delta_A(w_1))^q, (\delta_A(w_2^{-1}))^q \} = \min \{ (\delta_A(w_1))^q, (\delta_A(w_2))^q \}$$

Similarly it can be proved that $(\Delta_A(n(w_1, w_2)))^q \leq \max \{ (\Delta_A(w_1))^q, (\Delta_A(w_2))^q \}$

Now consider $(\delta_A(m(w_1^{-1})))^q$

$$(\delta_A(m(w_1^{-1})))^q = (\delta_A(m(cw_1^{-1})))^q \geq \min \{ (\delta_A(e))^q, (\delta_A(w_1))^q \}$$

$$= (\delta_A(w_1))^q \dots\dots\dots (1) \text{ Which will give}$$

$$(\delta_A(m(w_1)))^q = (\delta_A(m(w_1^{-1})))^q \geq (\delta_A(w_1^{-1}))^q \dots\dots\dots (2)$$

From (1) and (2), $(\delta_A(w_1^{-1}))^q = (\delta_A(m(w_1)))^q$ Similarly, $(\Delta_A(n(w_1^{-1})))^q = (\Delta_A(w_1))^q$

Hence 'A', is a q-rung ortho pair (m, n)- fuzzy subgroup of G.

Theorem 3.5 Let A and B be two q-rung orthopair (m, n) – fuzzy subgroup of G then

$A \cap B$ is a q-rung orthopair (m, n) - fuzzy subgroup of G.

Proof : For $w_1, w_2 \in G$

$$\begin{aligned} (\delta_{A \cap B}(m(w_1w_2^{-1})))^q &= \min \{ (\delta_A(m(w_1w_2^{-1})))^q, (\delta_B(m(w_1w_2^{-1})))^q \} \\ &\geq \min \{ \min \{ (\delta_A(w_1))^q, (\delta_A(w_1^{-1}))^q \}, \min \{ (\delta_B(w_2))^q, (\delta_B(w_2))^{-q} \} \} \\ &= \min \{ \min \{ (\delta_A(w_1))^q, (\delta_B(w_1))^{-q} \}, \min \{ (\delta_A(w_2))^q, (\delta_B(w_2))^{-q} \} \} \\ &= \min \{ \delta_{A \cap B}(w_1), \delta_{A \triangle B}(w_2) \} \end{aligned}$$

Similarly it can be shown that

$$(\Delta_{A \cap B}(n(w_1, w_2^{-1})))^q \leq \max \{ (\Delta_{A \cap B}(w_1))^q, (\Delta_{A \cap B}(w_2))^q \}$$

Hence $A \cap B$ is a q-rung orthopair(m, n) fuzzy subgroup of G.

Theorem 3.6 Let “A’ be a q-rung orthopair (m, n) – fuzzy subgroup of G, e be the element in G then $(\delta_A(m(xw)))^q = (\delta_A(w))^q$ and $(\Delta_A(n(xw)))^q = (\Delta_A(w))^q$ for all $w_1w_2 \in G$. If and only if $(\delta_A(x))^q = (\delta_A(e))^q$ and $(\Delta_A(x))^q = (\Delta_A(e))^q$

Proof Suppose $(\delta_A(m(xw)))^q = (\delta_A(w))^q$ and $(\Delta_A(n(xw)))^q = (\Delta_A(w))^q$ for all $w_1w_2 \in G$

Specifically, $w = e$, it follows that $(\delta_A(x))^q = (\delta_A(e))^q$ $(\Delta_A(x))^q = (\Delta_A(e))^q$

Suppose $(\delta_A(x))^q = (\delta_A(e))^q$ and $(\Delta_A(x))^q = (\Delta_A(e))^q$

Since $(\delta_A(mw))^q \leq (\delta_A(e))^q$ for all $w_1w_2 \in G$, $(\delta_A(m(xw)))^q \geq \min \{ (\delta_A(x))^q, (\delta_A(w))^q \}$ and

$$= \min \{ (\delta_A(e))^q, (\delta_A(w))^q \} = (\delta_A(w))^q \text{ for all } w_1w_2 \in G$$

Also, $(\delta_A(w))^q = (\delta_A(x^{-1}w))^q \geq \min \{ (\delta_A(x))^q, (\delta_A(xw))^q \} = (\delta_A(w))^q$ for all $w_1w_2 \in G$

Hence, $(\delta_A(mw))^q = (\delta_A(xw))^q$ for all $w \in G$. Similar, $(\Delta_A(xw))^q = (\Delta_A(mw))^q$ for all $w \in G$

Theorem 3.7 Let “A’ be a q-rung orthopair (m, n) – fuzzy subgroup of G, e be the element in G then $H = \{ w \in G / (\delta_A(mw))^q = (\delta_A(e))^q, (\Delta_A(mw))^q = (\Delta_A(e))^q \}$ is a subgroup of G.

Proof: By definition of H, it follows that $e \in H$. Hence, H is a non – empty subset of G. Let $w_1, w_2 \in H$. Then $(\delta_A(mw_1))^q = (\delta_A(e))^q = (\Delta_A(mw_2))^q$ and $(\Delta_A(mw_1))^q$

$$= (\Delta_A(e))^q = (\Delta_A(mw_2))^q. \text{ we have}$$

$$(\delta_A(m(w_1w_2^{-1})))^q \geq \min \{ (\delta_A(w_1))^q, (\delta_A(w_2^{-1}))^q \} = \min \{ (\delta_A(w_1))^q, (\delta_A(w_2))^q \}$$

$$= \min \{ (\delta_A(e))^q, (\delta_A(e))^q \} = (\delta_A(e))^q. \text{ By theorem 3.3}$$

$$(\delta_A(e))^q \geq (\delta_A(m(w_1w_2^{-1})))^q \text{ therefore } (\delta_A(m(w_1w_2^{-1})))^q = (\delta_A(e))^q$$

Similarly it can be shown that $(\Delta_A(m(w_1w_2^{-1})))^q = (\Delta_A(e))^q$ thus $w_1w_2^{-1} \in H$.

Hence it is a subgroup of G.

Theorem 3.8 Let “A’ be a q-rung orthopair (m, n) – fuzzy subgroup of G, then there exists an element $x \in G$ such that $(\delta_A(x))^q \leq (\delta_A(mw))^q$ and $(\Delta_A(x))^q \leq (\Delta_A(mw))^q$ for all $w_1w_2 \in G$.

Proof : Let K_1 denote the set of all elements in G with the lowest membership degree, and K_2 denote the set of all elements in G with the highest non-membership degree, if

$$K_1 = \{ w^1 \in G / (\delta_A(mw^1))^q \leq (\delta_A(w))^q \quad \forall w \in G \} \text{ and}$$

$$K_2 = \{ w^{11} \in G / (\delta_A(mw^{11}))^q \leq (\Delta_A(w))^q \quad \forall w \in G \}$$

To show $K_1 \cap K_2 \neq \emptyset$. Let $w^1 \in K_1$ and $w^{11} \in K_2$

$$\text{Clearly } w^1 = w^{11} \in p \text{ for some } p \in G \text{ and } (\delta_A(mw^1))^q \geq \min \{ (\delta_A(w^{11}))^q, (\delta_A(p))^q \}$$

Since, $(\delta_A(w^{11}))^q$ is the lowest membership degree either $(\delta_A(w^{11}))^q = (\delta_A(w^1))^q$ or

$$(\delta_A(mp))^q = (\delta_A(mw^1))^q \text{ if } (\delta_A(mw^{11}))^q = (\delta_A(mw^1))^q, \text{ it follows that}$$

$w^{11} \in K_1$ therefore $K_1 \cap K_2 \neq \emptyset$ and $x = w^{11}$. if $(\delta_A(mp))^q = (\delta_A(mw^{11}))^q$, consider $w^{11} = w^1 * p^{-1}$ then $(\Delta_A(mw^{11}))^q \leq \max \{ (\Delta_A(w^{11}))^q, (\Delta_A(p^{-1}))^q \}$. Since $(\Delta_A(mw^{11}))^q$ is the highest non-membership degree, then $(\Delta_A(nw^1))^q = (\Delta_A(nw^{11}))^q$ or $(\Delta_A(np^{-1}))^q = (\Delta_A(nw^{11}))^q$

if $(\Delta_A(nw^1))^q = (\Delta_A(nw^{11}))^q$ it follows that $w^{11} \in K_2$, $K_1 \cap K_2 \neq \emptyset$ and $x = w^1$

if $(\Delta_A(np^{-1}))^q = (\Delta_A(nw^{11}))^q$, then it follows that $(\delta_A(mp))^q = (\delta_A(mw^1))^q$ and $(\Delta_A(mp^{-1}))^q = (\Delta_A(mw^{11}))^q$, which means $p \in K_1 \cap K_2$ and $x = \emptyset$.

Theorem 3.9 : Let G be a cyclic group and let A be a q-rung orthopair (m, n) – fuzzy subgroup of G. Then the generators of G posses membership and non-membership degree in K.

Proof: let W_1, W_2 be two generators of G. Since W_1 is a generator, $W_2 = W_1^r$ for some

$$r \in \mathbb{N}. (\delta_A(mw_2))^q = (\delta_A(mw_1^r))^q \geq (\delta_A(w_1))^q \text{ and } (\Delta_A(mw_2))^q = (\Delta_A(mw_1^r))^q \leq \Delta_A(w_1)^q \rightarrow (3)$$

Since w_2 is a generator, $w_1 = w_2$ for some $S \in \mathbb{N}$.

$$(\delta_A(mw_1))^q = (\delta_A(mw_2^s))^q \geq (\delta_A(w_2))^q \text{ and } (\Delta_A(nw_1))^q = (\Delta_A(nw_2^s))^q \leq \Delta_A(w_2)^q \rightarrow (4)$$

From (3) and (4) $(\delta_A(mw_1))^q = (\delta_A(w_2^s))^q$ and $(\Delta_A(nw_1))^q = (\Delta_A(w_1))^q$

Some Orders of Q-Runge Orthopair (M, N) – Fuzzy Subgroups

Definition 4.1: Let 'A' be a q-rung orthopair fuzzy subgroup of group G. For $W \in G$, the least positive integer 'r' such that $(\delta_A(mw^r))^q = (\delta_A(e))^q$ and $(\Delta_A(nw^r))^q = (\Delta_A(e))^q$ is called the order of W in A, denoted as order of q-rung orthopair(m, n)-fuzzy subgroup.

Remark 4.2 The order of an element in a q-rung ortho pair (m, n) – fuzzy subgroup A of G is always less than or equal to its order in G. Also the order of an element and its inverse in 'A' are equal.

Theorem 4.3: Let A be a q-rung orthopair (m, n) – fuzzy subgroup of G, and 'e' be the identity element in G. Let $w_1 \in G$, and let 'u' be a (t) be in satisfying. $(\delta_A(mw_1^u))^q = (\delta_A(e))^q$ and $(\Delta_A(nw_1^u))^q = (\Delta_A(e))^q$ then order of q-rung orthopair (m, n) – fuzzy subgroup divides 'u'.

Conclusion: This objective of this article introduce the concept of q-rung orthopair (m, n)- fuzzy subgroups of finite groups and examines their fundamental properties. Additionally, it develop and analyzes key concepts such q-rung orthopair (m, n)- fuzzy subgroup, there by providing deeper insights in to the structure of q-rung orthopair (m, n)- fuzzy subgroups.

Future work : The research may explore concepts such as q-rung orthopair (m, n)- fuzzy homomorphism and q-rung orthopair (m, n)- fuzzy isomorphisms..

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